WHAT IS THE PERMISSIBILITY SOLUTION A SOLUTION OF? – A QUESTION FOR KROEDEL

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ABSTRACT: Kroedel has proposed a new solution, the *permissibility solution*, to the lottery paradox. The lottery paradox results from the *Lockean thesis* according to which one ought to believe a proposition just in case one's degree of belief in it is sufficiently high. The permissibility solution replaces the Lockean thesis by the *permissibility thesis* according to which one is permitted to believe a proposition if one's degree of belief in it is sufficiently high. This note shows that the epistemology of belief that results from the permissibility thesis and the epistemology of degrees of belief is empty in the sense that one need not believe anything, even if one's degrees of belief are maximally bold. Since this result can also be achieved by simply dropping the Lockean thesis, or by replacing it with principles that are logically stronger than the permissibility thesis, the question arises what the permissibility solution is a solution of.

KEYWORDS: permissibility solution, lottery paradox, Thomas Kroedel

Kroedel[1] has proposed a new solution, the *permissibility solution*, to the lottery paradox.[2] The lottery paradox shows that a plausible thesis, viz. the *Lockean thesis*,[3] leads to inconsistency when combined with other theses about belief and about degrees of belief. The Lockean thesis says that an ideal doxastic agent ought to believe a proposition just in case her degree of belief for the proposition is sufficiently high. The permissibility solution replaces the Lockean thesis by the *permissibility thesis* according to which one is permitted to believe a proposition if one's degree of belief in it is sufficiently high. This note shows that the

[1] Thomas Kroedel, "The Lottery Paradox, Epistemic Justification and Permissibility," *Analysis* 52 (2012): 57-60.

[2] Henry E. Jr. Kyburg, *Probability and the Logic of Rational Belief* (Middletown, CT: Wesleyan University Press, 1961), 197 and, much clearer, Carl Gustav Hempel, "Deductive-Nomological vs. Statistical Explanation," in *Scientific Explanation, Space and Time*. *Minnesota Studies in the Philosophy of Science* 3, eds. H. Feigl and G. Maxwell (Minneapolis: University of Minnesota Press. 1962), 163f.

[3] Richard Foley, "Belief, Degrees of Belief, and the Lockean Thesis," in *Degrees of Belief, Synthese Library* 342, eds. F. Huber and C. Schmidt-Petri (Dordrecht: Springer, 2009), 37-47, John Hawthorne, "The Lockean Thesis and the Logic of Belief," in *Degrees of Belief*, eds. Huber and Schmidt-Petri, 49-74.

epistemology of belief that results from the permissibility thesis and the epistemology of degrees of belief is empty in the sense that one need not believe anything, even if one's degrees of belief are maximally bold. Since this result can also be achieved by simply dropping the Lockean thesis, or by replacing it with principles that are logically stronger than the permissibility thesis, the question arises what the permissibility solution is a solution of.

In order to discuss Kroedel's proposal[4] it will prove useful to formalize the Lockean thesis in various flavors. For the sake of simplicity let us assume that there is a context-independent threshold c that specifies just how high sufficiently high is. Let us also assume that the threshold c is the same for all propositions under consideration.

Let a be the ideal doxastic agent, and B_a her belief relation, and $\Pr_a$ her degree of belief function. $\mathbf{O}(\cdot)$ is the operator for obligation, and $\mathbf{O}(\cdot \mid \cdot)$ is the operator for conditional obligation. The permissibility operators can be introduced as the duals of the obligation operators: $\mathbf{P}(\cdot) = \neg\mathbf{O}(\neg\cdot)$ and $\mathbf{P}(\cdot \mid \cdot) = \neg\mathbf{O}(\neg\cdot \mid \cdot)$. $\leftrightarrow$ is the material biconditional.

Locke 1 For all propositions (that are expressible in the underlying language) A,

$B_a(A) \leftrightarrow \Pr_a(A) > c$.

Locke 2 For all propositions A, $\mathbf{O}(B_a(A) \leftrightarrow \Pr_a(A) > c)$.

Locke 3 For all propositions A, $\mathbf{O}(B_a(A)) \leftrightarrow \mathbf{O}(\Pr_a(A) > c)$.

Locke 4 For all propositions A,

$\mathbf{O}(B_a(A) \mid \Pr_a(A) > c \wedge X)$ and $\mathbf{O}(\Pr_a(A) > c \mid B_a(A) \wedge Y)$.

Locke 1 is logically stronger than Locke 2 which in turn is logically stronger than Locke 3. Locke 1 is logically stronger than Locke 4, whatever the exact nature of X and Y. We will see that Locke 4 is the best formalization of the Lockean thesis, as the lottery paradox does not arise for Locke 2 or Locke 3.

X and Y are "admissible" propositions. What counts as admissible will depend on the underlying deontic logic, among others (see the appendix). For present purposes X can be assumed to be information about $\Pr_a$ and B_a that is consistent with $\Pr(A) > c$ and does not conflict with any of the norms mentioned

[4] Kroedel, "The Lottery Paradox," Thomas Kroedel, "The Permissibility Solution to the Lottery Paradox – Reply to Littlejohn," *Logos & Episteme* 4 (2013): 103-111, Thomas Kroedel, "Why Epistemic Permissions Don't Agglomerate – Another Reply to Littlejohn," *Logos & Episteme* 4 (2013): 451-455.

below. Similarly for Y, except that Y is information about B_a and $\Pr_a$ that is consistent with $B_a(A)$.

The Lockean thesis is of philosophical interest, because it allows us to derive the epistemology of belief from the epistemology of degrees of belief. (Strictly speaking it is the doxastology of belief and of degrees of belief, but I will follow standard usage.) Unfortunately the Lockean thesis results in paradox. It violates our expectations on the epistemology of belief once we start to fill in the details of our epistemology of degrees of belief. The latter will include the following, among others. For all real numbers x and y, and for all propositions A and C (in some algebra of propositions) over some non-empty set of possible worlds W that are jointly inconsistent in the sense that $A \cap C = \emptyset$:

1. $\mathbf{O}(\Pr_a(A) \geq 0)$
2. $\mathbf{O}(\Pr_a(W) = 1)$
3. $\mathbf{O}(\Pr_a(A \cup C) = x+y \mid \Pr_a(A) = x \wedge \Pr_a(C) = y)$ and

   $\mathbf{O}(\Pr_a(A) = x \mid \Pr_a(A \cup C) = x+y \wedge \Pr_a(C) = y)$ and

   $\mathbf{O}(\Pr_a(C) = y \mid \Pr_a(A \cup C) = x+y \wedge \Pr_a(A) = x)$. Etc.

This formalization is incomplete, as there are many further conditional obligations. It may also seem somewhat unorthodox. However, this formalization is logically weaker, even once completed, than the standard formulation of the probability calculus without the operators for obligation and conditional obligation. It is so in the exact same way that Locke 4 is logically weaker than Locke 1.

I assume $\mathbf{O}(\cdot)$ to be equivalent to $\mathbf{O}(\cdot \mid T)$ for the trivial or tautological (action) sentence T. Only action sentences will be allowed in the first argument place. While I have not done so, the reader should also feel free to replace '$\mathbf{O}$' by '$\mathbf{O}_a$', as these norms are directed at our ideal doxastic agent a, and justified by being the means to attaining *her* doxastic goals.[5] Given (a complete version of) the norms 1-3 it makes sense to additionally assume that c is a real number not smaller than 1/2, but smaller than 1.

When we add the Lockean thesis to our epistemology of degrees of belief we get an epistemology of belief. For instance, from Locke 4 and (a complete

[5] James M. Joyce, "A Non-Pragmatic Vindication of Probabilism," *Philosophy of Science* 65 (1998): 575-603, James M. Joyce, "Accuracy and Coherence: Prospects for an Alethic Epistemology of Partial Belief," in *Degrees of Belief,* eds. Huber and Schmidt-Petri, 263-297.

version of) 1-3 we can derive that the ideal doxastic agent ought to believe the tautological proposition W, and that she ought to believe every logical consequence of any belief of hers. That is, for appropriate choices of X and Y:

Taut $\mathbf{O}(B_a(W)|X)$

Closure For all propositions A and C with $A \subseteq C$, $\mathbf{O}(B_a(C) \mid B_a(A) \wedge Y)$.

With Kroedel[6] we will supplement Locke 4 with Littlejohn's Low,[7] except that we formalize it as a conditional obligation:

Low For all propositions A, $\mathbf{O}(\neg B_a(A) \mid \Pr_a(A)<1-c \wedge X)$,

where X is assumed to be information about $\Pr_a$ and B_a that is consistent with $\Pr(A) < 1-c$ and does not conflict with any of the norms mentioned above. Given Low we can derive that the ideal doxastic agent is not permitted to believe the contradictory proposition $\emptyset$, and that she is not permitted to believe the negation of any belief of hers. That is, for appropriate choices of X and Y,

Contr $\mathbf{O}(\neg B_a(\emptyset) \mid X)$

Neg For all propositions A, $\mathbf{O}(\neg B_a(\neg A) \mid B_a(A) \wedge Y)$.

We expect these consequences to be part of epistemology of belief. Unfortunately there are other norms we expect to be part of our epistemology of belief that we cannot derive. Indeed, there are norms we expect to be part of our epistemology of belief that are precluded by Locke 4 in the presence of Low and (a complete version of) 1-3. The lottery paradox shows the following one to be an example.

Conj For all propositions A and C, $\mathbf{O}(B_a(A \cap C) \mid B_a(A) \wedge B_a(C) \wedge Y)$.

(Y is again appropriately chosen information about B_a and $\Pr_a$ that is consistent with $B_a(A) \wedge B_a(C)$ and does not conflict with any of the norms mentioned above. I will assume this to be the case for the remainder of this note without explicitly mentioning it any longer.) The reason is that adding Conj to Locke 4 and Low and (a complete version of) 1-3 results in a conflict of norms for many seemingly reasonable distributions Pr of the ideal doxastic agent's degrees of belief.

Lottery 1 $\mathbf{O}(B_a(\text{Ticket 1 loses}) \mid \Pr)$, and

[6] Kroedel, "Another Reply."

[7] Clayton Littlejohn, "Lotteries, Probabilities, and Permissions," *Logos & Episteme* 3 (2012): 509-514.

Lottery 2 $\mathbf{O}(B_a(\text{Ticket 2 loses}) \mid \text{Pr})$, and ... and

Lottery 100 $\mathbf{O}(B_a(\text{Ticket 100 loses}) \mid \text{Pr})$

Lottery 101 $\mathbf{O}(B_a(\text{Tickets 1, ..., 100 all lose}) \mid \text{Pr})$

Lottery 102 $\mathbf{O}(B_a(\text{Tickets 1, ..., 100 do not all lose}) \mid \text{Pr})$

Lottery 1 follows from Locke 4 and can be read as follows: given that her degrees are what they are, the ideal doxastic agent ought to believe that ticket 1 loses. Similarly for Lottery 2, ..., Lottery 100, and Lottery 102. Lottery 101 follows from Lottery 1, …, Lottery 100, and Conj (in conditional deontic logic[8]). Together Lottery 101 and Lottery 102 and Conj imply

Lottery $\mathbf{O}(B_a(\emptyset) \mid \text{Pr})$

However, the following consequence of Low and (a complete version of) 1-3

anti-Lottery $\mathbf{O}(\neg B_a(\emptyset) \mid \text{Pr})$

implies the negation of Lottery:

non-Lottery $\neg\mathbf{O}(B_a(\emptyset) \mid \text{Pr})$

In other words, in the presence of seemingly minimal theses about degrees of belief and about belief, Locke 4 implies a contradiction.

The lottery paradox also arises if we formulate the Lockean thesis as Locke 1. Interestingly, though, the lottery paradox does not arise if we formulate the Lockean thesis as Locke 2 or Locke 3, even if 1-3 are strengthened to the standard formulation of the probability calculus and Conj is analogously strengthened as follows ($\supset$ is the material conditional):

For all propositions A and C, $B_a(A) \wedge B_a(C) \supset B_a(A \cap C)$.

What is paradoxical about the lottery paradox is that the epistemology of belief that we get from Locke 4 and Low and (a complete version of) 1-3 does not meet our expectations. In order to resolve the inconsistency at least one of the above mentioned principles has to be given up. Different philosophers have made different recommendations.[9] However, until recently, none has replaced the

[8] See Bas C. van Fraassen, "The Logic of Conditional Obligation," *Journal of Philosophical Logic* 1 (1972): 417-438.

[9] e.g. Henry E. Jr. Kyburg, "Conjunctivitis," in *Induction, Acceptance, and Rational Belief*, ed. M. Swain (Dordrecht: Reidel, 1970), 232-254, Richard C. Jeffrey, "Dracula Meets Wolfman: Acceptance vs. Partial Belief," in *Induction, Acceptance, and Rational Belief*, ed. Swain, 157-185.

Lockean thesis with an alternative thesis that would allow us to derive the epistemology of belief from the epistemology of degrees of belief. Maybe this is not possible. Then we need to explain why our expectations, as formulated in Conj and Contr and Closure and Taut, are misguided. Or maybe it is possible. Then we need to replace Locke 4 with a different thesis that does not preclude Conj. Either way, more has to be done if we do not merely want resolve the inconsistency, but solve the paradox and obtain an epistemology of belief.

Leitgeb[10] and Lin and Kelly[11] have recently proposed substitutes for the Lockean thesis. Their substitutes do not merely allow for Conj (and the other principles), their substitutes logically imply Conj (and the other principles) when conjoined to the epistemology of degrees of belief as formulated in Low and (a complete version of) 1-3 (that includes norms for conditional degrees of belief).

Leitgeb 's solution to the lottery paradox[12] may be termed the *stability solution*. It replaces the Lockean thesis by the thesis that an ideal doxastic agent ought to believe a proposition B just in case there is a proposition C implying B such that the agent's degree of belief for C conditional on any A consistent with C is greater than c. Lin and Kelly's solution[13] may be termed the *sufficiency solution*. It replaces the Lockean thesis by the thesis that the ideal doxastic agent ought to believe a proposition just in case this proposition is implied by, i.e. a (not necessarily proper) superset of, the set of most plausible possible worlds. According to Lin and Kelly[14] the ideal doxastic agent considers a possible world to be more plausible than another possible world if, and only if, her degree of belief in the former is sufficiently higher than her degree of belief in the latter. The most plausible worlds are those for which there is none that is more plausible. Both the stability solution and the sufficiency solution derive an epistemology of belief from the epistemology of degrees of belief that meets our expectations as they are formulated in Taut and Contr and Closure and Conj and still other principles that date back to Hintikka[15] and Alchourrón, Gärdenfors and Makinson.[16]

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[10] Hannes Leitgeb, "Reducing Belief Simpliciter to Degrees of Belief," *Annals of Pure and Applied Logic* 164 (2013): 1338-1389.

[11] Hanti Lin and Kevin T. Kelly, "Propositional Reasoning that Tracks Probabilistic Reasoning," *Journal of Philosophical Logic* 41 (2012): 957-981.

[12] Leitgeb, "Reducing Belief."

[13] Lin and Kelly, "Propositional Reasoning."

[14] Lin and Kelly, "Propositional Reasoning."

[15] Jaakko Hintikka, *Knowledge and Belief. An Introduction to the Logic of the Two Notions* (Ithaca, NY: Cornell University Press, 1961). Reissued as Jaakko Hintikka *Knowledge and Belief. An Introduction to the Logic of the Two Notions*, prepared by V.F. Hendricks and J. Symons (London: King's College Publications, 2005).

What about the permissibility solution proposed by Kroedel?[17] The latter arises by replacing the Lockean thesis with the *permissibility thesis* according to which the ideal doxastic agent is permitted to believe a proposition given that her degree of belief in this proposition is sufficiently high. More specifically, it results by adding the formalization High 5 instead of Locke 4 to Low and (a complete version of) 1-3.

High 5 For all propositions A, $\mathbf{P}(B_a(A) \mid \Pr_a(A)>c \wedge X)$.

An alternative formalization of the permissibility thesis works with obligations instead of conditional obligations and so avoids specifications of admissibility:

High 4 For all propositions A, $\Pr_a(A)>c \supset \mathbf{P}(B_a(A))$.

However, there may be Is-Ought problems with High 4.[18] This is perhaps clearest when we reformulate High 4 in terms of what is forbidden, $\mathbf{F}(\cdot) = \neg\mathbf{P}(\cdot)$:

High 4 For all propositions A, $\mathbf{F}(B_a(A)) \supset \Pr_a(A)\leq c$.

The Munich born poet Christian Morgenstern, well known for the (ridiculing of) philosophical theses in his poems, explains better than I ever could what is problematic about the Is-Ought problem and High 4. The following is the last verse of the poem *Die unmögliche Tatsache*, which is part of *Palmström*,[19] and which I translate as "The impossible fact:"

Und er kommt zu dem Ergebnis:

»Nur ein Traum war das Erlebnis.

Weil«, so schließt er messerscharf,

»nicht sein kann, was nicht sein darf!«

In addition High 4 has consequences that are presumably not welcome by Kroedel,[20] such as

High 4-1 For all propositions A, $\Pr_a(A)>c \wedge \mathbf{O}(B_a(\neg A)) \supset \mathbf{P}(B_a(A))$.

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[16] Carlos E. Alchourrón, Peter Gärdenfors, and David Makinson, "On the Logic of Theory Change: Partial Meet Contraction and Revision Functions," *Journal of Symbolic Logic* 50 (1985): 510-530.

[17] Kroedel, "The Lottery Paradox," Kroedel, "Reply," Kroedel, "Another Reply."

[18] David Hume, *A Treatise of Human Nature*, ed. L.A. Selby-Bigge (Oxford: Clarendon Press, 1739/1896).

[19] Christian Morgenstern, *Palmström* (Berlin: B. Cassirer, 1910).

[20] Kroedel, "The Lottery Paradox," Kroedel, "Reply," Kroedel, "Another Reply."

Finally, everything I am going to claim about High 5 below is also true for High 4. The same is true for High 4-5, which is logically weaker than High 4 (in the deontic logic *SD*4[21])

High 4-5 For all propositions A, $\mathbf{O}(\mathrm{Pr}_a(A)>c) \supset \mathbf{P}(B_a(A))$.

Even High 4-5 (and High 4, if we adopt the deontic logic *SD*4) has consequences that are presumably not welcome by Kroedel,[22] such as the permission to believe a proposition if one's degree of belief is sufficiently high even if one already believes its negation:

High 4-5-1 For all propositions A, $\mathbf{O}(\mathrm{Pr}_a(A)>c) \wedge \mathbf{O}(B_a(\neg A)) \supset \mathbf{P}(B_a(A))$.

For these reasons, and because High 5 does not lead to an inconsistency, I assume that High 5 is a charitable formalization of the permissibility thesis, and the permissibility solution as intended by Kroedel.[23] It is perhaps worth noting that the inconsistency is also avoided if we replace Locke 4 by

High 2 For all propositions A, $\mathbf{O}(\mathrm{Pr}_a(A)>c \supset B_a(A))$.

High 3 For all propositions A, $\mathbf{O}(\mathrm{Pr}_a(A)>c) \supset \mathbf{O}(B_a(A))$.

The inconsistency is not avoided if we replace Locke 4 by one of

High 0 For all propositions A, $\mathrm{Pr}_a(A)>c \supset \mathbf{O}(B_a(A))$.

High 1 For all propositions A, $\mathrm{Pr}_a(A)>c \supset B_a(A)$.

Adding High 5 instead of Locke 4 to Low and (a complete version of) 1-3 avoids the inconsistency. It does not solve the lottery paradox, though. Our expectations as formulated in Taut, for instance, are not met, as the permissibility solution does not deliver an epistemology of belief according to which an ideal doxastic agent ought to believe the tautological proposition. While Low implies that our ideal doxastic agent is not permitted to believe the contradictory proposition, she is not required to believe the tautological – or, for that matter, *any* – proposition if we add High 5 to Low and (a complete version of) 1-3.

Nor are our expectations as formulated in Closure met, as the permissibility solution does not deliver an epistemology of belief according to which an ideal doxastic agent ought to believe every logical consequence of all her beliefs. The

[21] See Jan Wolenseńki, "Deontic Logic and Possible Worlds Semantics: A Historical Sketch," *Studia Logica* 49 (1990): 273-282.

[22] Kroedel, "The Lottery Paradox," Kroedel, "Reply," Kroedel, "Another Reply."

[23] Kroedel, "The Lottery Paradox," Kroedel, "Reply," Kroedel, "Another Reply."

ideal doxastic agent need not even believe a single logical consequence of any of her beliefs. Nor need she obey Conj and believe the conjunction of any two propositions she believes.

Indeed, suppose our ideal doxastic agent has one of the boldest Jamesian degree of belief functions, one that assigns to each proposition the maximal degree of belief or else the minimal degree of belief.[24] It is compatible with this and High 5 and Low and (a complete version of) 1-3 (even in their logically stronger formulations) that the ideal doxastic agent's belief relation is the most cautious of all Cliffordian belief relations, the one that suspends judgment with respect to all propositions.[25] In other words, the epistemology of belief that results from the epistemology of degrees of belief on the permissibility solution is, in this precise sense, empty.

Replacing the Lockean thesis by High 5 (and Low) resolves the inconsistency. This much is true of the permissibility solution. However, this much is also true if we simply drop the Lockean thesis and with it the epistemology of belief, as Jeffrey[26] recommends. It also true if we bite the bullet and deny Conj, as recommended by Kyburg;[27] or, as recommended by Spohn,[28] if we develop two parallel epistemologies, viz. the epistemology of belief and the epistemology of degrees of belief. Indeed, this much is true even if we adopt High 3 or High 2, both of which are logically stronger than High 5.

However, replacing the Lockean thesis by High 5 does not solve the paradox, as our expectations on the epistemology of belief remain not being met. While the Lockean thesis may not give us the epistemology of belief we have expected, it at least gives us an epistemology of belief. The permissibility solution does not give us an epistemology of belief that we did not expect. But that is only because, much like the recommendation by Jeffrey,[29] it does not give us an epistemology of belief at all.

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[24] William James, "The Will to Believe (1896)," in *The Will to Believe and Other Essays in Popular Philosophy*, ed. F. Burkhardt, F. Bowers, and I. Skrupskelis (Cambridge, MA: Harvard University Press, 1979), 291-341.

[25] William K. Clifford, "The Ethics of Belief (1877)," in *The Ethics of Belief and Other Essays*, ed. T. Madigan (Amherst, MA: Prometheus Books, 1999), 70-96.

[26] Jeffrey, "Dracula Meets Wolfman."

[27] Kyburg, "Conjunctivitis."

[28] Wolfgang Spohn, "A Survey of Ranking Theory," in *Degrees of Belief*, ed. Huber and Schmidt-Petri, 185-228.

[29] Jeffrey, "Dracula Meets Wolfman."

Postscriptum on conditional obligations

According to the logic of conditional obligations, the following rule of inference preserves the designated value (is truth-preserving, if one thinks that conditional norms have truth values), and hence preserves deontic validity:[30]

L From $\mathrm{P}(C|D)$ and $\mathbf{O}(D|C)$ and $\mathbf{O}(A|D)$ infer $\mathbf{O}(A|C)$.

L says that conditional obligations are transitive if the condition C is permissible given the "middleman" D. The more conditions are permissible given various middlemen, the fewer assumptions about admissibility are needed. It is in this sense that what counts as permissible will depend on the underlying deontic logic. Suppose the underlying deontic logic included the axiom schema: $\mathbf{P}(C \mid D)$ or $\vdash \mathbf{O}(\neg C \mid D)$, $\vdash$ specifying derivability from a complete version of 1-3, Low, Locke 4 for empty X and Y. Then no assumptions about admissibility would be needed.[31]

[30] My preferred logic of conditional obligations is sketched in Franz Huber, "New Foundations for Counterfactuals," *Synthese* 191 (2014): 2180ff.

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