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ARITHMETIC AS PROPAEDEUTIC TO THEOLOGY: THE BRETHREN OF PURITY[*]

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Abstract:
In the 10th century, the Brethren of Purity conceived a henological arithmetic which they believed could explain the mathematical structure of the cosmos (as a macro-anthropos), and could lead the student to the discovery of the real substance of his own soul (as a micro-cosmos), a discovery which is the first step towards knowledge of metaphysical and theological truth.

Key words: Brethren of Purity, henology, theology, arithmetic, one.

Some years ago I was asked to review the Ikhwān al-Safā's (in English the "Brethren of Purity's") epistle *On Logic*.[1] At the time I already knew some Arabic logic by al-Fârâbi and Ibn Rushed, and I was anxious to know how a tenth century (fourth century in the Hegirian calendar) Ismailia esoteric fraternity of lettered urbanities would deal with such a technical subject as logic. It should be no surprise that the *Epistle on Logic* is a loose commentary on Aristotle's *Organon* which includes some Stoician doctrines, and for theological reasons passes over Aristotle's discussion of contingency and modality in the *De Interpretatione*.

But the *Epistle on Logic* is only one of fifty-two epistles (one for each week of the solar calendar). These fifty-two epistles constitute an encyclopedia of what a learned man—not a confirmed philosopher—of southern Iraqi is supposed to know in the tenth century.

Except for the rather deceiving *Etymologia* of Isodorus from Sevillia, nothing comparable existed at the time in the western Latin world. The avowed intention of the Brethren of Purity was:

> "… to study all the sciences of existent beings that are in the world, be they substances, accidents, abstract entities, simple or compound, and to inquire into the principles and the quantity of that species, kinds, and properties, and into their arrangement and order, as they are present, as well as into the process of their originating and growing out of one cause and of one origin, by one Creator…"[2]

In other words, they intended to present a complete henological cosmology originating from *one* cause, *one* origin, and *one* creator. But how to build such a cosmology? And how to start building it?

The Brethren of Purity distinguishes four kinds of philosophical sciences: "the first is the propaedeutic sciences, the second is the logical sciences, the third is the natural sciences, and the fourth is the theological sciences."[3] Here it

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[1] *Epistles of the Brethren of Purity*, *On Logic*, an Arabic Critical Edition and English Translation of the EPISTLES 10-14, Edited and Translated by Carmela Baffioni, foreword by Nader El-Bizri, Oxford UP, 2010.

[2] Epistles of the Brethren of Purity, *On Arithmetic and Geometry*, an Arabic Critical Edition and English Translation of Epistles 1 & 2 by Nader El-Bizri, Oxford University Press with The Institute of Ismaili Studies, 2012, p. 65.

[3] Id., *Ibid*., p. 65.

is important to make a comparison with philosophical-scientific categories in the medieval western Latin world. There, from Cassiodorus onward and later Lanfranc, the logical sciences are part of the *trivium* (grammar, rhetoric, and logic), and the disciplines of the *trivium* constitute, for the western Latin world, the propaedeutic sciences. In the 12th century, learning the disciplines of the *trivium* made you a Master of Art. After the *trivium*, the disciplines of the *quadrivium*[4] (arithmetic, geometry, astronomy, and music) may be learned. The *quadrivium* is not propaedeutic.

For the Brethren of Purity, contrary what would be thought in the later Latin world, the *quadrivium* is propaedeutic: "The propaedeutic sciences are of four kinds: the first is arithmetic, the second is geometry, the third is astronomy, and the fourth is music."[5] Arithmetic "…is the study of the properties of numbers and the qualities of the existents that conform to them, following what Pythagoras and Nicomachus[6] recorded in this regard,"[7] while geometry "…is the science of mensuration by means of proofs recorded in the book of Euclid."[8] Astronomy "…is the science of the stars,"[9] and music "…is knowledge of the composition of sounds."[10]

Arithmetic, they say, "…is the easiest science to acquire,"[11] an assertion few of my students would agree with. But we have to keep in mind that this arithmetic is propaedeutic to a henological cosmology and theology. As we shall see, the Brethren of Purity claimed that the world had been created in a mathematical way. To know this order leads you to discover the real substance of your soul, makes you capable of understanding the metaphysical truth, and—last but not least—makes you capable of understanding the theological truth.

But how can we build a henological arithmetic? The Brethen's first observation is that any number may be reduced to *one*—which is not a number—in the following way:

> "…if one is taken from ten, then nine remains; and if one is taken from nine, eight remains; and when one is taken from eight, seven remains; and similarly ones are taken away until only one remains."[12]

Once again we have to remember that arithmetic as here conceived is the ground for a cosmological henenology. The consequence is that the *zero*—an Indian invention which an Arabic mathematician will only later introduce into the Western world—cannot be conceived by the Brethren of Purity: "…nothing can be removed from one, because (by definition) a part cannot be taken from it."[13] A further consequence of this logic is that there are no negative numbers.

The Pythagorean numerology developed by the Brethren of Purity is dependent on the language they use. They make the observation that:

> "Twelve single words […] suffice to encompass all the names of the numbers, namely, [the numbers] from one to ten are covered by ten words; and one word, 'hundred'; and one word, 'thousand'; so there are twelve single words in all. The other names [for numbers] are derived from these, or combined from them, or are a repetition of them."[14]

This would not be true in either English or French. In French, for instance, the first ten numbers do not encompass the following

[4] As Nader El-Bizri writes in his "Introduction" that the classical *quadrivium* curriculum was "conceptually attributed to Pythagoras of Samos, and 'institutionally' elaborated by Anicius Manlius Severinus Boethius," p. 2.
[5] Epistles of the Brethren of Purity, *On Arithmetic and Geometry*, p. 66.
[6] Nicomachus of Geresa wrote an *Introduction to Arithmetic* (Nicomachi Geraseni Pythagorei Introductionis arithmeticae, ed. R. Hoche, Leipzig, Teubner, 1886) and the *Theological Principles of Arithmetic* (Theologumena arithmetcae, ed. V. de Faco, Leipzig, Teubner, 1922).
[7] Epistles of the Brethren of Purity, *On Arithmetic and Geometry*, p. 66.
[8] Id., *Ibid.*, p. 66.
[9] Id., *Ibid.*, p. 66.
[10] Id., *Ibid.*, p. 66.
[11] Id., *Ibid.*, p. 67.
[12] Id., *Ibid.*, p. 68.
[13] Id., *Ibid.*, p. 68.
[14] Id., *Ibid.*, p. 69.

numbers all the way up to sixteen (*seize*), neither do they encompass twenty (*vingt*), thirty (*trente*), and so on.

But there is another thing at stake: the last word for numbers is the word 'thousand'—there is no word for the *infinite*, nor for the *indefinite*. Did the Brethren of Purity have the notion of the infinite? In a way, like in the French translation of the *Arabic Nights* (*Les Milles et une Nuits*) the idea of an impossibly large number is captured by "a thousand-and-one."

In any case, as the Brethren of Purity have told us, any number can be reduced to *one*. In this way the relation of the unit *one* to all the numbers is like the relation of the Creator—or the platonician *en*—to all existence.[15] The Brethren now need to solve the classical problem in henology: how can we move from *one* to *multiplicity*? The solution is evident: "Plurality is an aggregate of ones," and "the first of the plural numbers is two,"[16] and "One, which precedes two, is the source and principle of all numbers…"[17] *One*—as the principle of all numbers—is not a number; *two* is the first of the numbers.

If *one* is the source and principle of all numbers, the number *four* is also a very important number: "…most of natural things were established by the Creator (…) in fourfold order."[18] In this way the number *four* is like prime matter.[19]

The various assertions of the Brethen's about the meaning of specific numbers has to be considered in relation to the science of their time and their geographic situation.

The first two lines of argument are taken from Aristotle's physics: there are "…four natures: heat, cold, dampness, and dryness"; then there are "…four elements: fire, air, water, and earth." The third is taken from antique medicine: there are "…four humors: blood, phlegm, and the two biles, yellow bile and black bile."

These three arguments are, of course, completely obsolete for a 21st century reader. From Lavoisier and Einstein to the discovery of neutrinos and Higgs bosons, we know that the real "natures" and "elements" are not what Aristotle thought they were. And modern medicine has nothing to do with the antique theory of "humors."

The fourth argument provides some clues about the region where the Brethren of Purity lived: there are "…four seasons: spring, summer, autumn, and winter." In the twelfth century, Ibn Tufayl, though he lived in Spain, knew that under the equator there were only two seasons, a dry one and a wet one. The notions of spring, summer, autumn, and winter depended on the place where you lived.

The fifth line of argument cannot even agree with the meteorological science of Herodotus: there are "…four winds: the east wind, the west wind, the south wind, and the north wind." Winds do not restrict their directions to the cardinal points. In Southern France the "Mistral" is a south wind, the "Tramontane" a south-east wind…

The sixth argument seems very Pythagorean: there are "…four orientations, upward, downward, right and left…" but another Pythagorean, the Jewish philosopher Philo of Alexandria, claimed in the first century PCN that the number of orientations proves the importance of the hexade (number *six*): men and animals can move upward, downward, right and left, but also forward and backward…

The seventh one argues that there are "…four *cardines*," which is still the basis of our geography. But these four cardines are "…the ascendant, descendant, *medium coeli*, and *imum coeli*…" This proves that the Brethren of Purity lived a long time before the invention of the compass. Their vision of the world was determined by the place where the sun is rising (like some old Arabic maps), and was not already northward directed.

The eighth and final argument is that there are "…four [sub-lunar] beings: metals, plants,

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15 Cf. Nader El-Bizri' "Introduction," p. 17.
16 Epistles of the Brethren of Purity, *On Arithmetic and Geometry,* p. 68.
17 Id., *Ibid.*, p. 68.
18 Id., *Ibid.*, p. 71.
19 Cf. Nader El-Bizri' "Introduction," p. 17.

animals, and humans." This eludes the long discussed theological question about devils and angels: are they sub-lunar beings or not?

But, whatever objections we may make to their arguments, we must consider that for the Brethren of Purity, the Creator had established natural things in a fourfold order. This is the reason why the number *four* is related to prime matter. As a consequence, they think that the set of numbers[20] from one to four (1=Creator, 2=Active agent, 3=cosmic universal soul, 4=prime matter) is the origin of all numbers, and that the Creator, the active agent, the cosmic universal soul, and prime matter are the origin of all existence in the sub-lunar world. Indeed, as all conceivable numbers could be named with only twelve entities (from one to ten, plus the numbers "hundred" and "thousand") in a perfectly Pythagorean combinatory way, the first four numbers are "the source of all the numbers." For example, five and six are mere additions: "…when you add one to four, the total is five," "…when you add two to four, the total is six," Let's take the case of seven: "…when you add three to four, the total is seven," You can proceed again in a perfectly combinatory way: "…when you add one and three to four, the total is eight." And again "…if you add two and three to four, the total is nine. Finally you come to the Pythagorean Tetractys: "…when you add one and two and three to four, the total is ten."[21] And, in order to name all the numbers, you just need two more words, "hundred" and "thousand."

Numbers have special properties:

One—which is not a number—is the source of all the numbers, generating all numbers, both odd and even.[22] But *one* itself is neither odd nor even.

The Brethren of Purity had a problem conceiving the infinite (Aristotle had similar problems), and they think that a special property of the number *two* is "…that it generates half of the numbers, the even numbers as opposed to the odd numbers."[23] In post-Cantorian mathematics, half of the infinite is still infinite. But we are not allowed to quarrel with the Brethren of Purity for not having read Cantor…

Another special property of the number *two* is that "…it is absolutely the first whole number."[24] The number *two* is like the Active Intellect.[25] Similarly the number *three* has the special property of being "…the first odd number"[26] (which incidentally means, just as we said a few lines ago, that *one* is not an odd number, nor of course an even number). Furthermore three has the special property that "…it generates a third of the numbers, some odd, some even."[27] A third of the infinite is still, in post-Cantorian mathematic, an infinite… Nevertheless the number *three* is like the cosmic universal soul.[28]

The Brethren of Purity conceived of the number *four* in a typically Neopythagorean way, that is to say, through spatialization: "A special property of [the number] four is that it is the first perfect square."[29] It is just as if the units were stones, and that you need to arrange them on the sand to see which geometrical figure they corresponded with. It is worth noticing that only two stones (in the Euclidian sense) can define a straight line, while with four stones you can make a square—but also a rectangle, a diamond, or any quadrilateral figure. The choice of a "perfect square" provides some clues about the way the Brethren of Purity saw the world (as a world with regularity and symmetry). The world is mathematical.

The number *five* also has some special properties: "it is the first circular number, also called 'spherical.'"[30] With five stones on the sand, you can draw an infinite number of figures. The two regular figures are the

[20] Assuming that *one* is not really a number.
[21] Epistles of the Brethren of Purity, *On Arithmetic and Geometry*, p 72.
[22] Id., *Ibid.*, p. 75.
[23] Id., *Ibid.*, p. 75.
[24] Id., *Ibid.*, p. 75.
[25] Cf. Nader El-Bizri, p. 17.
[26] Id., *Ibid.*, p. 76.
[27] Id., *Ibid.*, p. 76.
[28] Cf. Nader El-Bizri, p. 17.
[29] Epistles of the Brethren of Purity, *On Arithmetic and Geometry*, p. 75.
[30] Id., *Ibid.*, p. 76.

pentagon and the circle. As children learn at school, a perfect pentagon first requires the drawing of a circle with a compass, and then drawing the five heads of the pentagon. So the circle has a technical—and maybe an ontological—priority to the pentagon. So the Brethren's longing for regularity and symmetry may be the reason why the circle was the specific geometrical form of the number *five*.[31]

The number *six* has a very interesting special property: it is the first perfect number, in the sense that it is equal to the sum of all its divisors:

> "…a special property of the number six is that it is the first perfect number; namely, if the divisors of a number add up to itself, it is called a perfect number, and six is the first of them. Six has a half, which is three, and a third, which is two, and a sixth, which is one, and if these divisors are added up, the sum is equal to six. No number before six has this property, but after it is the number twenty-eight, then 496, and after that 8,128, and all are perfect numbers."[32]

So 6/2=3, 6/3=2, and 6/6=1, so 3+2+1=6. That proves that *six* is a perfect number. As an exercise you might try to prove that 28, 496, and 8,128 are also perfect numbers, and to discover the following ones.[33]

It is hard to find a special property of the number *seven*. It is a prime number, so it has no chance at all to be a perfect number. And the regular geometrical figure associated with it—the heptagon—is not easy to construct, even with a compass. Nevertheless, the Brethren of Purity think that the special property of *seven* is that "…it is the first complete number."[34] The Brethren of Purity were Shiites, but there are some references to the Jewish Law, as well as to the Holy Scriptures of Christianity and the Koran. In Genesis, God completed creation in six days, and on the seventh day he rested. The seventh day is a day of completion. Furthermore the very old notion of a seven-day week is determined by the 28-day lunar calendar. So there are four weeks (7×4=28) in a month, and each week is a complete entity. But there is another mathematical reason for *seven* to be complete: "If the first odd number is added to the second even number, or the first even number is added to the second odd number, the sum is seven."[35]

Accordingly 3+4=7 and 2+5=7. We may wonder whether there are any other complete numbers…

The number *eight* leads to a three-dimensional representation: the special property of the number *eight* is that "it is the first perfect cube."[36] As 8=4+4 (and 4 represents a perfect square), a perfect cube is drawn by two squares in two different—but symmetrical, equally distant—planes. But if you throw four stones on the sand, and you can imagine another plane of sand which is exactly parallel to the first one and upon which you then throw another four stones, are you sure to have a cube?

The Brethren of Purity just suppose that there is symmetry and regularity in the world. The world—as a macro-anthropos—is a mathematical creation.

This is even clearer with the first special property of the number *nine*, which "…is the first odd perfect square."[37] Indeed, nine is an odd number, and if you arrange three ranks of three stones (3x3=9) with perfect regularity and symmetry, you will get a perfect square.

But this again requires perfect regularity and symmetry. Once again the world is mathematical.

Nine has another special property: "…it is the last of the ranks of units."[38] Which means that the Brethren of Purity adopted a decimal

[31] I have not yet found an explanation for why *five* can be called "spherical," which supposes a third dimension that will appear only with numbers *eight* and *nine*.
32 Id., *Ibid.*, p. 77–78.
[33] Good luck! I finally succeeded with 28, but I prefer not to try with 436 and 8,128, and I'll let you discover the following perfect number by yourself.
[34] Epistles of the Brethren of Purity, *On Arithmetic and Geometry*, p. 76.
[35] Id., *Ibid.*, p. 78.
[36] Id., *Ibid.*, p. 76.
[37] Id., *Ibid.*, p. 76.
[38] Id., *Ibid.*, p. 76.

system and gave up the duodecimal system (which still bears some importance), as we will see with number *twelve*.

In my opinion, the reason for the importance of the number ten is very close to the reason found in Pythagoras' *Tetractys*. Its special property is just that "…it is the first number of the ranks of tens."[39] Once again, we are in a decimal system…

Eleven is a difficult case. As a prime number it seems it might not represent a perfect number. Furthermore it is not a complete number. We can hardly conceive a regular geometrical figure corresponding to it. So its special property is that "…it is the first deaf number."[40] In other words, the special property of *eleven* is that it has no special properties.

Finally, number *twelve* "…is the first abundant number."[41] But what does it mean to be an *abundant* number? As we said, every conceivable number has a name that is a combination of the names of the twelve numbers (from one to ten, then the numbers "hundred" and "thousand"). And even if we do not know whether the number of the numbers is infinite or not, the number of the numbers is still abundant. But there may be other reasons. The Brethren of Purity did not live far from old Babylon, with its duodecimal system. The Chaldeans composed its solar calendar year with twelve months, corresponding more or less to the cosmological signs of the Zodiac. The number of hours of daylight is twelve. And even as far as in Belgium, eggs are sold in 12-pieces boxes (in time of economic crisis—i.e. when you are not in abundance—you would buy six eggs, which is half of twelve, or if you really suffer from crisis, you would buy four eggs, which is a third of twelve). So *twelve* is a number of abundance.

Now we come to the central question. How can such a conception of arithmetic be propaedeutic to theology?

Man is a micro-cosmos, and the world is a macro-anthropos. When a man learns the mathematical structure of the world, he learns about himself as well. He learns about his own soul, its origin, its location before its embodiment, when it was joined with the body. The knowledge of numbers is the main door to knowledge of the soul, which is the prerequisite to knowledge of theological truth.

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[39] Id., *Ibid.*, p. 76

[40] Id., *Ibid.*, p. 76.

[41] Id., *Ibid.*, p. 76.