Symbolic Logic and Appraisal of Argument

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According to a recent textbook on symbolic logic,

> Logical studies comprise today both logic proper and metalogic...the aim of logic proper is to develop methods for the logical appraisal of reasoning, and the aim of metalogic is to develop methods for the appraisal of logical methods. In pursuing the aims of logic, it has been fruitful to proceed systematically, that is, to construct formal axiomatic systems of various kinds. These logical systems provide the immediate subject matter for metalogical investigations.
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> —Bas C. van Fraasen, *Formal Semantics and Logic* (New York: Macmillan, 1971)

Suppose we begin by taking this claim literally, asking ourselves: To what extent does symbolic logic, as presented today in "standard texts," provide adequate guidance for the appraisal of arguments, by means of formal systems, and for the appraisal of the formal systems themselves? It seems to me, after a number of years of teaching symbolic logic from a variety of textbooks, that both of these frequently asserted *claims* go well beyond what is in fact offered in practice.

The problem addressed here is a twofold one: the difficult nature of the relationship between logical form as it is represented in formal languages, and logical form as it occurs in natural language; and the difficulties in appraising logical systems by standard metatheoretic methods. From the standpoint of instruction what is important about these difficulties is, not that they exist, but how they are handled, and whether the students are given an honest understanding of present limitations of formal logic and some help in dealing with those limitations.

In our current understanding of the subject, logical appraisal of an individual piece of reasoning in natural language is supposed to take place by isolating the elements of logical structure "on which the argument turns" or, put another way, by determining the "formal" elements on which we estimate the correctness of the piece of reasoning depends.[1] These elements of logical structure ("syncategoretic" vs. "categorical" elements, as the medievals called them) are to be distinguished from the words tied by meaning relations (more or less directly) to the world. Once isolated, we take our understanding of how the logical elements function together in arguments from a *formal system* in which their conjoint operation has been systematically studied.

Thus our appraisal of reasoning is tied directly to our ability to isolate the functional elements determining the logical effect. This is normally done by "rewriting," "paraphrasing," or "translating" the given sentence(s) in the formal language. Once we have done this, we can ordinarily (though not always) apply techniques resembling in some respects those of calculation — mechanical, formal procedures — to assess the correctness of the reasoning.[2] Notice that the techniques or tests which we use apply to the sentences as represented in the formal language, not to the originals directly. Thus however good our tests may be, they will only be as significant, with respect to the original reasoning, as the "translation" is faithful, i.e., as the isolation of the crucial elements of

logical form has been successful. For if the validity of an argument depends upon its "logical form," the question of *which* form it has is as important as whether a given form is valid.[3]

If the above fairly represents current logical theory, a normal expectation would then be that in teaching symbolic logic with the aim of imparting the skill to appraise reasoning from a logical standpoint, a great deal of emphasis would of necessity be laid upon the problems of *translation* from natural language into formal language. This is reasonable, since otherwise any specific evaluation of an argument would be subject to rejection on the grounds that the representing argument form was not *the* form of the argument in question. This would have the effect of putting the young logician out of the evaluation business in short order.

Of course if a proffered "translation" of an argument were rejected on these grounds, one could still argue that the apprentice logician has only to demand that the critic offer a revised translation which *does* capture "the" logical form, and then apply formal tests to that version. Thus he is still prepared to evaluate arguments. Note, however, that the selection of alternative translations may be itself the subject of considerable informal argument, and that in the end one might face an argued claim that no existent formal language is adequate to capture the logical subtlety of a particular argument. So there are problems aplenty in application here with which the aspiring logician will need help.

When we look at a variety of modern texts, however, we find this reasonable expectation (emphasis on translation) very far from being met. In the first place, the vast array of arguments encountered in everyday life are simply dichotomized into two very broad classes: the so-called "deductive" arguments and the "inductive" arguments. The former class is usually characterized in terms of a relationship between premises and conclusion in which the premises provide *"absolutely conclusive* grounds" or as those in which it is "absolutely impossible for the premises to be true unless the conclusion is true also." Inductive arguments are characterized as "those whose premises provide some grounds for their conclusions."[4] After this division, symbolic logic texts immediately dispense with any consideration of the latter category and concentrate exclusively on the first.

Is the distinction between valid and invalid the primary and most significant logical distinction in appraising arguments? Or is it a gross distinction which only serves to pick out a small portion of the arguments we encounter in everyday life because of their tractability to manipulation? Within the great realm of those arguments not formally valid but having some weight, are there not important distinctions?[5] Would it not be appropriate to test the classification, to survey arguments in a variety of fields, for a starter, to see whether a legal argument, or an historical argument, for example, have distinctive logical characteristics (as opposed to simply employing the specialized *concepts* by which we normally recognize and classify them)?

Further, as we shall note below when discussing metalogic, it needs to be kept in mind that the delimitation of deductive arguments in terms of the concepts of "necessity" and "impossibility" will yield a determinate class only to the extent that these concepts are not, in Wittgenstein's useful phrase, "family concepts."

Next, on taking up the study of deductive arguments, a standard text will give a more or less careful definition of a standard formal language and deductive apparatus. In a great step forward over older axiomatic treatments, the student

is usually provided with truth-table tests and natural deduction proof techniques, perhaps also truth-tree (semantic tableaux) methods.

Since the formal language itself will usually have been introduced as an abbreviation and simplification of natural (English) language, the student might expect that he has now arrived at his goal and is ready to tackle actual, concrete arguments in the natural language and subject them to "logical appraisal." Perhaps to his surprise, he is likely to find that the standard text has little interest in this task. For example, three recent texts, by well known logicians, consider the formal languages and their deductive apparatuses almost exclusively, devoting at most a brief section to the problem of translation and hence, application. Benson Mates' text gives eight pages out of 230 to translation and dealing with natural language argument.[6] Gerald Massey's gives twenty of 158 pages on sentential logic to "logical analysis of natural languages," fifteen out of 125 pages on quantification theory.[7] In following basically the same approach, Richmond Thomason defends it thus:

> The formal languages of classical logic were devised to account for mathematical reasoning, and serve very well to express mathematical material, but very often it is difficult or impossible to render colloquial English in these languages.[8]

Is the real goal of studying symbolic logic, then, learning to "logically appraise," not argument in general, but *mathematical* reasoning? If so, there are several problematic aspects to this. First, I would venture to suggest that most courses in symbolic logic are taught in philosophy departments, and primarily aimed at students in humanities and social sciences, whereas the counterpart courses sponsored by mathematics departments and usually entitled "mathematical logic" are supposed to serve mathematics and natural science majors. As a general rule the students in symbolic logic will not have had extensive firsthand experience with mathematical reasoning and particularly not with the sophisticated reasoning procedures of the differential and integral calculus, which are at the root of so many of the interesting problems in this area. They may be expected to have little initial interest or appreciation, certainly not enough to motivate them to take a demanding course. Moreover, most will never go on to take enough mathematics to see the point of many of the logical subtleties they learn.

In the second place, if the aim is really to teach students the logical theory of *mathematical* reasoning, would it not be better to provide for practice with some important mathematical structures, for which the student may some day find application in his line of work, than to offer logical structures whose intended uses will always remain something of a mystery to him?

Third, the student is rarely told, as honestly and openly as Thomason tells him, that he is going to be studying essentially mathematicized logical theories of mathematical reasoning, but rather something far more general is proffered, indicating a wide range of applications and/or a tight connection with everyday argumentation. Smuggled in, as it were, under the ancient rubric of its being a basic "liberal art" is a highly technical theory with limited application. As Bar-Hillel put it in opening a symposium on formal logic and natural languages,

> I challenge anybody here to show me a serious piece of argumentation in natural languages that has been successfully evaluated as to its validity with the help of formal logic.... The customary applications are often careless, rough and unprincipled, or rely on reformulations of the original linguistic entities under discussion into different ones... through processes which are again mostly unprincipled and ill understood.[9]

But even if the student were given adequate help and training in translating natural language expressions into formal language, he would still be left to discover for himself the place that deductive arguments have in the world of argumentation outside mathematics, because inductive logic is simply dismissed as not of concern. Perhaps this is because, as Quine suggests, it is thought of as coextensive with the theory of knowledge. This means, however, that the role played by deductive arguments in such important areas as scientific and legal argumentation is weakly, if at all, grasped by the student.

Moreover, focusing on the single concept of validity defined in the usual way, as Toulmin pointed out some time ago[10] and as recent evidence (see below) confirms, causes the student to lose sight of the fact that other concepts are relevant to logical appraisal of an argument besides this one. If one keeps in mind the general purpose of argument, for example, as aiming at establishing or explaining or accounting for some proposition in terms of others, it is clear that different structures which count as proofs succeed in generating this desired illumination in quite different ways. Although it is hard to pin down precisely, the recent work by Kreisel, Prawitz, and Hintikka suggests that there is a concept of information communicated by an argument which needs to be developed here.[11] Although mostly this work is highly proof-theoretical, it is clearly relevant to teaching introductory symbolic logic.

So should we say the student should have learned, as promised, to "logically appraise" arguments? It is clear that he has been handed some tools which *may* permit him in *some* instances to do so. But unfortunately in place of a genuine understanding of how to *use* these tools, he has simply been given *more* tools, all essentially equivalent in power and all relating to sentences in the formal language. It is not unfair, I think, to suggest that this device of piling on additional technical methods was originally inspired by Quine's *Methods of Logic*,[12] but as attention has drifted away from natural language arguments the idea has caught on. See, for example, recent texts by LeBlanc and Wisdom and by Lambert and van Fraasen.[13]

Here one can imagine the objection being raised that after all symbolic logic texts devote considerable time to metalogic, i.e., "to the appraisal of logical methods," and it is here where the student learns the strength and limitations of his formal techniques. Let us turn to consider metalogic, then.

In the appraisal of arguments, so we have said, we evaluate an argument by isolating its logical form and then testing the formalized argument by specific techniques. But how are we to be sure that the tests or techniques themselves are such as to properly dichotomize arguments into valid and invalid? For example, if an argument requires as a principle of inference a certain schema, we may or may not recognize intuitively the validity of that schema. And even if we did, could we trust our intuitions?

In the history of logic, from Aristotle down, a great deal of effort has gone into the attempt to establish the evaluating principles themselves. Peirce listed eight types of evidence to which appeal has been made on this issue: to direct dicta of consciousness, to psychology, to the usages of language, to metaphysical philosophy, to history, to everyday observation, to mathematics, and to process of dialectic.[14] While a number of these survive in use by one group or another — *e.g.*, appeal to language usage by ordinary-language logicians, such as Strawson, to dialectic by Lorenzen, to direct dicta of consciousness by most mathematical logicians — it is currently the fashion in texts to appeal to only one type of evidence: the mathematical methods of metalogic, comprised of proof theory

or semantics, deriving from Hilbert and Tarski and their followers.[15]

Few would wish to claim that metalogical methods are irrelevant to the appraisal of logical systems. My point, however, is that it does not necessarily tell us all that we would like to know about the effectiveness of our logical systems *as tools.* That fact is not, by itself, an objection, since we can hardly deny importance to a method solely on the grounds that it does not give us the whole truth, if the truth it does give us is not trivial. The objection arises because, first, the fact that metalogical methods give us only *partial* appraisal is not made clear, especially to students. Rather, its yield is passed off as though it were much more than it is, and little is attempted in the way of supplementing metalogical methods. Further, the attempt to use only metalogical methods to appraise logical systems leads to some paradoxical results which go counter to the very aim of introducing the student to metalogic. Each of these points deserves some elaboration.

To remind oneself of just how incomplete appraisal of logical systems by metalogical methods is, consider as an example the proofs of expressive and deductive completeness for the standard sentential calculus. It is useful to know that with a certain limited set of connectives we can express all possible truth functions, but for many purposes it is more important to know that not all important arguments involving unanalyzed sentences are truth-functional in character. In the standard text, the former is elaborated, the latter skipped over hurriedly. Similarly for deductive completeness of the calculus: provability of all truth-table valid tautologies is significant, but the student needs to know that we can hardly maintain that truth-table tautologies are exactly coextensive with the class of (sentential) "logical truths," because the latter concept is an open one, subject to extension and revision.

In fact, a more useful, as well as a truer picture would be presented if the student were given more help in recognizing where the standard formal language fails to represent an argument properly, so that tests on its formal stand-in misfire. If the student has any initiative he will quickly discover this for himself, anyway, as soon as he tries to formalize an argument from his philosophy readings — one from Descartes or Aquinas involving modalities, for example, or a legal or ethical argument with adverbial qualifications of actions. His reaction is likely to be that he has been tricked into learning a useless subject.

Of course this sad result could be avoided by providing supplementary material, stressing the existence of important variant logics such as strict implication, entailment logics, intuitionist logics, and the various other modal logics. What needs to be communicated is that different formal logics are available, which analyze the notion of validity in terms of *different* notions of possibility and necessity or which do not require exactly the same reductions to canonical forms (e.g., tense logic). For any given argument, therefore, the question of whether 'A implies B' is not necessarily settled even when a particular formal system yields a direct answer: A can materially but not strictly or intuitionistically imply B, so the question of whether to accept this particular argument depends on considerations outside the formal system. Logicians all know this, so why do they not make this clear to their students?

This point deserves a great deal more emphasis and exploration than we can give it here, but it is crucial to add one more note which goes back to our discussion of the quick dismissal of inductive arguments. The basic reasons for this traditional short shrift for induction appear to be two: first, practically, there is plenty of technical material in deductive logic alone to occupy two semesters

or terms, let alone the usual one term course; and second, what can be said about inductive arguments in logical appraisal is much less susceptible (apparently) to rigorous formulation and justification than in the realm of deductive logic, because the former is not *formal* enough. But it is just here that one needs to emphasize that the choice of formal systems in which to evaluate arguments is not itself a problem to be settled by formal means, and the overall rigor is more apparent than real.

Once this point is grasped, the whole range of argument-evaluation begins to take on the appearance much more of a continuum than a neat dichotomy. Realization of this would do much, I believe, to change the whole approach to appraisal of arguments by modern texts. And the emphasis on selection of the best formal system in dealing with diverse concrete examples would do much to help keep logic from falling into the problems that have so bedeviled mathematical pedagogy in the "new math" movement, of teaching disconnected abstractions.[16]

I might also add that it would be useful to supply supplementary material on the historical development of logic, so that the student could grasp some sense of what range of arguments logic could previously treat and what range is now open. But this point deserves separate treatment, so I will leave it at this.

Let us turn now to examine the claim made earlier that exclusive reliance on mathematical methods of metalogic also generates certain unintended paradoxical effects. One of the central aims of instruction in formal logic traditionally has been taken to be that of making students self-consciously or reflexively aware of the patterns of argument which they themselves use and hear constantly. The idea here is that this effect will occur naturally from the practice of formal techniques of analysis and evaulation of sample arguments, although to my knowledge this effect has never been demonstrated. At any rate this constitutes one of the arguments for retaining instruction in logic in the philosophy curriculum.

But the recent avoidance of dealing with concrete argument, and shift of emphasis to metalogic, often serves to create just the opposite momentum. This is due to the fact that, if one is primarily interested in the metalogical appraisal of systems there is a natural tendency to take it up immediately after developing a fragment of logic, usually right after the sentential calculus. The result of this ordering of materials is that the student is now *using* arguments in the metalanguage which are far stronger and more complex than those which he has presumably learned to appraise formally. This is pushed to an extreme in Thomason's and Massey's texts, where semantic completeness is proved for sentential logic through adaptation of Henkin's technique of "maximal consistent sets." The student is thereby involved not only in the use of predicate logic and argument by mathematical induction, neither of which he has studied formally, but also must employ non-constructive set theory. Not to belabor the point, the beginning student is hardly in a position to appreciate the great subtlety and strength of the non-constructive set theoretic axioms, so he must be given a very cursory, dogmatic run-through on this material, usually relegated to an appendix, and the problematic aspects must be kept under cover rather than exposed to critical examination. So the critical, careful examination is kept focused on the object language to the deliberate exclusion of attention to the informal metalanguage arguments which are actually being used—the very opposite of the reflective, self-conscious attitude towards what we do, which philosophy generally tries to engender and likes to think of logic as fostering.

So far I have mainly discussed the problem of logical evaluation of argument

as it appears in teaching symbolic logic from standard texts. Although I did not discover Michael Scriven's article until after I had prepared the bulk of this paper, I believe I have shown some detailed evidence which, sadly enough, lends weight to his biting remark to the effect that logic courses reveal such a "startling lack of match between pronouncement and practice...to legitimate an FTC action for misleading advertising."[17]

But the problem here addressed, while troublesome enough to anyone who takes teaching seriously, has more extensive ramifications than so far seen. It is a significant fact, for example, that the dichotomy between techniques of formal logic and the evaluation of concrete arguments is reflected both in the history of logic and in current research practices in logic. The two general histories of logic now widely used[18] both study logic as evolving through its own internal dialectic and pay little attention to its historical interaction with argument processes.

Moreover, it is interesting to note how seldom one sees the intervention by logicians in ongoing arguments in other fields. Mathematicians sometimes use mathematics to do so — one quickly thinks of splendid examples such as G. H. Hardy's classic letter to *Science* clarifying an argument in genetics, or von Neumann's famous corrective reformulation of the proof of equivalence of formalisms in quantum mechanics — but one strains to find instances where logicians have used the elaborate weaponry of formal logic in a similarly decisive manner. So perhaps it is fair to say that the attitude found in texts represents or parallels the attitude logicians take towards concrete arguments in their research, too. If so, this too is an interesting fact which needs more exploration.

1. These terms and phrases, and similarly those occurring in the next paragraph and throughout the paper, occur so widely in contemporary texts that documentation seems tedious and unnecessary except in selected cases.

2. Cp. J. van Heijenoort, "Preface" to *From Frege to Godel* (Cambridge, Mass.: Harvard University Press, 1967), p. vii, for this type of formulation.

3. On this point, compare the stronger position taken by Y. Bar-Hillel in "Argumentation in Pragmatic Languages," *Aspects of Language* (Jerusalem: Magnes Press, 1970).

4. These formulations are taken from I. Copi, *Symbolic Logic,* (4th ed., New York: Macmillan, 1973), pp. 3-4, but variants in other texts convey the same distinction.

5. For the argument against taking these as central, see S. Toulmin, *Uses of Argument* (Cambridge: Cambridge University Press, 1958).

6. *Elementary Logic* (2d ed., New York: Oxford University Press, 1972).

7. *Symbolic Logic* (New York: Harper and Row, 1970).

8. *Symbolic Logic* (New York: Macmillan, 1970), p. vi.

9. "Formal Logic and Natural Languages (A Symposium)," *Foundations of Language* V (1969), p. 256.

10. Toulmin, *op. cit.*

11. This idea runs through much of Georg Kreisel's work, from the early "On the interpretation of non-finitist proofs," *Journal of Symoblic Logic* 16-17 (1951-52), to the later "Survey of Proof Theory: II," in *Proceedings of Second Scandinavian Logic Symposium* (Amsterdam: North Holland Publishing Company, 1971). See also the *Collected Papers of Gerhardt Gentzen* (Amsterdam: North Holland Publishing Company, 1969) and Dag Prawitz, *Natural Deduction* (Stockholm: Almqvist and Wiksell, 1965); J. Hintikka, *Logic, Language-Games and Information* (Oxford: Oxford University Press, 1973).

12. (1st ed., 1950; 3d ed., New York: Holt, Rinehart & Winston, 1972).

13. *Deductive Logic* (Boston: Allyn and Bacon, 1972) and *Derivation and Counterexample* (Encino and Belmont: Dickenson, 1972) respectively.

14. C. S. Peirce, *Collected Papers* Vol. 2 (Cambridge, Mass.: Harvard University Press, 1960), sec. 208.

15. See the classic Hilbert-Ackerman text *Principles of Mathematical Logic* trans. of the 2d ed. *Grundzuge der Theoretischen Logik,* 1938 (New York: Chelsea, 1950).

16. See Morris Kline, *Why Johnny Can't Add* (New York: St. Martin's Press, 1973).

17. Michael Scriven, "Philosophy of Education: Learning Theory and Teaching Machines," *Journal of Philosophy* LXVII (1970), p. 901.

18. I refer to I. M. Bochenski's *History of Formal Logic* (Notre Dame, Ind.: University of Notre Dame Press, 1961), and W. and M. Kneale's *Development of Logic* (Oxford: Clarendon Press, 1962).