



Ancient Philosophy

Mathematics as Paideia in Proclus

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ABSTRACT: I examine one aspect of the central role which mathematics plays in Proclus's ontology and epistemology, with particular reference to his *Elements of Theology*. I focus on his peculiar views about the ontological status of mathematical objects and the special faculties of the soul that are involved in understanding them. If they are merely abstract objects that are "stripped away" from sensible things, then they are unlikely to reorient the mind towards the intelligible realm, as envisioned by Plato in the *Republic*. Thus, in order to defend the function of mathematics as a prodaideutic to dialectic, Proclus rejects Aristotelian abstractionism in favor of an elaborate account in terms of *Nous* projecting images of its Forms through the medium of the imagination. In metaphorical terms, he replaces the Aristotelian image of the soul as a blank tablet with that of a tablet that has always been inscribed and is always writing itself, while also being written on by *Nous*. The mediating function of mathematics for understanding the higher realities is grounded in the fact that its central principles of Limit and Unlimited have a universal provenance in Proclus's whole system of reality.

Introduction

Alexander of Aphrodisias established abstractionism as an Aristotelian dogma about mathematical objects, but for later Neoplatonists this proved difficult to reconcile with the educational function of mathematics in Plato's philosophical curriculum. Thus Proclus, for example, rejected abstractionism as a basis for the ascent to the realm of Forms, and proposed an alternative view based on the typical Neoplatonic hierarchy of *Nous*, Soul, and Nature. At the highest noetic level, geometrical Forms are unextended and indivisible, so that only at the level of Soul can they become available for study by the geometer when they are embodied in the intelligible matter supplied by the imagination. Proclus also accepted that geometrical forms can be embodied in sensible matter, though they never have the exactitude necessary for science, nor could they ever acquire it through abstraction. Thus the diagrams used by the geometer are products of the imagination, which are really projections by the higher intellect onto a lower level so as to facilitate the study of geometrical objects. Proclus seems to accept that the human intellect can never attain the Platonic goal of studying geometrical Forms in their pure and unextended form as paradigms. Although such a goal can be achieved only by divine *Nous*, yet it becomes for Proclus the guiding rationale for his whole system.

Most people who have written about Proclus's commentary on the first book of Euclid's *Elements* have treated it purely from the point of view of the history and philosophy of mathematics. I hope to supplement this rather narrow perspective by discussing Proclus's broader purpose in preparing such a commentary. Along with being a fervent Pythagorean, this late Neoplatonic thinker took very seriously the curriculum for educating the philosopher ruler in Plato's *Republic*. For Proclus the importance of mathematics in that curriculum could hardly be over-emphasized because he took it as self-evident that it provides the key which opens the door to the realm of intelligible being. Ultimately, according to him, this realm must be studied by theology, but the model for this science seems to be derived from mathematics, and particularly from geometry.

I. Division of the Sciences

At the beginning of the first Prologue to his Euclid Commentary, Proclus discusses mathematical being in terms of its intermediate status between partless realities, which are the most simple, and divisible things which are the least simple of all things. It is noteworthy that, contrary to Plato's methodological approach in the *Republic*, Proclus draws extensive conclusions about the ontological status of mathematical objects. According to him, such objects must be superior to the things that move about in matter, since mathematical propositions are unchangeable, stable, and incontrovertible. On the other hand, the discursive procedure of mathematics, dealing with its objects as extended and as dependent on higher principles, implies for Proclus that such objects are inferior in being to that indivisible being which is completely grounded in itself.

Having outlined this ontological division, Proclus argues that it reflects exactly Plato's distinction between different types of knowing. For instance, intellect (nous) is held to correspond to indivisible realities because of their purity and freedom from matter. By contrast, opinion (doxa) corresponds to divisible things at the lowest level of sensible natural objects; whereas understanding (dianoia) goes with intermediate things such as the forms studied by mathematics. While accepting it as inferior to intellect, Proclus claims that understanding is more perfect, more exact, and purer than opinion:

For it traverses and unfolds the measureless content of Nous by making articulate its concentrated intellectual insight, and then gathers together again the things it has distinguished and refers them back to Nous. (1)

Despite his appeal to Plato's authority and the use of familiar language from the *Philebus* (58-59), what we have here is something unique to Proclus; namely, epistemological and ontological grounds for composing his *Elements of Theology* after the model of Euclid. The crucial point for this project is that (mathematical) understanding is seen as a means by which the unitary content of intellect is reflected in a multiple form that makes it accessible to our discursive thinking. Even though he insists that mathematical objects are multiform images which only imitate the uniform patterns of being, one suspects that Proclus finds them indispensable for obtaining access to the completely unitary objects of intellect. At least, this is a natural way of interpreting his metaphor about mathematical 'standing in the vestibule' of the primary forms, announcing their unitary and undivided and generative reality (*in Eucl.* 5.2-3).

My interpretation is supported by Proclus's use of Limit and Unlimited both as mathematical principles and as principles of being as a whole. In his ontology (cf. *ET*. Prop 89), these are the two highest principles after the indescribable and incomprehensible causation of the One. By contrast with the immobile One, these transcendent principles *do* give rise to an ordered procession (proodos) of things that appear in their appropriate divisions. The objects of Nous, for instance, by virtue of their inherent simplicity, are the primary participants of Limit and Unlimited; since their unity, identity, and stable existence are derived from the former, while their variety, fecundity, and otherness are drawn from the latter. While mathematical objects are further offspring of Limit and Unlimited, Proclus insists (*in Eucl.* 6.7 ff.) that secondary principles are also involved in the generation of this intermediate order of things.

Although he does not specify what secondary principles he has in mind, the examples suggest that they are principles in arithmetic and geometry which are seen as reflections of the primary principles of Limit and Unlimited. In the mathematical order of being, he says, there are ratios proceeding to infinity (reflecting the Unlimited) but controlled by the principle of the Limit. For instance, number is capable of being increased indefinitely, yet any number you choose is finite. Likewise, magnitudes are indefinitely divisible, yet the magnitudes distinguished from one another are all bounded, and the actual parts of the whole are limited. From these examples it is clear that the Unlimited is reflected in the multiplicity of number and in the divisibility of the continuum, both of which are controlled by some principle which reflects the Limit. Since mathematics presents such an accessible model for the characteristic activity of these principles, it is no wonder that Proclus finds it so indispensable for understanding how they function also in the intelligible realm, which is the subject-matter of his theology.

But, alongside the ontological path towards this higher goal, Proclus (*in Eucl.* 10.15 ff.) seems to be following a parallel epistemological path when he addresses the question of which faculty (kriterion) pronounces judgment in mathematics. On this question, once again, he defers to the authority of Plato whose *Republic* he takes to be making firm connections between forms of knowing and knowable things. I think, however, that Proclus is clearly going beyond Plato's text in assigning intellection (nous) exclusively to intelligibles (noeta), while connecting understanding (dianoia) with 'understandables' (dianoeta) as its exclusive objects. In spite of its tautological appearance, this latter connection is not made explicitly in the *Republic*, and I think it has been argued successfully by Myles Burnyeat (2) that precisely this question is left open by Plato himself. But Proclus is on firmer textual ground when he attributes to Plato the parallel connections between belief (pistis) and perceptibles (aistheta), and between conjecture (eikasia) and images (eikasta). Similarly, he is correct in saying that Plato finds the same relation between conjecture and perception as between understanding and intellection, since the first apprehends images of the second in each case. In the *Republic*, conjecture is illustrated in terms of images of sense objects that are reflected in water or some other reflective surface, which means that such images of images have the lowest place in Plato's ontology.

One might argue that the corresponding relation between understanding and intellection implies that 'understandables' are images of intelligibles, but the fact remains that Plato nowhere draws this seemingly obvious conclusion. However, Proclus *does* draw this conclusion and upon it he builds his own theory about the mode of being of mathematical and the special cognitive faculty which grasps them. Since mathematical objects do not have the status of partless and indivisible objects, nor of perceptible and changeable objects, he takes it to be obvious that they are essentially 'understandables' and that understanding is the faculty which judges them (*in Eucl.* 11.10-16). But he insists on

offering this as a commentary on the *Republic* (533d) where Socrates describes the knowledge of 'understandables' as being more obscure than the highest science, though clearer than the judgment of opinion. By way of interpreting this passage, Proclus explains that mathematical understanding is inferior to intellectual insight because it is more explicative and discursive, though it is superior to opinion on account of the stability and irrefutability of its ideas. Furthermore, the fact that mathematical sciences begin from hypotheses makes them inferior to the highest knowledge, yet their preoccupation with immaterial forms makes their knowledge more perfect than sense perception. Here we find some useful pointers for Proclus's own views about the differences between mathematics and the highest science of intelligible things, which may be called dialectic or theology.

II. The Ontological Status of Mathematical Objects

A careful reading of Proclus's First Prologue to his commentary on Euclid reveals that one of the great obstacles to his attempted reconciliation between the views of Plato and Aristotle was the traditional dispute over what mode of being should be assigned to mathematical objects. In dealing with this question, he adopts a quasi-dialectical approach which consists in setting out alternative views, presenting objections to them, and then setting out his own position. But it is not an open-ended dialectical inquiry because Proclus has already (*in Eucl.* 12.7-9) made up his mind to follow Plato by assigning to mathematical objects an existence (*hupostasis*) prior to sensible objects, as is shown by the processional order (*proodos*) of things. Therefore, it is for the purpose of refutation that he sets out the alternative views about mathematical objects as being derived from sense objects, either by abstraction (*kata aphairesin*) or by collection from particulars (*kata athroisin tôn merikôn*) to one common definition. The second view seems to envisage some process of induction whereby mathematical universals are grasped through the experience of particulars, thereby treating mathematical objects as dependent upon such sensible particulars.

According to Proclus, however, soul is the only possible source of the certainty and precision that one finds in mathematics. Behind this assumption one can spy his own metaphysical principle that the character of the cause is transferred to the effect. Such a principle seems to ground his next objection to the effect that one cannot explain the stability of unchangeable ideas if they are held to be derived from things which are continually changing from one state to another. In this context (*in Eucl.* 12.26-13.6) he takes it as agreed that anything which results from changing things receives from them a changeable character. So how could it be possible for exact and irrefutable ideas to emerge from what is inexact and uncertain? Once again the rhetorical question betrays the dogmatism lying behind the ostensible dialectical inquiry, as Proclus moves quickly to his conclusion (*in Eucl.* 13.6 ff) that the soul must be posited as the generatrix of mathematical forms and ideas. Citing Plato as his authority, he asserts that the soul can produce such mathematical objects because their paradigms already subsist in her, so that she can bring forth these projections (*probolai*) of previously subsisting forms. In support of this purportedly Platonic view about the mode of being of mathematical objects, Proclus argues that the soul must have these standards in herself by which she can judge whether her offspring are fertile or merely 'wind eggs'. If that were not the case, he asks rhetorically, how could she produce such a variety of ideas? Here a sceptic might object that this is simply begging the question, since there is no guarantee that the soul has any internal standard by which to judge the correctness of what is produced by imagination. It is obvious, however, that Proclus is not troubled by any sceptical doubts when he concludes that the birth-pangs and subsequent offspring of the soul must yield manifestations (*ekphaneis*) of eternal forms abiding in her.

Against the possibility that the soul gets mathematical forms from itself alone, Proclus objects (*in Eucl.* 15.22) that this would prevent them from being images of intelligible forms. But, he demands rhetorically, how can they fail to receive their share of the filling up of being from the "firsts", given their intermediate position between indivisible and divisible natures? It seems that Proclus is here begging the question by assuming that mathematical objects have such an intermediate status, though if he were merely describing the self-evident character of these objects then perhaps he could not be convicted of *petitio principii*. Yet it does seem that he is assuming too much from his own metaphysics when he objects that, if mathematical objects come only from the soul, then the forms in Nous cannot retain their primacy as the preeminent patterns of all things. Here the Platonic language, combined with the rhetorical style of argument, gives the distinct impression that Proclus already has adopted a clear metaphysical framework into which mathematical objects must fit.

In relation to the second possibility that these objects come from Nous alone, he asks (*in Eucl.* 15.26 ff) how the self-activating and self-moving character of the soul is to be preserved, if one accepts that it receives its ideas from elsewhere, like things that are 'other-moved'. What Proclus finds objectionable here is any conception of the soul as purely passive like matter which, although potentially all things, does not generate any of the forms embodied in it. Without further argument he takes this to be an insuperable objection to the second possibility, so that only the third remains; namely, that the soul gets its ideas both from herself and from Nous. Proclus describes (*in Eucl.* 16.5-8) the soul in metaphorical terms as being "the full complement of forms", which themselves are constituted from intelligible patterns but which enter spontaneously upon the "stage of being". The purpose of such metaphorical language is to replace the Aristotelian conception of soul as a blank tablet with the notion of a tablet that has always been inscribed and is always writing itself, and is also being written on by Nous; cf. *in Eucl.* 16.8-10. As Proclus explains, the ground for this conception of soul is that it is an unfolding of the Nous which presides over it, so that soul become a likeness and external replica of Nous. The explanation seems to involve two reflecting levels of being such that, as he says, Nous is identical with everything in an intellectual way, whereas soul is everything in a soul-like manner. Thus, Proclus says, if Nous reflects everything paradigmatically then soul does so in an image fashion, or if Nous is everything in a concentrated way then soul is everything discursively.

Conclusion

For Proclus, therefore, the essence of the soul consists in such forms as self-moving numbers, living figures, and invisible circles. But he explicitly warns us (*in Eucl.* 17.5 ff) against assuming that number here means a plurality of monads or that the notion of interval involves bodily extension. Instead all of these forms must be seen as intelligible paradigms of visible numbers, figures, ratios, and motions. By way of vindicating such a conception, Proclus appeals (*in Eucl.* 17.9 ff) to Plato's *Timaeus* which seems to construct the soul in terms of mathematical forms that are thereby established as the causes of all things. He refers specifically to those passages (*Tim* 35b-c, 36a-b) which seem to make the principles of numbers and of figures fundamental in the composition of the soul, and its primary circular motion the principle for all other motions.

But it is not necessary to discuss whether or not his interpretation of Plato holds up, since the important thing is that such 'evidence' is used to conclude (17.22-24) that the mathematical ideas "filling up" souls are both substantial and self-moving. According to

him, it is by projecting and unfolding these ideas that the understanding brings into being all the mathematical sciences. Thus Proclus is clearly committed to saying that these sciences and their objects somehow depend for their existence on the soul, and yet that such objects are themselves real and substantial. Although such a view of the relation between mathematics and the soul places him squarely within the Platonic tradition, his "superrealism" is peculiar to late Neoplatonism. (3) (4)

Notes

(1) *In Eucl.* 4.12-14: translation by G. Morrow

(2) M. Burnyeat, "Platonism and Mathematics: A Prelude to Discussion", pp. 212-40 in *Mathematics and Metaphysics in Aristotle*, ed. A. Graeser. Bern & Stuttgart: Haupt, 1987.

(3) By 'superrealism' here I mean simply that independent mathematical objects are also constitutive of the intellects engaged in mathematics.

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