ΠΑΙΔΕΙΑ

Modern Philosophy

Leibniz's Theory of Space in the Correspondence with Clarke and the Existence of Vacuums (1)

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ABSTRACT: It is well known that a central issue in the famous debate between Gottfried Wilhelm Leibniz and Samuel Clarke is the nature of space. They disagreed on the ontological status of space rather than on its geometrical or physical structure. Closely related is the disagreement on the existence of vacuums in nature: while Leibniz denies it, Clarke asserts it. In this paper, I shall focus on Leibniz's position in this debate. In part one, I shall reconstruct the theory of physical space which Leibniz presents in his letters to Clarke. This theory differs from Leibniz's ultimate metaphysics of space, but it is particularly interesting for systematic reasons, and it also gave rise to a lively discussion in modern philosophy of science. In part two, I shall examine whether the existence of vacuums is ruled out by that theory of space, as Leibniz seems to imply in one of his letters. I shall confirm the result of E. J. Khamara ("Leibniz's Theory of Space: A Reconstruction," *Philosophical Quarterly* 43 [1993]: 472-88) that Leibniz's theory of space rules out the existence of a certain kind of vacuum, namely extramundane vacuums, although it does not rule out vacuums within the world.

Introduction

It is well-known that a central issue in the famous debate between Gottfried Wilhelm Leibniz and Samuel Clarke is the nature of space. Leibniz and Clarke, who did not only take a Newtonian standpoint, but was even assisted in designing his answers to Leibniz by Sir Isaac Newton himself, (2) disagree on the ontological status of space rather than on its (geometrical or physical) structure. Closely related to the disagreement on the ontological status of space is a further disagreement on the existence of vacuums in nature: While Leibniz denies it, Clarke asserts it.

In this paper I shall focus on Leibniz's position in the debate about these issues. In the first part I shall try to reconstruct the theory of physical space which Leibniz presents in his letters to Clarke. In the second part I shall examine, whether the existence of vacuums is ruled out by that theory of space, as Leibniz seems to imply in one of his letters (see below).

To focus exclusively on the correspondence with Clarke is a confinement I am aware of. The theory which I am going to reconstruct differs from Leibniz's ultimate metaphysics of space, (3) but it is particularly interesting for systematic reasons and it also gave rise to a lively discussion in modern philosophy of science.

Leibniz's Theory of Space in the Correspondence with Clarke

In his letters to Clarke Leibniz trys to "confute the fancy of those who take space to be a substance, or at least an absolute being." (4) To this end, he does not only try to point out the incoherence of Newton's idea of absolute space, but also gives his own positive account of space. As he repeatedly states, the word 'space' denotes, "in terms of possibility, an order of things, which exist at the same time, considered as existing together." (5) Moreover, space is a mere "ideal thing." (6) But what does that mean?

In his fifth letter, par. 47, which starts with his explanation of "how men come to form to themselves the notion of space," Leibniz later on gives "a kind of definition":

> ... *place* is that, which we say is the same to A and, to B, when the relation of the co-existence of B, with C,E,F,G, etc. agrees perfectly with the relation of the co-existence, which A had with the same C,E,F,G, etc. supposing there has been no cause of change in C,E,F,G, etc. It may be said also, without entering into any further particularity, that *place* is that, which is the same in different moments to different existent things, when their relations of co-existence with certain other existents, which are supposed to continue fixed from one of those moments to the other, agree entirely together. And *fixed existents* are those, in which there has been no cause of any change of the order of their co-existence with others; or (which is the same thing,) in which there has been no motion. Lastly, *space* is that, which results from places taken together.

As has been noticed by several commentators, (7) the general procedure Leibniz adopts here is as follows: First, he defines the relation *B is at time t_2 at the same place as A has been at time t_1* in terms of "relations of co-existence" of A and B to different physical objects C,E,F,G, etc., which are supposed to have been "fixed" from t1 until t2 (or from t_2 until t_1). Since the relation of being at the same place is an equivalence relation, he can define places as equivalence classes under this relation. (8) Finally, he defines space as the set of all places.

Although the general strategy thus seems to be quite clear, some basic questions are still unanswered:

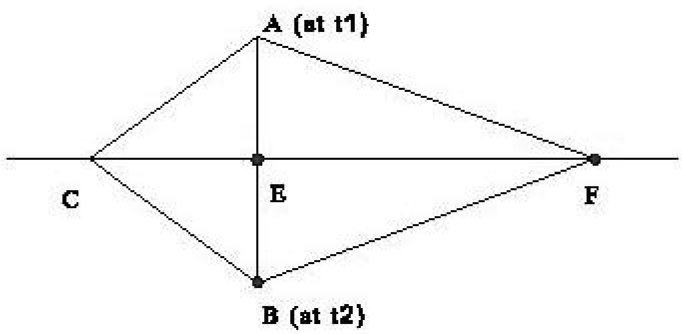
> (a) Which "relations of co-existence" between A and B, on the one hand, and the "fixed" objects C,E,F,G,etc., on the other hand, did Leibniz have in mind?
>
> (b) How many "fixed" objects are necessary to determine the unique place of A at t_1 and B at t_2, respectively?
>
> (c) Is it possible to explain what it means that C,E,F,G, etc. are "fixed" without referring to places in space?

I shall now try to answer these questions, in order to explain the theory in more detail. However, I do not claim that thereby all problems of the theory will be solved.

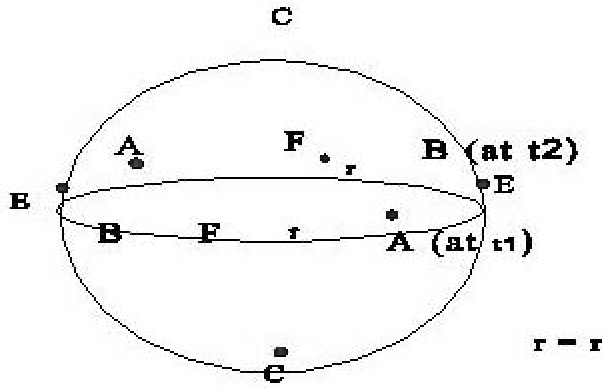
(*ad a*) In order to explain "how men come to form to themselves the notion of space," Leibniz starts by saying that "[t]hey consider that many things exist at once and they observe in them a certain order of co-existence, according to which the relation of one thing to another is more or less simple. This order, is their *situation* or distance." However, as the reader is told later, the "situation" of an object is not a relation, but a mere attribut or affection of that object. (9) Thus, one might think that Leibniz thought of distances as the relevant "relations of co-existence" and, presupposing only distances between all physical objects, had in mind the following definition of being at the same place: *B is at t_2 at the same place as A is at t_1 iff B has at t_2 the same distances to "fixed" objects C,E,F,G, etc. (which are distinct from A and B) as A has at t_1.*

Of course, this would be an appealing idea, if it worked, since the only structures we would have to presuppose in constructing space are (measurable) distances between physical objects in time, *i.e.* a distance function on physical objects (and instances of time). Unfortunately, not every distance function suffices to yield an adequate concept of being at the same place. (10) A trivial example for a distance function which is insufficient, is the so-called discrete metric d_D. (11) This function does not even determine the unique place of objects A, B *at the same time t_1*, if A has at t_1 the same distances to distinct "fixed" objects C,E,F,G, etc. as B has (since $d_D(A,C)=d_D(B,C) \wedge d_D(A,E)=d_D(B,E) \wedge ...$ does not imply dD(A,B)=0). Hence, it does not determine the unique place of such objects A, B *at different times t_1,t_2* at all. Consequently, Leibniz has to presuppose either, that the distances between physical objects define a stronger distance function, or more complicated *geometrical* "relations of co-existance." An example for a stronger distance function would be a so-called Euclidean metric.

(*ad b*) Even with a Euclidean metric we are in danger to identify different places, if we do not consider a sufficient number of "fixed objects" which satisfy a certain condition, as the following example shows: Let us take three "fixed" objects C,E,F in a Euclidean plane, which are on the same straight line. Let us suppose, the distances between an object A, which is distinct from C,E,F, and C,E,F are the same as the distances between an object B and C,E,F. Then, the following situation may occur:



If we say that B is at the same place as A, since A has the same distances to C,E,F, as B has, we obviously identify different places. However, this would not be possible, if C,E,F were not on the same straight line. But, it would have been possible as well, if we had considered only two "fixed" objects (*e.g.* C and E). (12) -Whereas in a Euclidean plane we need three "fixed" objects which are not on the same straight line, in the projective plane **P2** (which is a simplification of the unit sphere **S2**) even three "fixed" objects, which are not on the same line, are insufficient. This is exemplified by the following picture:



Consequently, the number of "fixed" objects, which is needed to gain an adequate concept of being at the same place solely by means of distances, depends on the distance function which is defined by the actual distances between physical objects.

(*ad c*) The word 'fixed', however, cannot simply mean 'not in motion', since motion for Leibniz is always relative motion. There may be cases in which we suppose that objects C,E,F,G, etc. continue to be "fixed" during a certain time period, although the distances of C,E,F,G, etc. to some object W change. Bertrand Russell who assumed that Leibniz understood that fact and therefore confused his theory with a reference to absolute motion, wrote in his book on Leibniz:

> [W]e may accept the following definition: 'Place is that which is the same in different moments to different existent things, when their relations of coexistence to certain other existents ... agree entirely together.' But when he [i.e., Leibniz] adds that these other existents 'are supposed to continue fixed from one of these moments to the other,' he is making a supposition which, on a relational theory, is wholly and absolutely devoid of meaning. (13)

However, as Edward J. Khamara has pointed out, for Leibniz an object C may change its distances to other objects and still be "fixed" if the *cause* for the change of distances does not lie in C. (14) For example, a man who stands at a traffic light and is passed by a car, continues to be "fixed" although his distances to the car change constantly. Thus, Leibniz's theory of space needs to be supplied by an adequate *causal theory of motion* which I cannot discuss here. (I do not even claim that there is such a theory.) But, though such a theory is a necessary supplement to Leibniz's theory of space, it is not part of the core of that theory, as Khamara has plausibly argued. (15)

Leibniz's Theory of Space and the Existence of Vacuums

So far we have seen, that for Leibniz space is an abstraction from distances between physical objects in time, which is made up by the human mind and thus only ideal. To be precise, it is a set of equivalence classes of ordered pairs consisting of physical objects and instants of time (not just a set of equivalence classes of physical objects, since an object which moves from one place to another would then be a member of different equivalence classes). Each equivalence class, called place, contains at least one pair. Therefore, only that can be considered to be a place, where a physical object has been at some time t. Without physical objects and instants of time there are no places and thus no space. However, it is not necessary, that a place contains an ordered pair ⟨A,t⟩ (where A is a physical object) *for every instant of time t*. There may have been times which are not represented in the ordered pairs of a place, meaning that there have been no objects at that place at those times or, in other words, that the place was void at those times. And, if there may have been or may be void places at a time t, there may as well have been or be collections of such places at t. Hence, the existence of vacuums is not generally ruled out by Leibniz's theory of space. To the contrary, the theory even provides us with a way to define exactly the notion of a void place and the notion of a vacuum: *A place P is void at a time t iff there is no physical object A such that* $\langle A,t\rangle \in P$*; a space V is a vacuum at a time t iff every place* $P \in V$ *is void at t.*

To put the matter in a slightly different way, like Khamara did, we may say: Leibniz's general denial of the existence of vacuums does not belong to the core of his theory of space. (16) But, unlike Khamara, I do not believe that Leibniz's reasons for denying the existence of vacuums were only theological. There are some passages in the correspondence with Clarke in which Leibniz distinguishes different kinds of vacuums, and there is at least one passage in which Leibniz seems to deny the existence of a certain kind of vacuums referring to his theory of space as a reason:

> I have demonstrated, that space is nothing else but an order of the existence of things, observed as existing together; and *therefore* the fiction of a material universe, moving forward in an empty space cannot be admitted. (17)

Although we already know that the existence of vacuums is not *generally* ruled out by Leibniz's theory of space, we can take this passage as a reason to ask whether certain kinds of vacuums are ruled out by that theory. Leibniz himself distinguishes two kinds of vacuums: (a) vacuums within the world, and (b) extramundane vacuums. (18) Unfortunately, Leibniz does not tell us what the difference is between vacuums within the world and extramundane vacuums (at least not in the correspondence with Clarke). But, on the basis of what has been said in the first part of this paper, I am able to explain that difference, thus reducing it to a distinction between void places within the world and void extramundane places: *A void place P is within the world, iff for all pairs* $\langle A,t\rangle \in P$ *the object A is a proper part of the universe*. As we have seen above, there is no problem with the existence of such places. However, *a void place P is extramundane, iff there is a pair* $\langle A,t\rangle \in P$ *such that A is either the whole universe or a physical object which is neither identical with nor part of the universe*. The latter alternative is ruled out by the definition of the universe: All physical objects belong to the universe, so that there can be no physical objects which are neither identical with nor parts of the universe. The former alternative, however, is ruled out by the definition of the universe in combination with the definition of the concept of being at the same place: (19) There can be no "fixed" physical objects distinct from the universe which can serve as a reference frame for determining the place of the universe at the time t.- Therefore, we can conclude that although the existence of vacuums is not ruled out in general by Leibniz's theory of space, there is a certain kind of vacuums whose existence is ruled out by that theory, namely extramundane vacuums. This is probably what Immanuel Kant had in mind when he wrote in the observation on the antithesis of the first antinomy of pure reason:

> The proof of the infinitude of the given world-series and of the world-whole, rests upon the fact that, on the contrary assumption, an empty time and an empty space must constitute the limit of the world. I am aware that attempts have been made to evade this conclusion by arguing that a limit of the world in time and space is quite possible without our having to make the impossible assumption of an absolute time prior to the beginning of the world, or an absolute space extending beyond the real world. With the latter part of this doctrine, as held by the philosophers of the Leibnizian school, I am entirely satisfied. (20)

Notes

(1) I would like to thank Eric Watkins and Rainer Noske for valuable comments and suggestions.

(2) *Cf.* A. Koyré/I.B.Cohen (1962)

(3) For an account of Leibniz's ultimate metaphysics of space as well as its development see, for example, M.Gueroult (1946), A.T.Winterbourne (1982), and G.A.Hartz/J.A.Cover (1988).

(4) Leibniz's third letter, par. 5. In quoting from the Leibniz-Clarke Correspondence I follow the text of H.G.Alexanders edition (see *Bibliography*).

(5) Leibniz's third letter, par. 4. *Cf.* his fourth letter, par. 41, and his fifth letter, par. 29.

(6) Leibniz's fifth letter, par. 33; see also par. 104.

(7) See, for example, A.T.Winterbourne (1982), p.203, K.L.Manders (1982), p.578ff., E.J.Khamara (1993), p.474 and *passim,* and R.Athur (1994), p.237.

(8) This interpretation is confirmed by the following passage from the same letter and paragraph: "I have here done much like Euclid, who not being able to make his readers well understand what *ratio* is absolutely in the sense of geometricians; defines what are the *same ratios*. Thus, in like manner, in order to explain what *place* is, I have been content to define what is the *same place*."

(9) *Cf.* A.T.Winterbourne (1982), p.203

(10) I will assume here that a distance function d assigns a non-negative real number to every pair of objects and satisfies the following conditions: (i) $d(x,y)=0$ iff $x=y$, (ii) $d(x,y)=d(y,x)$, and (iii) $d(x,y)\leq d(x,z)+d(z,y)$.

(11) dD is defined as follows: $dD(x,y):=0$, if $x=y$, and $dD(x,y):=1$, if $x\neq y$.

(12) *Cf.* E.J.Khamara (1993), p.479.-Generally, if one wants to construct n-dimensional Euclidean space in the way described above, one needs n+1 "fixed" objects which do not lie within the same n-1-dimensional subspace. Yet, what it means for objects C,E,F,G, etc. to lie in a m-dimensional space can be defined in terms of distances between C,E,F,G, etc. Hence, an analysis of the distances between all physical objects could *in principle* provide one with the knowledge of how many dimensions space has, which one at least needs to know in order to postulate a sufficient number of "fixed" objects when defining the concept of having the same place.

(13) B.Russell (1951), p.121; see also pp.84-7.

(14) *Cf.* E.J.Khamara (1993), p.474

(15) Op. cit., p.481

(16) See E.J.Khamara (1993), p.480.-Yet, Leibniz may have had a different opinion; in par. 62 of the his fifth letter he says: "I don't say that matter and space are the same thing. I only say, there is no space, where there is no matter." For Leibniz's denial of the existence of vacuums see also his fourth letter, par. 4 and 7-8, and his fifth letter, par. 33-35.

(17) Leibniz's fifth letter, par. 29 (italics added by the author).

(18) *Cf.* Leibniz's fifth letter, par. 33

(19) It is important to consider this alternative since according to Clarke and the Newtonians the universe can move through (absolute) space and, therefore, have been at

places which are void later on; see the passage from par. 29 of Leibniz's fifth paper quoted above.

(20) *Critique of Pure Reason* A431/B459

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