

# COMPLEMENTARITY AND ANTINOMY

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**ABSTRACT:** In this study we present some contributions of the logician and philosopher Petre Botezatu (27.02.1911-01.12.1981), who turned the idea of complementarity, formulated by Niels Bohr for the interpretation of the wave-particle structure of the quantum world, into an ordering principle of his work. Thus, he understood general logic as a synthesis in which the style of classical logic is complementary to the style of the 20th century logic. He didn't give up either the mathematical modelling of logical language or the conceptual description through natural language. Thus, *natural operational logic* was created. Then Petre Botezatu assessed the achievements and failures of deduction in order to build the notion of *methodological antinomy*, and formulated five antinomies of axiomatization and five antinomies of formalization. The main purpose of this study is to present and interpret them.

**KEYWORDS:** complementarity, determination, determinism,  
natural operational logic

The main purpose of this paper is the presentation and interpretation of some original contributions of the philosopher and logician Petre Botezatu (27.02.1911-01.12.1981), PhD, Professor at the Faculty of History and Philosophy, "Alexandru Ioan Cuza" University of Iași, Romania. He was considered "the Romanian logician with the highest number of achievements, who, above all, through his natural, operational, logic, created a whole new domain, classical-symbolical, with multiple applications in education and in scientific thinking and practice."<sup>1</sup> He directed his first researches towards the elaboration of his PhD thesis, *Cauzalitatea fizică și panquantismul (The Physical Causality and the Panquantism)*, defended in 1945. In other words, texts of general epistemology and philosophy guided his steps in the first ten formative years and then, until the end of his career, his work on logic twinned with metalogical and epistemological interpretations.

However, his PhD thesis remained a manuscript as he was uninterested in publishing it, explaining his reservations through the fact that he should have reconsidered his opinion on *the value of determinism in modern physics*. Yet,

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<sup>1</sup> Alexandru Surdu, *Contribuții românești în domeniul logicii în secolul XX* (București: Fundația "România de Măine," 1999), 14.

studying some of his main published works, we have noticed that many of the ideas found in that thesis constantly incited Petre Botezatu. Actually, evaluating his own scientific activity in a *Curriculum Vitae* written on February 10, 1981, he considered that in his thesis he had “demonstrated that two fundamental confusions were made during the passionate discussion around Heisenberg’s relations: between indetermination and indeterminism and between quantitative causality and qualitative causality. In quantum mechanics there is indetermination, not indeterminism and what has become impossible is the quantitative determination of causal relations, which exist in their qualitative form (various kinds of particle collision). Determinism exists under a new form and the controversies around it can be mainly explained by the conflict between certain forms of realism and idealism.”<sup>2</sup>

Petre Botezatu came to these conclusions from the 1940s, after he analysed the main results obtained by the quantum physicists and their philosophical interpretations. Thus, Werner Heisenberg, who was working in Copenhagen under the direction of Niels Bohr, formulated interpretations on Louis de Broglie’s wave mechanics under the influence of some positivist and phenomenological ideas that became the creed of the *Wiener Kreis* philosophical school. Here it was considered that physical theory should only rely on quantities whose values can be directly observed and should avoid any representation for which there would be certain physical elements inaccessible to experience. Animated by this ‘Copenhagen spirit’, which from certain points of view reminded of the spirit of the classical conception of energetism, Heisenberg’s *quantum mechanics*, sometimes called *matrix mechanics* as it was mathematized, was presented under the form of a pure formalism which rejected any image of microphysical world, explaining all phenomena observable at the atomic scale by means of mere algebraic calculations.

In this way an opposition was reached, difficult to reconcile: on the one hand, wave mechanics was trying to obtain a representation of the microphysical phenomena within space and time, providing a clear and intelligible image of the wave-particle association, and on the other hand, quantum mechanics was elaborating a formalism capable of providing with accuracy the future evolution of the phenomena experimentally observed. Secondly, quantum physics was *interpreted probabilistically*, without any causal mechanism. The wave from wave mechanics becomes the solution of an equation with partial derivatives, solution made possible by a mathematical instrument, and the particle has no longer a permanent localization in space, but exists in a state of potential in an entire region of space and is statistically distributed between several states of motion. Therefore Niels Bohr introduced the notion of *complementarity*: wave and

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<sup>2</sup> Petre Botezatu, *Curriculum Vitae* (manuscript), 3.

particle are two complementary aspects of reality, a reality which otherwise cannot be intelligibly described from the microphysical point of view.

Petre Botezatu adopted the idea of complementarity, presented it in his PhD thesis<sup>3</sup> and turned it into an *ordering principle* of his work. In Chapter II, Part III of his thesis, invoking Niels Bohr's opinions, he detailed specifications that were later on presented and explored by philosophers of physics. Petre Botezatu maintained that the idea of complementarity was a result of "the insufficiency of the concepts created by the common intuition when they are applied in microphysics."<sup>4</sup> Namely, pairs of notions, such as: wave and particle, causality and space-time, which, in the description of microcosm, exclude and complete one another at the same time. In our opinion, the insufficiency was due to the fact that quantum physicists did not operate a *fundamental distinction*: macrophysical phenomena are observed in a natural space-time world, while microphysical phenomena are observed under experimental conditions and their description depends on instruments, on their power and precision.

Prior to 1950 Petre Botezatu became preoccupied with building a *general theory of reasoning*. First, he analysed the logical operations of the Aristotelian syllogistic and he discovered that Aristotle had determined the logical operation subjacent to syllogism according to the actual steps of thinking – *the transfer of a property between two classes related to each other by inclusion* – which represents a typical operation of thought. Hence the idea that it can be generalized so that it operates between other mutually related logical objects. Thus Petre Botezatu defined the *transitive logical operation*. Then, making good use of Kant's and Goblot's ideas on mathematical constructivity, he also defined the *constructive logical operation*: composing an object from other objects. Thus the natural logic was founded.

The new conception was presented in the paper "Teoria raționamentului întemeiată pe structura obiectelor" ("The Theory of Reasoning Based on the Structure of Objects"),<sup>5</sup> held at Alexandru Ioan Cuza University of Iași in 1958, and later on, in the same year, in the paper "La logique et les objets," XII International Congress on Philosophy, Venice-Padua (12–18 September 1958)<sup>6</sup>. The study "Les

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<sup>3</sup> Petre Botezatu, *Cauzalitatea fizică și panquantismul*, ed. Teodor Dima (Iași: Editura Universității "Alexandru Ioan Cuza", 2002).

<sup>4</sup> Botezatu, *Cauzalitatea fizică*, 161.

<sup>5</sup> In *Analele științifice ale Universității „Al. I. Cuza” din Iași. Științe sociale*, V (1959): 183-198.

<sup>6</sup> In *Atti del XII Congresso Internazionale di Filosofia V* (Firenze: Sansoni, 1960), 77-83.

raisonnements transitifs”<sup>7</sup> followed and after that the volume *Schiță a unei logici naturale* (*Sketch of a Natural Logic*)<sup>8</sup> provided the complete theoretization.

Almost at the same time, the project of a (theoretical) natural logic became of interest to other logicians and epistemologists, especially in France, the homeland of rationalism: Jean Piaget’s, *genetic operational logic*,<sup>9</sup> René Poirier’s *organic logic*,<sup>10</sup> and Robert Blanché’s *reflexive logic*.<sup>11</sup> Petre Botezatu’s idea of a natural logic developed independently of the aforementioned authors’ achievements, having different roots, as we previously showed. Also, its basis was the idea that *the relation between mathematical logic and traditional logic is a functional one*, which ultimately would be proven by means of metalogical investigations.

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*The fundamental idea of complementarity* suggested Petre Botezatu that in methodology every success leads to a failure, that one cannot have unlimited success. “It is in fact a broadened lesson of the complementarity of aspects, and quantum mechanics rendered us familiar to that.”<sup>12</sup> In *Valoarea deducției* (*The Value of Deduction*), from which we’ve just quoted, he made the balance of the achievements and failures of deductions in order to formulate epistemological interpretations: the methodological objectives are not all compatible; they moderate each other so that advancing on one direction means retreating from another. Certainly, he commented, no obstacle can prevent us from advancing ever so far on a certain path, but this has a price; that is, sacrifices in another sector. As far as logic is concerned, opposite tendencies can always be balanced by choosing strategies by means of which maximum benefits are associated with minimum losses.

These reflections inspired him the term *methodological antinomy*, in the Kantian sense of simultaneous presence of two contradictory theses that appear to be equally justified. The following opposite statements, for instance, can be held: (1) language is formalizable and (2) language is not formalizable. The antinomy is resolvable but not like in the case of paradoxes, through a theory of levels or types, but by *differentiating the points of view*. A theory is (relatively) formalizable from

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<sup>7</sup> In *Acta logica* 1 (1960): 59-81.

<sup>8</sup> Petre Botezatu, *Schiță a unei logici naturale. Logică operatorie* (București: Editura Științifică, 1969).

<sup>9</sup> Jean Piaget, *Traité de logique. Essai de logistique opératoire* (Paris: A. Colin, 1948). Edition reviewed for symbolism and J.-B. Grize’s mathematical ideas (Paris, 1972).

<sup>10</sup> René Poirier, *Logique et modalité du point de vue organique et physique* (Paris: Hermann, 1952).

<sup>11</sup> Robert Blanché, *Raison et discours. Défense de la logique réflexive* (Paris: Vrin, 1967).

<sup>12</sup> Petre Botezatu, *Valoarea deducției* (București: Editura Științifică, 1971), 168.

the *synthactical*, i.e. its internal construction, point of view, but it is not relatively formalizable from the *semantic*, i.e. its interpretation, point of view.

As deductive devices diversified and improved, the number of antinomies increased and Petre Botezatu grouped them into *five antinomies of axiomatization* and *five antinomies of formalization*. We will present, explicate and comment them briefly, and our approach will reveal some of the discussions *en vogue* 50 years ago on the achievements and failures of deductive devices. Many of them are still topical.

1. The **antinomy of simplification**: *the simplification of bases leads to the complication of construction*. The extension of the axiomatic method revealed the fact that for a given theory, larger or smaller classes of axioms and primitive (undefined) terms can be selected. Among other requirements, logicians wanted the axiomatized theories to be as unified as possible, to have axioms independent from one another, and the consequence was a minimal demonstrative basis: a number of axioms and primitive terms as small as possible. For example, for the propositional logic, numerous axiomatic variants were proposed:<sup>13</sup> Frege, in 1879, used two functors (negation and implication) and six axioms (Lukasiewicz reduced them to five) and Whitehead-Russell, in 1910, used also two functors (negation and disjunction) and six axioms (Bernays reduced them to four). The French logician Jean Nicod, in a study published at Cambridge,<sup>14</sup> showed that if the idea of incompatibility – Sheffer’s functor – is taken as the main functor, all Russell’s axioms can be reduced to just one. But it has five variables and 43 signs! Nicod himself admitted that such an axiom was uncomfortable for demonstrations. To explain its antinomy, Petre Botezatu took an example given by Alfred Tarski<sup>15</sup> who noted dissimilarities between the ‘methodological’ value and the ‘didactical’ value of two axiomatization variants for the arithmetic of real numbers. The first system, which presented the set of real numbers as an ordered Abelian number, had four primitive terms and nine axioms. The system was methodologically superior, being the simplest axiomatic construction of the entire arithmetics. But this formal advantage was accompanied by didactical disadvantages – complex and difficult definitions and demonstrations. The second system characterized the set of real numbers as an ordered body structure and needed six primitive terms and twenty axioms. It had methodological disadvantages – neither the terms nor the

<sup>13</sup> Cf. Joseph Dopp, *Notions de logique formelle* (Louvain: Publications universitaires de Louvain, 1965), 261-275.

<sup>14</sup> Jean Nicod, “A Reduction in the Number of the Primitive Propositions of Logic,” *Proceedings of Cambridge Philosophical Society* 19 (1917): 32-41.

<sup>15</sup> Alfred Tarski, *Introduction à la logique* (Paris: Gauthier-Villars, 1960), ch. X.

axioms met the independence criterion. Instead, the system had didactical advantages: important chapters of real number arithmetics were simple to elaborate by elementary reasonings. The antinomy occurred between the concern for the independence of axioms and primitive terms and the facility of demonstrations. Therefore in axiomatization practice, the independence criterion is given less attention, being considered the one that ensures the elegance of construction, even if it is closely related to the condition of system non-contradiction.<sup>16</sup>

**2. The *antinomy of strength*:** *a system's increasing strength is accompanied by the degradation of its metatheoretical quality.* Adopting Jean Ladrière's explanations, Petre Botezatu showed that a system is stronger if it can be interpreted under the form of a broader theory. Thus, in arithmetics axiomatization, Tarski's system is stronger than Peano's because the first covers the arithmetics of real numbers, while the second only the arithmetics of natural numbers. But the construction of ever stronger systems has the shortcoming of satisfying less the metatheoretical conditions: non-contradiction, completeness, categoricity, decidability. *What is gained in broadness is lost in formal perfection.* For example, propositional logic is a modest axiomatic system, but with precious metatheoretical qualities: it is non-contradictory, complete and decidable. However, it has little strength: it does not include the notions of predicate, function, quantifier, etc. If the system is enriched with the necessary terms in order to get a stronger logic, its formal properties start to faint. Thus, the enlarged predicate calculus is neither complete (Gödel) nor decidable, and its non-contradiction cannot be proved. It was believed that the obstacles were due to the imperfection of the used formalisms and that they could be overcome by other formalisms. Ultimately, limitation theorems were demonstrated: Gödel's (1931) and Church's (1936). Extended, generalized and unified by subsequent theorems, they proved that the limits expressed requirements of formalisms. Non-Gödelian systems (Myhill, Church), which increase the system's strength almost to the limitation theorem, were elaborated. But they were either insufficiently equipped (Myhill's system doesn't have the negation operator and the universal quantifier) or too disconnected from the common intuition of certain operations. Once again Gödel was right – only the elementary systems have the property of decidability. This result was corroborated by Church's theorem – a stronger logic, such as predicate restrictive logic is not resolvable even if partial solutions can exist, hence the conclusion that a logical machine able to solve all the problems in mathematics can never exist

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<sup>16</sup> Cf. E.W. Beth, *The Foundations of Mathematics* (Amsterdam: North-Holland Publishing Company, 1959), § 32.

(Turing). The antinomy of strength keeps its balance by admitting Tarski's notion of "essentially undecidable theory."

In the following, we present briefly the other three axiomatization antinomies:

**3. The *antinomy of purity*:** *the purification of demonstration involves the insufficiency of the basis.*<sup>17</sup> Petre Botezatu commented *in extenso* this antinomy because it involves the problem of the relation between deduction and intuition, a dualism firmly proclaimed by Descartes. But the deductive method by its own structure, which was enunciated by Aristotle, aims for purity: each step complies with an inference rule applied to a statement that is considered true. For example, *Principia Mathematica* – the logic was constructed first and the mathematics resulted from it. In the process, the theory is enriched with constants and variables, but the deduction rules are the same. The intuitive arguments, judiciously or not accepted in the beginning, are now avoided. This is the plan for the perfect deduction (however, it has been noticed that this is an ideal, i.e., not available in its pure form because the *intuition cannot be completely eliminated*). Several logicians noted that any formal axiomatics is bordered by intuitive domains: at the bottom, by the concrete interpretations given to it through models, one of which being the model that made it possible; on top, by the previous scientific knowledge, which steps in with its indubitable truth and intuitive significance.<sup>18</sup> From formalized mathematics we arrive to logic, from logic, if it has to be formalized, to metalogic, and so on. In order to avoid the *fallacy of infinite regress*, reasoning must be stopped at some level and this is, no doubt, the level of intuition. Then, Petre Botezatu analyzed the subject matter of the ultimate foundation of theories relying on the results obtained at the *International Symposium of Science Methodology*, Warsaw, 1961. Here it was emphasized that the axiomatic method alone cannot verify the axioms.<sup>19</sup> *Formal thinking cannot exhaust the content of intuitive thinking*. Intuition resists formalization and this resistance takes various and unexpected forms. In order to demonstrate the theorem, it is not enough to establish the axioms and primitive terms alone; methods must be chosen that will make possible the deduction from the foundation to the theorem.

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<sup>17</sup> Botezatu, *Valoarea deducției*, 175–180.

<sup>18</sup> Robert Blanché, *L'axiomatique* (Paris: P.U.F., 1959), 65–66; Roger Martin, *Logique contemporaine et formalisation* (Paris: P.U.F., 1964), 189; E.W. Beth, Jean Piaget, *Epistémologie mathématique et psychologie* (Paris: P.U.F., 1961), ch. V, IX, X; Jean Ladrière, *Les limitations internes des formalismes* (Louvain: E. Nauwelaerts/Paris: Gauthier-Villars, 1957), ch. X.

<sup>19</sup> Cf. *The Foundations of Statements and Decisions*, ed. Kazimierz Ajdukiewicz (Warszawa: PWN-Scientific Publishers, 1965).

**4. The *antinomy of exactness*:** *the exactness of terms requires the idealization of objects.* Petre Botezatu considered that the deductive edification of a theory is not reduced to a mere arborescent organization of its sentences in order to give a unique and unitary system, but *in-depth changes* must be made, in the structure and the content under organization. Einstein, discussing the relation between theory and reality, paradoxically stated that “Since mathematical propositions address reality, they are uncertain, and since they are uncertain, they do not address reality.”<sup>20</sup> Indeed, mathematical propositions describe the properties of ideal objects. When applied to reality, the formal object and the empirical object must coincide; for example, a geometric circle and a physical circle; this is very difficult to accomplish. Hence, philosophers of science proposed various solutions. N.R. Campbell spoke about a ‘dictionary’ supposed to translate the concrete terms into the abstract ones. P.W. Bridgmann asked for operational ‘rules’ and Rudolf Carnap, for ‘correspondence rules.’<sup>21</sup> Petre Botezatu considered that such devices only conceal the difficulty, they do not get rid of it. Robert Blanché concluded that any axiomatics can be read in two different ways: an abstract, rational and formal way, and a concrete, empirical and material way.<sup>22</sup> Thus, geometry splits into pure, axiomatic geometry, without intuitive content, where truth means non-contradiction, and applied geometry, which is intuitive and deals with physical laws. The debates are still going on. Botezatu described various solutions to show that the opposition between exact and inexact concepts is a major achievement of contemporary logic. In this way the distance between theoretical and empirical thinking is better understood, but the antinomy is still there: the gain in exactness is the loss in expression.

**5. The *antinomy of abstractization*:** *the abstractization of structures involves the undetermination of theory.* Petre Botezatu was referring to the capacity of instruments to penetrate into the intimacy of complex phenomena in order to create new scientific theories: relativity, quantum theory, theory of heredity, cybernetics, etc. Due to the advance of the abstractization process, these theories ignore the particular properties of objects and then isolate some relations, leaving others aside. The selection and various combination of relations were fruitful approaches as they rendered obvious the fact that very different scientific fields can be unified, such as logic and electronics, measure theory and probability theory, etc. The structures are *multivalenced* and *plural*. Once a set of relations is selected, the abstractization process continues and the number of fundamental

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<sup>20</sup> Albert Einstein, *Mein Weltbild* (Berlin, 1955), 119.

<sup>21</sup> Rudolf Carnap, *Philosophical Foundation of Physics* (New York: Basic Books, 1966), § 24.

<sup>22</sup> Blanché, *L'axiomatique*, § 28.

relations decreases gradually but new forms are created. Thus, *theories progressively fail to correspond with reality*. For example, logic. Despite the multitude of formalisms, researches and achievements, logic still fails to express appropriately the natural line of thinking. Therefore, the process of structuring the theories represents an indubitable success and a failure at the same time. In a mathematical structure we can include more, but cover each sector less. *The gain in extension is the loss in intension*.

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Petre Botezatu considered that axiomatization antinomies are primordial as compared to formalization antinomies because the deductive method resorts to axiomatization in most of the cases. However, formalization represents the superior level deduction can reach, but here as well the progress in one direction means the withdrawal from another.

**6. The *antinomy of rigurocity*: the exactness of the demonstration imposes the complication of the demonstration.** Formalization requires that all the elements needed to create a new theory be mentioned, such as *rules for the formation* of correct formulas, *rules for the transformation* of one formula into another and *rules for the definition* of terms. Thus the derivation of theorems from axioms becomes entirely formal: one sequence of signs is turned into another even if the meanings are unknown. *Full proof* is obtained in this way, i.e., at least apparently, nothing relies on intuition any more. But full proof needs more, that is to go back from theory to theory until it reaches the basic logical theory. Now there will be no more elements of subjectivity and a superior level of rigurocity is achieved. But Petre Botezatu considered that this procedure had a practical disadvantage, not negligible at all: if any thought is formalized, the demonstration becomes excessively extended and complicated: “a gain in exactness and methodological accuracy comes with a loss in clarity and intelligibility.”<sup>23</sup> On the other hand, the risk of error is still not overcome. “The longer the demonstrations, the higher the risk of errors, and the increase in the number of elementary operations, as long as they are not performed by a machine, is rather more dangerous than useful.”<sup>24</sup> Hence the paradoxical situation in which mathematicians often avoid formalization, although they created it. Botezatu concluded that, given the current level of performance, *the operations cannot be certain and simple at the same time*.

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<sup>23</sup> Tarski, *Introduction à la logique*, 118, cf. Botezatu, *Valoarea deducției*, 188.

<sup>24</sup> Martin, *Logique contemporaine*, 187.

**7. The *antinomy of totalization*:** *closing a system means transcending the system.* Aristotle himself stated that a syllogism is correct only if “no exterior term is needed for the consequence to be necessary.”<sup>25</sup> Axiomatization and formalization continue this necessity: a formalism either is sufficient or denies itself. But it was proved that *self-sufficiency is unrealizable* because first of all it is impossible to create a language formalized within itself; a language (*object language*) is always created within another language (*metalanguage*), which is given. For example, the arithmetic language is created by means of the English language. Of course, a metalanguage can also be formalized using another metalanguage, in several formalization steps, but this has to stop somewhere and there the language will be informal. Botezatu concluded that “we cannot dispose of the intuitive in order to create the formal.”<sup>26</sup> On the other hand, it is common knowledge that Gödel proved that one of the undecidable propositions that affect stronger systems (containing arithmetic at least, e.g. *Principia Mathematica*) is the very one that states the consistency of the system. Hence, *the noncontradiction of a system cannot be proved through the means of the system itself*. So, a formalism cannot become a closed system. Two indispensable external references always accompany it: a reference to an informal basic language and a reference to a superior formalism. We may say that a given formal system grows on an intuitive substructure and continues in a stronger formal system and these connections cannot be cut when the problem of justification arises.<sup>27</sup>

**8. The *antinomy of consistency*:** *The consistency of a system involves the incompleteness of the system.* Petre Botezatu correlated this antinomy with the previous one and derived it from Gödel’s theorems. It was demonstrated that a formal system containing arithmetics, if it is noncontradictory, it contains undecidable propositions, such as Proposition *G* which states its own indemonstrability. But another troublesome aspect appears: Proposition *G*, although indemonstrable, is true, a fact which can be proved by metatheoretical means: it states that integers have a certain mathematical property that is well-defined and belongs to each number. Because the system contains a property that is undecidable and true at the same time, *the system of axioms is incomplete*. In other words, the system of axioms is not strong enough and not all theorems can be derived from it. Moreover, even if new axioms would be added to the system so that Proposition *G* could be demonstrable, another undecidable formula can be created, and then another one, and so on. So, if a formal system is consistent, it is

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<sup>25</sup> Aristotel, *Analitica primă*, I, 1, 24 b.

<sup>26</sup> Botezatu, *Valoarea deducției*, 190.

<sup>27</sup> Botezatu, *Valoarea deducției*, 191.

incomplete and remains incomplete. From the epistemological point of view, it can be concluded that full formalization is not the most beneficial way to organize a deductive theory; human spirit is free and overcomes the barriers of formalization, learning from practical and intellectual experience.<sup>28</sup>

**9. The *antinomy of interpretation*:** *the formalization of a system entails the relativization of interpretations.* A formal system is designed to interpret a certain theory, which in fact was originally its model. For example, Peano's system was created in order to give an interpretation to the arithmetics of real numbers. In other words, when creating a formalism, one starts from just one point, a model, and by means of that model many other points are reached: *a multitude of interpretations are created, all based on the same formal pattern.* At the epistemic level, there are structural similarities and connections between set theory, number theory, geometry and logic, which allow the theorems to be extended from one field to another. However, there exist a *lack of precision* which can be explained in the following way. An inconsistent (contradictory) system has no models, while a consistent system can have several models. If all the models of a system are isomorphic, that is structurally identical, the system is categorical or monomorphic; otherwise, it is noncategorical or polymorphic. The plurality and the irregularity of models generate difficulties when trying to determine the concepts. For example, Peano's axioms should be categorical and able to characterize univocally the natural numbers. But Thoralf Skolem (1933) proved that the sequence of numbers cannot be characterized by a finite system of axioms that would distinguish it from other sequences. Skolem created ordered sets of integer functions, sets that satisfy the axioms and yet belong to another type of order than the sequence of natural numbers. In 1950, Leon Henkin proved that any consistent formal system containing the theory of integers has irregular models and therefore cannot be categorical. Petre Botezatu gave the most conclusive example: set theory. Thoralf Skolem had already proved, in 1920, that it was impossible to create an absolutely categorical system. The theorem proved that any theory of the first-order predicate logic had an uncountable model even if it was conceived for a countable model, which it actually had. But the set theory belongs to this category, which means that, if it has a model, it certainly has an uncountable model as well. Cantor established a precise demarcation between the countable and the uncountable; now the boundary is disappearing. Analyzing the interpretations given to this state of facts, Botezatu made clear that, following axiomatization, formalization as well is not to be reduced to a mere translation from one language into another. Structural changes occur in the process and perhaps the most significant one is the

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<sup>28</sup> Botezatu, *Valoarea deducției*, 192.

loss of univocity. Depending on the standpoint, this can be an advantage or a disadvantage.<sup>29</sup>

10. *The antinomy of expressiveness: the semantization of a system involves the inexpressiveness of that system.* Because systems are created in view of interpretations, the syntactic construction is backed up by a semantic one. But in formal systems the formulas have only the property of being derivable (demonstrable) or not, while in the language of interpretations the statements can be true or false, hence truth is a semantic concept when it involves the sign-object relation and it is also a fundamental epistemological notion. Hence the following question arises: is it possible to introduce the concept of truth into the formalism of a system? Alfred Tarski (1936), preoccupied with this problem, proved that, given a strong enough formalism, *the theory of 'true' semantic predicate cannot be formalized in that system*, otherwise a Liar-type paradox would arise. In 1948 he proved that the same limitation is required for other semantic concepts as well, such as *definissability*. Botezatu's conclusion was that in strong formal systems metatheoretical (semantic) notions can be created, which means that the formal system has been transcended. It follows that formal systems are never self-reflexive whenever they have to be both comprehensive and expressive.

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These *methodological antinomies* formulated by Petre Botezatu express the idea that we cannot create a formal system possessing all the necessary qualities: purity and comprehensiveness, consistency and completeness, decidability, etc. We can advance in one direction if we withdraw from another. Or we can moderate our ambitions and then we may fulfil more objectives. In any field of study there are scholars who attempt to build a great system, complete and unique, but only partial and perfectible system are ever created, and this ensures the progress of knowledge. From the cognitive point of view, Petre Botezatu noted the *alternation between adjustment and assimilation*. Formalization is a process of *deductive assimilation*: the scientific objects are transfigured in order to be integrated into the sequence of deduction. Interpretation, that is the transition from the relations between signs to the relations between objects, is a process of *adjustment*. In other words, the construction and improvement of formal systems oscillate between these two opposite tendencies. Depending on the objectives, one tendency will prevail over the other.<sup>30</sup>

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<sup>29</sup> Botezatu, *Valoarea deducției*, 194.

<sup>30</sup> Botezatu, *Valoarea deducției*, 198-199.

The idea of methodological antinomy continued to preoccupy Petre Botezatu. For example, he tried to explain the growing up crisis of higher education formulating the *antinomy of accessibility* between *intuitive* and *structural* and the *antinomy of structuring*, that is knowledge structuring facilitates knowledge consolidation but makes knowledge renewal difficult. He presented them at his last public conference, held in the autumn of 1981 during “The Days of Iași University.” He pointed out that in the process of scientific teaching, the objectives of methodics cannot be fulfilled all at once.<sup>31</sup>

The two antinomies relied on two *fundamental laws*<sup>32</sup>: 1. *The law of the gap between the scientific-technical revolution and the change in mentality*: the adaptive effect is considerably delayed. New habits and a new perspective take time and need favourable circumstances in order to be structured and consolidated. 2. *The law of information distortion*: the accuracy in conveying the message is directly proportional to the cultural level of the senders and indirectly proportional to their emotional tension. Petre Botezatu warned of the fact that “Ignorance makes it difficult, if not impossible, for the scientific message to be received with accuracy. On the other hand, the emotional turmoil has the same negative effect.”<sup>33</sup> These two nomic relations generate antinomical situations. Two of them were formulated by Petre Botezatu, as previously showed.

The subject matter of antinomies has also preoccupied us. Certainly, the initial impulse came from Petre Botezatu,<sup>34</sup> and then we added the *antinomy of certainty*: “*certainty in experimental research limits the applicability of inductive methods*. The study of real phenomena requires that logic’s formal frame of reference becomes more flexible and enhanced. But the logical loss is the gnoseological gain.”<sup>35</sup> This conflict can be avoided by moderating the two natural characteristics of thinking: the need for abstractization, formalism, deduction, satisfied by logic, and

<sup>31</sup> Cf. Teodor Dima, *Privind înapoi cu deferență* (București: Editura Academiei Române, 2006), 189.

<sup>32</sup> Petre Botezatu, “*Homo Logicus*,” in his *Interpretări logico-filosofice* (Iași: Junimea, 1982), 334. This paper was published for the first time in *Analele științifice ale Universității „Al. I. Cuza” din Iași*, Philosophical Sciences XXV, III b (1979): 57–72.

<sup>33</sup> Botezatu, “*Homo Logicus*.”

<sup>34</sup> In a text evoking the personality of Petre Botezatu, we confessed that the idea of dealing with the problem of antinomies came to our mind when Petre Botezatu’s volume, *Valoarea deducției* (1971), was published. Here the author discusses the ten methodological antinomies. In that period we were elaborating our PhD thesis. See Teodor Dima, *Pseudo-Jurnal din Iași, strada Sărării, numărul 174*, in *Symposion* V, 1(9) (2007): 173–183.

<sup>35</sup> Teodor Dima, *Metodele inductive* (București: Editura Științifică, 1975), 129–130.

the need for concreteness, empirism, induction, closer to the spirit of experimental sciences.<sup>36</sup>

Bogdan Olaru, appreciating our contribution to the study of antinomies, observed that “Teodor Dima wrote on several occasions on antinomies and Romanian authors who dealt with the subject. Mainly, there are three constants of these researches: 1) the analysis of the interpretations of the Kantian antinomies given by some Romanian philosophers; 2) the assessment of Romanian contributions to the of antinomies of thinking; 3) his own interpretation of Kant’s antinomies.”<sup>37</sup>

Joining the analysis of the “temperate audacities” of reason, Bogdan Olaru admits there is a counterpart of theoretical antinomies in moral philosophy, considering them *moral dilemmas*; there are instances when two opposing theses, with the same subject but different predicates (*it is good/it is bad* from the moral point of view), are seemingly equally well-founded. Often, the solution to a moral dilemma is to admit there is not just one solution, unless there is agreement upon the fact that this is an interim solution or a solution indissolubly dependent on restrictions imposed by the context. He argued that “emergency exit through the formulation of some postulates is the least convincing. Only by appeasing the impulse to settle the dilemma by resort to *a priori* postulates one can be given the best chance to understand the nature of formal conflicts. And the chance to find that they are real, not apparent.”<sup>38</sup>

In conclusion, continuing and developing by original detailings the preference of modern and contemporary thinking for an interdisciplinary approach to rationality, tempted to transcend and even elude logicity, Petre Botezatu proved that the aporias, dilemmas and antinomies of knowledge are given new expressions once they are formulated, interpreted and commented.

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<sup>36</sup> We presented the antinomy of certainty for the first time in public on 23rd February 1973, when we defended our PhD thesis at the University of Bucharest.

<sup>37</sup> Bogdan Olaru, „Prolegomene la o critică a rațiunii temperate sau: despre câștigurile practice ale antinomiilor teoretice,” in *Revista de Filosofie* LVIII, 1–2 (2011): 103–116. A shorter version of the paper was published in the volume *De Dignitate Philosophiae*, ed. Cătălina-Daniela Răducu et al. (Iași: Terra Nostra, 2009), 307–326.

<sup>38</sup> Olaru, „Prolegomene la o critică a rațiunii temperate,” 116.