

COMING TO KNOW BY ASKING QUESTIONS: EXPLORING THE BORDERLINE OF LOGIC AND EPISTEMOLOGY*

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Abstract:

The paper explores the intricate interplay of two parallel developments: on the one hand, the Socratic turn in epistemology with its shifting focus on information retrieval, evidence-based reasoning, and the cognitive relevance of questions; and the advance of dynamic epistemic logic with its accent on knowledge-acquisition. Both are relevant for any realistic model of knowledge which pays due attention to learning. It is argued that the formal models are still wanting in some key respects, but the development of alternative and mutually complementing logical systems marks a promising trend for re-establishing the close links between epistemology and epistemic logic.

Key words: Socratic epistemology, dynamic epistemic logic, interrogative model of inquiry, evidence models, logic of questions.

1. At present, the relationship between epistemic logic and epistemology seems more worrisome than ever. These two subjects which were tightly knit half a century ago are now almost completely estranged. As Johan van Benthem reminds us, “epistemic logic started as a contribution to epistemology, or at least a tool in its *modus operandi* ... [but soon] became a sleepy backwater” (van Benthem 2006, 49–50). The “kiss of life” for the field was the intrusion of computer scientists who found that “human-oriented metaphors of knowledge, ignorance,

and communication turned out highly successful in understanding the behavior of complex interactive programs” (van Benthem, 50). This led to a gradual shift of interest to “logical dynamics”—the study of information update, communication, and interaction.

The exploration of such complex phenomena is impossible if we stick to the traditional divisions of philosophical logic into subfields as “modal logic,” “temporal logic,” “epistemic logic,” “doxastic logic,” “erotetic logic,” etc., since virtually “every meaningful task to be analyzed involves many of these things at once” (van Benthem, 58). The focus of a plethora of research contributions which appeared in the last decade are the “dynamic mechanisms that produce or modify knowledge and related epistemic attitudes like belief such as speech acts, communication, observation, learning, or even more radical belief revision” (van Benthem, 62). Since knowledge is a complex phenomenon which involves many different components (learning, questioning, inferring, etc.), it cannot be studied with the resources of pure-blooded epistemic logic (van Benthem, 63).

The focus of this paper is coming to know, i.e. learning. It turns out that questioning has a major role to play in this process. My aim shall be to trace the developments in epistemology which highlight the importance of questioning

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and locate the logical formalisms which are able to shed light on this process. Thus we can hope to locate some important but overlooked insights which are bound to play prominent role in the future.

1.1. The classical JTB account of knowledge equates it to justified true belief. Notwithstanding its noted flaws and shortcomings, most present day epistemologists strive to rectify it in some way or another. Still, there are radical approaches which aim to supplant the traditional definition with a more viable alternative. When we juxtapose the state of knowing on one hand and the process of acquiring knowledge (learning) on the other, one problem looms large: all three components of the JTB account are desiderata that, in principle, could be met while there is no way to ascertain this fact. These three desiderata are as follows.

(1) How are we supposed to find what X believes? There are circumstances when the overt behavior of X does not in any way correspond to what X believes or desires. Rational choice theory explored many different cases where such mismatch occurs. Of course, we could ask X what her beliefs are, but her answers could be inadvertently misleading. We have no privileged access to the content of our mental states and could not be expected to produce invariably true answers concerning their nature. (2) How are we supposed to find what is the truth? There are cases when the truth value of a particular claim is unknown. Imagine that X claims to know p. But is p true? If I know that p things seem pretty easy, but the prospects of finding out what is the case become rather bleak if I don't know p or even if I don't know that I know p. Thus, the existence of doxastic blindspots and the failure of the infamous KK-principle make the satisfaction of this second desideratum hard to ascertain. (3) How are we supposed to find whether X is justified to believe that p is the case? First, the standards for the required strength of justification that X applies could be completely different from our own standards. Second, it seems virtually impossible to find whether X believes p on the

basis of the justifying evidence which is accessible for X. The grounds for X's beliefs in general are not accessible even for X herself. Finally, should X be aware of the availability of this justifying evidence, or is its sheer availability enough?

All of these questions show that the traditional definition of knowledge does not correspond to an effectively applicable criterion for knowledge; so, the question of what knowledge *is* is completely separated from the question of what is in fact known. This state of affairs seems intolerable, so there are prominent attempts to find a way out.

1.2. The so-called “Socratic turn” in epistemology seeks to remedy the situation. Inaugurated by Jaakko Hintikka in the 1970s, its mature expression is provided in his book *Socratic Epistemology. Knowledge-Seeking by Questioning* (2007). The general idea of his approach is to render the process of knowledge-acquisition as effected by the exchange of questions and answers between two idealized agents (the student and the teacher, or the scientist and the nature). This is the basis of his “interrogative model of scientific inquiry,” according to which the research agenda (the questions posed) determines the outcome (the answers achieved).

According to Hintikka, when the logical positivists distinguished sharply between the context of discovery (the way we come to know) and the context of justification (the way we come to know that we know), they installed a completely distorted picture of the way science is practiced. Of primary importance is the way we achieve new knowledge, not the way we justify already available knowledge (Hintikka 2007, 20). Since the game of questions and answers effects an inflow of information, the principal concept of scientifically oriented epistemology is not knowledge but information, i.e. the worldly commodity on which our knowledge-claims are based. Since the question of knowledge relates to the domain-specific standards that determine which results of the process of belief formation are

reliable bases for action, it belongs to applied epistemology (Hintikka, 30). Moreover, knowledge is both domain-specific and context-sensitive, since to know p is to be able to eliminate the relevant alternatives of p on the basis of the available information and the success of this process of elimination depends not only on the amount of available information but also on the alternatives we are actually confronted with. Of primary importance is the fact that the alternatives are not to be seen simply as imagined variations in the obtaining state of affairs, they should be “real,” i.e. different states of affairs which in fact obtain in similar circumstances (Hintikka, 12). This gives to Hintikka's Socratic epistemology a noted externalist flavor.

Turning back to the traditional JTB account of knowledge, we can summarize Hintikka's critical comments in the following way: (1) what is important for epistemology is not the justification of already available knowledge but the accumulation of new knowledge on the basis of the available information; (2) information could be true or false, so the semantic properties of belief are irrelevant for the modeling of the process of knowledge-acquisition; (3) the attitudinal aspect of knowledge which is usually rendered as belief relates to the subjective (*a fortiori* relative) evaluation of the process of knowledge-acquisition and has nothing to do with general epistemology. In a nutshell this is a call for complete eradication of the JTB account and the venerable epistemological tradition which has grown around it.

1.3. Hintikka's way of thinking has greatly influenced a number of philosophers. One of them is Jonathan Schaffer, who developed the interrogative model into a full-fledged epistemological doctrine whose fundamentals were laid out in the award-winning paper “Knowing the Answer” (2007). For Schaffer, as for Hintikka before him, to know means to know the answer of a particular question. The question itself does not belong to the pragmatics of knowledge-attribution but is incorporated in the very semantics of the knowledge-claim.

According to Schaffer there are two types of propositional knowledge: (1) knowledge-that (which requires a factive complement and corresponds to the JTB account); and (2) knowledge-who/what/where/when/how/why/whether, or simply knowledge-wh (which requires an interrogative complement and corresponds to Hintikka's interrogative model of inquiry). According to the received reductive view (championed by Frege), the second type of propositional knowledge claims are reducible to the first which are basic: To know who has killed Kennedy is to know that Oswald is the murderer of Kennedy. This claim is substantial since the two types of knowledge-claims have different surface structure: knowledge-that is usually modeled as a binary relation between the knower and the known proposition, while knowledge-wh should be rendered as a ternary relation between a knower, a question, and the known answer to this question.

Schaffer has amassed considerable linguistic evidence which suggests that the reduction could be performed the other way around and in fact knowledge-that is simply a degenerate form of knowledge-wh, where the question is deemed irrelevant: (1) There are cases when the two types of expression are substitutable *salva veritate*: cf. “Moore knows that he has hands” and “Moore knows whether he has hands;” (2) we can construct expressions where the two types of knowledge-claims are coordinated: cf. “I know what time it is and that I have promised to make a dinner” where knowing what and knowing that appear on an equal footing; (3) it is common to say things like: “Before I asked myself why the sky is blue but now I know it.” In this case, the anaphoric reference clearly suggests that what is known is not simply a fact, but the answer of a particular question. These examples show convincingly that knowledge-wh is a genuine type of knowledge which is not simply a form of knowledge-that. This naturally raises the question what is the semantics of knowledge-wh claims. In fact, it seems that it can easily be distilled from the semantics of knowledge-that claims.

Indeed, Stalnaker has modeled propositional knowledge in the framework of possible-worlds semantics in the following way: take a collection of possible worlds W , a distinguished possible world w and accessibility relation R on W , where wRw' implies that w' is an epistemic alternative of w (i.e., what is known at w does not rule out the possibility that w' is the actual world). In this setting a proposition p is known at w iff p is true not simply at w but at all R -accessible worlds (i.e. at all epistemic alternatives of w). The same machinery can be applied for knowledge-wh: instead of starting with an accessibility relation R and defining on its basis the set $Q = \{w' | wRw'\}$ we can start directly with Q , which is interpreted as the domain of a particular question (the elements of Q represent the ways the world could have been which are consistent with the presuppositions of the question which corresponds to Q). Thus the question fixes a context for the knowledge-claim: the set of relevant alternatives which the knower should be able to rule out on the basis of the available evidence. To summarize Schaffer's theory, we may reiterate his principal insight: both knowledge-that and knowledge-wh are genuine but knowledge-wh is more fundamental since its tripartite structure cannot be recovered from the structure of knowledge-that.

§2. The short survey above has shown the importance of questions, information, and learning as the key to the proper understanding of knowledge. Therefore, it seems natural to assume that all these phenomena are important for the logic of knowledge, i.e. epistemic logic; so, it is of crucial importance to map their presence in this well-explored territory. The logic of knowledge appeared in the 1960s, dynamic logic – in the 1970s, logic of information started to grow in the 1980s, and the logic of questions reached its fully developed form in the 1990s. All of these different trends began to converge in the new millennium.

Hintikka inaugurated epistemic logic with his book *Knowledge and Belief: An Introduction to the Logic of the Two Notions* (1962). It was Hintikka again who paved the way for the

synthesis of epistemic and erotetic logic. In the paper “Interrogative Logic as a General Theory of Reasoning” (2002), he and his co-authors tried to pinpoint the strategic principles of deduction and interrogation which were presented as complementary activities in the process of discovery and learning. According to Hintikka, some kinds of knowledge statements can operate as desiderata (or presuppositions) for different kinds of questions. Let us take for example the question “Is it the case that S ?” Its desideratum can be expressed as $K(S \vee \sim S)$ and the possible answers of this question—respectively as KS and $K\sim S$. This seems interesting but what is missing from this formal framework is the very possibility to reason about the process of learning and to formalize the gradual accumulation of knowledge—Hintikka's framework is irreparably static.

2.1. A natural point of departure would be dynamic epistemic logic which is equipped with tools to formalize both processes and epistemic notions. A well-developed formalism of this kind is presented in Hans van Ditmarsch's “Prolegomena to Dynamic Logic for Belief Revision” (2005). The starting point of its exposition is the so-called doxastic epistemic frame – a pair of the form $(W, <^w)$, where W is a collection of possible worlds, and for each world w in W , $<^w$ is a plausibility ordering. When augmented with a valuation which assigns a subset of W to each atomic proposition in the formal language L , the frame gives rise to a model of L . Finally, a doxastic epistemic state would be a pair (M, w) , where M is a model and w is a distinguished element of W .

Obviously, the most important part of each model is the plausibility ordering. The formula $u <^w v$ could be interpreted to mean that, from the point of view of the information available at w , it is less probable that u is the actual world than that v is the actual world. The union of $\text{dom}(<^w)$ and $\text{cod}(<^w)$, Plaus^w , is the collection of those worlds which seem probable from the point of view of w . It is assumed that for each w , Plaus^w has a least element and can be embedded in a totally ordered set S (paradigmatically, this would be the set of natural numbers). It can be

shown that for each x in S , the embedding associates with $<^w$ an accessibility relation \rightarrow^x on W . These accessibility relations give rise to modal operators with the following semantics: $M, w \models \Box^x \phi$ iff for each w' in W , $w \rightarrow^x w'$ implies $M, w' \models \Box^x \phi$ (i.e. ϕ is true in all x -accessible worlds). The standard interpretation of $\Box^x \phi$ is as a degree of rational belief in the proposition ϕ (if $\Box^x \phi$ is satisfied in the model for each x in S , then ϕ is known, i.e. believed with arbitrary high degree of plausibility). This completes the exposition of the static part of the model.

In order to introduce dynamics in this setting we need to add to our language state transformers of the form $[\ast\phi]$, expressing binary relations between information states. The result of applying $[\ast\phi]$ to a formula ψ would be to evaluate ψ according to a different model with a different plausibility ordering. In order to clarify the construction of the alternative information states (M^\ast, s^\ast) , associated with $[\ast\phi]$, van Ditmarsch introduced so-called doxastic epistemic action models. These are triples of the form $A = (U, <, \text{pre})$, where U is a collection of basic epistemic actions, $\text{pre}(a)$ is a precondition for a , where a is an action a in U , and $<$ is a function which assigns to each action a in U a preference function $<^a$ (thus the action models are just mirror image of their doxastic counterparts – a point to which we shall return below). Now we have everything we need at our disposal.

Let us take now a doxastic epistemic model $M = (W, <^w, V)$, a doxastic epistemic action model $A = (U, <^a, V)$, and assume that for some w the precondition for the execution of a is satisfied: $M, s \models \text{pre}(a)$. The result of executing a in (M, s) would be $M^\ast A = (W^\ast, <^\ast, V^\ast)$, where W^\ast is the collection of all pairs (u, b) such that $M, u \models \text{pre}(b)$, $V^\ast(\phi)$ iff u belongs to $V(\phi)$, and $<^\ast$ is defined in an obvious way by pooling together the plausibility orders of M and A . It can be rigorously established that, defined in this way, the state transformer $[\ast\phi]$ respects some of the key properties listed in the AGM postulates for belief revision, as codified in the seminal paper of Alchourrón, Gärdenfors and

Makinson (1985). Nevertheless, several problems remain unsolved (and even unaddressed). First of all, the separation of the doxastic and the action component of the epistemic system seems unwarranted: beliefs are not idle representations but state transformations of the cognitive system (as made evident by the fact that the two parts of the model are isomorphic). Moreover, the framework does not capture the fact that the dynamism of belief is triggered by the available evidence which is accumulated by means of questions and answers. Finally, the system deals only with atomic acts of learning and so does not capture the fact that, in general, the process of learning is a sequence of basic acts of knowledge-acquisition.

The last defect was (at least to some extent) rectified in the paper “Dynamic Epistemic Logic and Temporal Modality” (2011). Its author, Audrey Yap, proposes to add a relation R_a for each action, so that $wR_a w'$ implies that $w' = (w, a)$ and $M, w' \models \text{pre}(a)$. By means of these relations, a new modal operator P_a is introduced by stating that $M, w \models P_a \phi$ iff there is a w' such that $wR_a w'$ and $M, w' \models \phi$. Intuitively, this means that ϕ was established on the basis of a before the state (M, w) was reached. Thus we get a system with a rudimentary history which is able to track the way through which some belief was reached.

2.2. An alternative approach which conceives available evidence as the trigger of knowledge dynamics is sketched by Johan van Benthem and Eric Pacuit in “Dynamic Logics of Evidence-Based Belief” (2011). The basic building blocks of their model are the so-called evidence models $M = (W, E, V)$: just as before, W is a collection of possible worlds, V is a valuation function, and E (the evidence relation) associates possible worlds with subsets of W . The intuition behind it is that wEX means that the information available at w is consistent with the propositions that hold at the elements of X . Moreover, it is postulated that the relation E satisfies the postulates of consistency (which means that for no w the empty set is associated with w by E) and triviality (which means that

each w is E -connected with W , i.e. the space of the available alternatives is known from the outset).

A pair of modalities can be introduced in this setting: $\Box\phi$ (with the interpretation “there is available evidence that ϕ is the case”) and $B\phi$ (with the interpretation “it is believed on the basis of the available evidence that ϕ is the case”). The semantical clause for $\Box\phi$ is simple: $M, w \models \Box\phi$ iff there is a set X with $w \in X$ such that for each w' in X , $M, w' \models \phi$. For the explication of $B\phi$, we need the concept of maximal family with the finite intersection property. A family F of subsets of W has the finite intersection property (FIP) iff $\bigcap F$ is nonempty; it has maximal FIP if it has FIP but no proper extension of F does. The condition for $B\phi$ is as follows: $M, w \models B\phi$ iff for each maximal FIP family F which is contained in $E(w) = \{w' \mid \exists X \subseteq W, w \in X \wedge w' \in X\}$ and for each w' in $\bigcap F$, $M, w' \models \phi$. These operators have relativized or conditional counterparts: $M, w \models \Box^\phi\psi$ iff there is an evidence set X in $E(w)$ such that the intersection of X with the support set of ϕ is nonempty and for any world w' which belongs to this intersection we have $M, w' \models \psi$; $M, w \models B^\phi\psi$ iff for each maximal FIP set F in $E(w)$ and each world w' which belongs both to $\bigcap F$ and the support set of ϕ , $M, w' \models \psi$. Thus both the availability of evidence and the believability of propositions based on this evidence can be relativized to prior knowledge.

As far as evidence dynamics are concerned, van Benthem and Pacuit propose four dynamic modalities expressing different operations on evidence models: evidence addition $[+\phi]$, evidence removal $[-\phi]$, evidence upgrade $[\uparrow\phi]$ and evidence combination $[\#]$. They act on the evidence relation in the following way: $E^{+\phi}(w) = E(w) \cup \{[\phi]_M\}$; $E^{-\phi}(w) = E(w) \setminus \{X \mid X \subseteq [\phi]_M\}$; $E^{\uparrow\phi}(w) = \{X \cup [\phi]_M \mid X \in E(w)\} \cup [\phi]_M$; and $E^\#(w)$, which is the smallest set that contains $E(w)$ and is closed under nonempty intersection. These operations affect both $\Box\phi$ and $B\phi$ (and their conditional counterparts) in a predictable way.

What is even more interesting is that van Benthem's evidence models can be put into one-

to-one correspondence with the doxastic epistemic models of van Ditmarsch (which were called “plausibility models” by van Benthem). A plausibility model is triple $M = (W, R, V)$, where R is reflexive and transitive. The evidence model generated by M is $EV(M) = (W, E, V)$, where E is the collection of all nonempty downward- R -closed sets. In the other direction, given an evidence model (W, E, V) , we can construct a plausibility model $ORD(M) = (W, R, V)$, where wRw' iff for each X in E , if w' belongs to X , then w belongs to X . Moreover, the following pair of equalities hold: (a) $ORD(EV(M)) = M$, $EV(ORD(M)) = M^\#$, where $M^\#$ is produced by evidence combination. Thus a faithful translation between the two alternative conceptual frameworks can be constructed (to the best of my knowledge this possibility has not been fully explored).

2.3. Up to this point, the role of questions in the process of accumulating evidence has not come to the fore. They become topical in the paper “Toward a Dynamic Logic of Questions” (2012). Johan van Benthem and Stefan Minica introduced in this paper a new framework for the analysis of knowledge-claims—the epistemic issue model. An epistemic issue model is a quadruple $M = (W, \sim, \approx, V)$. The newcomers, \sim and \approx , are equivalence relations. When $w \sim w'$ we shall say that w and w' are epistemically indistinguishable, which means that what is known is compatible (or incompatible) with w and w' . Analogously, when $w \approx w'$ we shall say that w and w' are conceptually indistinguishable, which means that the questions posed so far provide no occasion to discriminate between w and w' . For example, if I know just that B is between A and C , I cannot say whether A is on the left (case w) or A is on the right (case w')—these cases are epistemically indistinguishable from my point of view. In the same way, if I ask whether B is between A and C , the positive answer of this question does not provide information whether A is on the left or A is on the right—the aforementioned cases are conceptually indistinguishable. On the basis of these relations

we can introduce a pair of modal operators—the well-known knowledge modality K and the issue modality Q : $M, w \models K\phi$ iff for each $w', w \sim w'$ implies $M, w' \models \phi$ and $M, w \models Q\phi$ iff for each $w', w \approx w'$ implies $M, w' \models \phi$. As noted in Baltag, Boddy, and Smets (2018), the issue modality is intended to capture the information carried by the answers of all “active” questions.

Now let us put $R(\phi, M) = \{(w, w') \mid [\phi]_w = [\phi]_{w'}\}$. On the basis of this relation between formulas and models, we can introduce four different operations on the basic epistemic issue model: (a) Questioning: $M_{\phi?}$ with $\approx_{\phi?} = \approx \cap R(\phi, M)$; (b) Announcement $M_{\phi!}$ with $\sim_{\phi!} = \sim \cap R(\phi, M)$; (c) Refinement: $M_{?}$ with $\approx_{?} = \approx \cap \sim$; and (d) Resolution $M_{!}$ with $\sim_{!} = \sim \cap \approx$. Just as before, these state transforming operations generate corresponding dynamic modalities which interact with the static knowledge and issue modalities. For example, it can be proved that $[\phi?]K\psi$ is equivalent (with respect to the semantics above) to $K[\phi?]\psi$. This particular equivalence means that you can come to know ψ simply by asking the question whether ϕ only if you know that ψ would be the case if you ask the question whether ϕ . The most interesting feature of the epistemic issue models is the interaction between the relations \sim and \approx . In the paper that I've just mentioned, Baltag, Boddy, and Smets greatly simplify the model by assuming that \approx is included in \sim , i.e. that if a pair of worlds are conceptually indistinguishable then they are epistemically indistinguishable, which implies that questions function as “epistemic filters”: each cognitive agent can know only the answers of her questions—just as Schaffer insisted (Schaffer's work in epistemology and van Benthem's work in logic are the chief sources of the formal system presented by Baltag, Boddy, and Smets).

§3. The short overview above shows that the Socratic turn in epistemology and the dynamic turn in epistemic logic are closely allied developments. Much remains to be done before we have at our disposal a logical system which is able to model the real-world process of learning in its full complexity. Nevertheless, the systems we have just explored strengthen the

conviction that, in order to obtain a firm conceptual grip on knowledge, we need to come to terms with a host of interrelated cognitive phenomena. This would probably force us into even fewer explored territories, as suggested by the promising new trend of inquisitive semantics which has also entered the domain of epistemic logic (see Ciardelli 2014). What it suggests is that in order to better understand these phenomena, we shall need to augment and modify one of our most basic concepts—the concept of proposition. Although such radical approaches are beyond the scope of this paper, their sheer availability shows what is at stake. Time shall judge what is the best way to take.

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