ΠΑΙΔΕΙΑ

Logic and Philosophy of Logic

# On Universal Grammar and its Formalization

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**ABSTRACT:** This paper sketches or signals some ideas, results, and proposals connected with the theoretical issues related to the categorial approach to language which originated from the first author (1985, 1989, 1991, 1998) and which form the basis for further research by the second author. The main aims are the following: 1) to bring into common use some Polish ideas concerned with classical categorial grammar; 2) to take into consideration a universal and simultaneously formal-logical perspective; 3) to consider Peirce's well-known differentiation of linguistic objects, i.e. their twofold ontological status as *tokens* (concretes) and *types* (abstract objects) and, according to this, to consider the biaspectual formalization of language dealing with the two main orientations in the controversy between nominalism and Platonism; 4) to characterize language according to Frege's ontological canons, according to which each expression of language corresponds to its denotation. All of these factors make possible not only the syntactic characterization of language but also the introduction of syntactic and semantic definitions of a true expression and its denotation. These notions correspond here to the old classical, but not necessarily standard, understanding of semantic concepts. The paper is divided into four sections: the first contains a brief characterization of the categorial approach to syntax; the second presents two strains of this approach; the third touches on certain general semantic issues connected with the notion of truth; and the last gives some final remarks.

## 1. General characteristics of the categorial approach to syntax

At the beginning of this paper, to avoid misunderstanding, we explain that considerations concerning universal grammar should be understood as a theoretical and very general, formal-logical approach to the logic of language. The paper treats logical syntax and logical semantics, conceptualized as a theory providing general principles for generating languages from the so-called classical categorial grammar. In this section we outline some universal ideas of such a *categorial approach.*

### 1.1. Some ideas concerning logical syntax of language

The first ideas pertaining to the formalization of language syntax appear only in the Twentieth Century and have been provided by Rudolf Carnap (1934). The term 'logical syntax' introduced by Carnap is understood here in the narrower sense, namely as a field of the logic of language to which belong issues connected with the classification of expressions and their syntactic structure. We are mainly interested in the formalization of the *categorial approach* to logical syntax, which can be regarded as a formal theory of logical syntax elaborated in the spirit of the Polish tradition, which is here associated with: (1) Leśniewski-Ajdukiewicz's theory of semantic categories, known today as the theory of syntactic categories, and (2) Tarski's axioms for metascience.

As for theory (1), it was built by Leśniewski, not without the influence of Husserl's idea of pure grammar (1900-1901), for the languages of his protothetics (1929) and ontology (1930) systems. Ajdukiewicz (*Die syntaktische Konnexität*, 1935) significantly improved it with the help of index-assignation. Leśniewski-Ajdukiewicz's theory can be regarded as a theory for the classification of linguistic expressions and takes into consideration some factors according to which compound expressions should: a) have unique categorization, like simple words; b) have functor-argument structure; c) be reducible to atomic categories; d) satisfy the principle of syntactic connection; and (e) satisfy the rule of substitutability.

Leśniewski-Ajdukiewicz's theory is not, however, a formal theory, while Tarski's axiomatic approach to metascience (2) — which can be found in his famous paper on the concept of truth (1933) — provides the first formal foundations of metascience and thus also metalanguage in accordance with Leśniewski and Ajdukiewicz, Łukasiewicz and Post's ideas. This approach lay the groundwork for the first axiomatic, deductive base for language syntax and recursive grammar. In order to present such a grammar, Tarski gives the first deductive theory of strings in which the *relation of concatenation* of strings is a primitive concept characterized by axioms and serves to generate concatenations of strings from the vocabulary of a given language. This theory points a way for a formalization of the general theory of classical categorial grammar used by the first author.

The theory of Wybraniec-Skardowska (1985, 1991, 1998) not only explicates certain ideas of the numerous researchers of the theory of syntactic categories including Bocheński (1947), Hiż (1960, 1967, 1968), Kubiński (1960), Geach (1971), Cresswell (1973, 1977), Marciszewski (1977, 1988), van Benthem (1984, 1988), Buszkowski (1989, 1997) but as an axiomatic theory it contains a comprehensive formal formulation which adheres to the original assumptions of approach (1). They are slightly different than in Bar-Hillel (1950) and Lambek (1958, 1988) or in the combinatory logic of Curry (1961, 1963). Let us observe that although Bar-Hillel coined the term 'categorial grammar' neither he nor his co-workers took into account the mentioned aspects (a) - (e) of the theory of syntactic categories because it has been adopted for computer language orientations; the syntactic description of language expressions has been replaced by the mechanical procedure of determining syntactic structures. Wybraniec-Skardowska's theory can be regarded as a formal theory in the certain sense of a universal grammar referring to the categorial approach. It will be denoted by ***TCG*** and formalize very generally both approaches (1) and (2) to categorial syntax and gives new proposals to categorial semantics. The syntactic researches initiated by the first author were inspired by her teacher, Jerzy Słupecki, a representative, like the famous Alfred Tarski, of the Warsaw Logical School; and the semantic ones were inspired by Suszko (1958, 1960, 1964), Lewis (1970), Montague (1970, Stanosz and Nowaczyk (1976), van Benthem (1984,1986, 1988), Buszkowski (1989) and Andrzej K. Rogalski. Let us however notice that any way to a formalization of semantic problems was opened by Tarski's famous paper (1933) on the notion of truth.

### 1.2. Certain conventions pertaining to language

Prior to presenting general ideas of the theory of categorial grammar it seems indispensable to define the following conventions:

- language is characterized mostly syntactically and only partially semantically;
- language analysis will not concern spoken language;
- language is considered and formalized as a construct of a double ontological nature: as the language of *tokens* (at the *token* level) and the language of *types* (at the *type*-level), according to the distinction *types-tokens* of Peirce (1931-1935);
- *tokens* are understood as concrete, material, empirical objects perceived by sight and may be inscriptions — but do not necessarily have to be inscriptions — on a paper, a notice-board, a sign-board, a stone, etc.; they may be some configurations of such things as stones, leafs and even stars, or smoke signals, or illuminated advertisements, or the so-called "live pictures" during Olympic ceremonies or other shows, and so on;
- *types* are understood as sets of *tokens* established by an *indiscernibility* (*equiformity*) *relation*, *i.e.* as some abstract beings; *tokens* are some physical realizations or representatives of *types*;
- the *relation of indiscernibility* (*equiformity*) of *tokens* is determined by some pragmatic objectives and not by physical similarity, and can be understood very broadly; we will assume that indiscernibility is an equivalence relation;
- the formal-logical characterization of language comprises two levels of considerations: the *token*-level and the *type*-level; it assumes the set-theoretical formalization;
- the syntactic characterization of language considers approach (1) and (2) referring to the theory of syntactic categories of Leśniewski-Ajdukiewicz on the one hand, and to Tarski's axiomatization of metascience on the other;
- the above-mentioned characterization of language allows us to treat it as language generalized by a classical categorial grammar and its aim is to generate *concatenations* from a vocabulary of a given language that would be its *functor-argument expressions* (see b)) and to assess which of them are well-formed ones, using index-assignation and the principle of syntactic connection (see d));
- an analysis of the syntactic correctness of a functor-argument expression, i.e. testing whether the principle of syntactic connection holds for it, should lead to creating an algorithm;
- all well-formed expressions of language are assigned to suitable syntactic categories by means of indices, *i.e.* expressions with the same or indiscernible indices have the same categorization (see a)).

Such characterization of language is essential because:

> (i) it takes into account the common linguistic practice both in syntax and semantics — on the *type*-level, and in pragmatics — on the *token*-level (in pragmatics this practice consists in the functionality of language, the use of *tokens* in linguistic contexts in the process of communication, in syntax by means of *types* we formulate rules of the grammatical correctness of expressions, and in semantics for *types* we can define such terms as "denotation" and "truth"),
>
> (ii) it allows us to solve some semantic and philosophical problems;
>
> (iii) in connection to the ideology of Frege's ontology and Suszko's (1958, 1960, 1964) ideas anticipating researches in categorial semantics, this characterization

gives a new, not necessarily standard view on how to develop categorial semantics in the most general manner.

### 1.3. Intuitive foundations of *TCG*

Let us consider the intuitive background of the theory ***TCG*** as a theory of categorial syntax, and the same theory of categorial grammar. A strict and axiomatic presentation of the theory in the space of this paper is impossible. Thus we refer the reader to the book (1991) and paper (1989) of the first author (cf. also1998).

The simplest syntactic characterization of any language $L$ gives the following ordered system:

($L$) $< U_{\mathrm{L}}, \sim, V_1, \mathrm{c}, W_1; S >$

consisting of:

$U_L$ - the *universe* class of all linguistic objects (broadly understood inscriptions);

~ - the *indiscernibility* (equiformity) relation in $U_L$ given on the *token*-level;

$V_1$ - the *vocabulary* of all simple words as a subset of $U_L$;

$c$ - the ternary relation of the *concatenation* on $U_{\mathrm{L}}$ to generate from $V_1$:

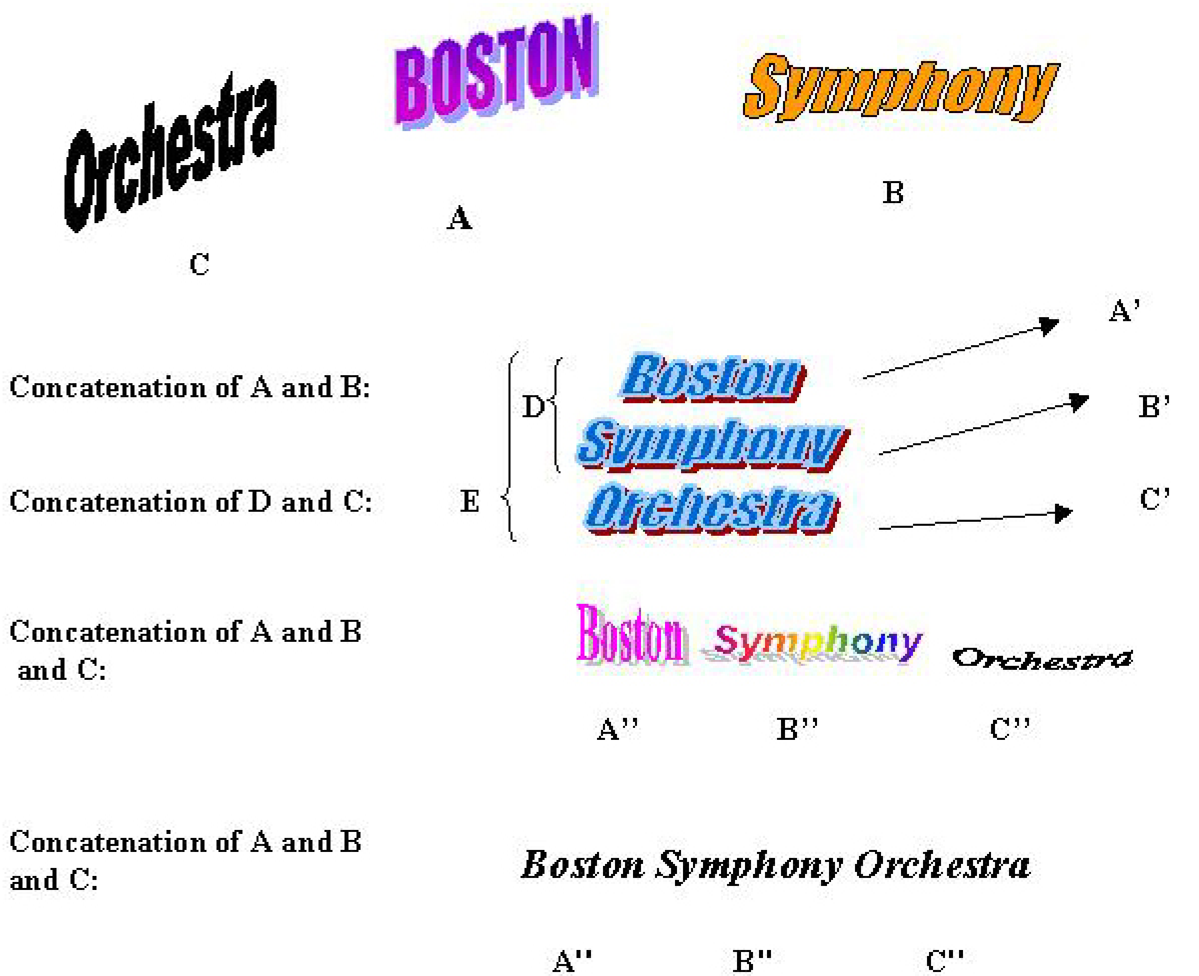
$W_1$ - the *set of all words* of which a subset is the set:

$S$ - the *set of all well-formed expressions* (for short *wfes*).

As we mentioned, all the notions of the discussed theory ***TCG*** are understood very generally. Regardless of whether the *token*-level or the *type*-level is considered as the primitive level of formalization of language, we assume that in ***TCG*** the universe $U_L$ and the vocabulary $V_1$ are primitive notions. The same pertains to the relations of *indiscernibility* ~ and *concatenation c*. We assume axiomatically that $V_1$ is a subset of $U_L$, and $V_1$ and $U_L$ are non-empty sets. The vocabulary $V_1$ can be exactly established and closed as e.g. in formalized languages, or open as e.g. in natural languages. The relations of *indiscernibility* ~ and *concatenation c* for *tokens* are understood very broadly too. If we apply them to types, then *indiscernibility* is a simple identification relation and *concatenation c* is a set-theoretical function of juxtaposition of two types in a new one. On the *token*-level, the *relation of indiscernability* is characterized axiomatically as an equivalence relation. Intuitively it can, but does not have to be, the relation of empirical similarity if the pragmatic aim is, for instance, to compose an advertisement or a special program/invitation, as we can see in *Example 1* regarding the different *token* words "Boston".

The *relation of concatenation c* considered on the *token*-level may, but does not have to be, understood as a sequencing of two similar *tokens*, if the pragmatic aim is the same (see *Example 1*). Intuitively, *concatenation* of two written *tokens A* and *B*, for example an English one (resp. Semitic one), is written *token D** that is made up by adding to *A**, *indiscernible* from *A*, on the right side (resp. on the left side), the written *token B**, *indiscernible* from *B*. The *concatenation* can (see *Example 1*) also be a relation of the non-linear juxtaposition of two *tokens*, which thus form a single *token*, e.g. in the hieroglyphic script and mathematical formulas. We assume two axioms stating, respectively, that: concatenations of two pairs of *tokens* with the first and second elements pairwise indiscernible yield two indiscernible *tokens*; and that a *token* which is indiscernible from the concatenation of two *tokens* is also their concatenation. In this way the relation of concatenation defined on *tokens* is not a set-theoretical function.

***Example 1.*** This example shows the functioning of the relation of indiscernibility and the relation concatenation in concrete cases. We see that the following quadruples of tokens: **A**, **A'**, **A''**, **A''** ; **B**, **B'**, **B''**, **B''** and **C**, **C'**, **C''**, **C''** are indiscernible, and concatenations of tokens **A** and **B** and **C** are different but indiscernible from the *tokens* words "Boston Symphony Orchestra".




As for the *set* $W_1$ *of all words*, it is defined as the smallest set of objects of $U_L$ containing the vocabulary $V_1$ and closed under concatenation *c*.

The above-given syntactic characterization of any language $L$ is typical for any formal grammar, but if we want to describe any language in formal theory we must define the notion of the *set S of all well-formed expressions*. This is the basic task of the formal theory of categorial syntax axiomatically characterized from a universal point of view. In order to define this notion on the ground of the theory ***TCG,*** the formal characteristics requires the consideration of an ordered system more complex than ($L$), denoted here by ($L^k$) ($k$ = 1, 2):

$$(L^k) < U_L^k, =^k, c^k, V_1^k, V_2^k, W_1^k, W_2^k, \iota^k, E_1^{sk}, E_2^{sk}, r_1^k, r_2^k, E_1^{ck}, E_2^{ck}; S^k, Ct^k(S^k), B^k, F^k >,$$

where $k$ = 1, when language is characterized on the *token*-level as the language of *token* objects, and $k$ = 2 for language characterized on the *type*-level as language of *type* objects. The sign "$=^1$" denotes here the relation ~ of indiscsernibility among *tokens* while the symbol

"$=^2$" — the ordinary relation = of identification of *types*. In ($L^k$) beside notions which correspond to the notions of the system ($L$) we distinguish the following primitive or defined notions of ***TCG***:

$V_2^k$ -- the *auxiliary vocabulary* of $L^k$,

$W_2^k$ -- the *set of all auxiliary words* of $L^k$,

$\iota^k$ -- the *relation of indication of indices to words* of $W_1^k$,

$E_1^{sk}$ -- the *set of all simple expressions*,

$E_2^{sk}$ -- the *set of all basic indices*,

$r_1^k$ -- the *relation of the formation of functor-argument expressions* of $L^k$,

$r_2^k$ -- the *relation of the formation of functoral indices*,

$E_1^{ck}$ -- the *set of all functor-argument expressions* of $L^k$,

$E_2^{ck}$ -- the *set of all functoral indices*,

$S^k$ -- the *set of all well formed expressions* (*wfes*),

$Ct^k(S^k)$ -- the family of all syntactic categories,

$B^k$ -- the *set of all basic wfes*,

$F^k$ -- the *set of all functoral wfes*.

If we want to build the theory ***TCG*** as the theory of a description of all notions of the system ($L^k$) ($k$ = 1, 2), on two levels, we have to establish what is the first level of formalization of language $L$. If we begin with the formalization on the *token*-level then the terms listed in the system ($L$) and taken with the superscript 1 have already been characterized. The remaining terms in ($L^1$) are new: primitive or defined ones of the theory ***TCG***, and all dual notions of the system ($L^2$) pertaining to *types* are defined on the *type*-level. Conversely, if we begin with the formalization of the language of *types*, then we first characterize, in a similar way as before, the notions of system ($L^2$) on the *type*-level and then the dual notions of system ($L^1$) on the *token*-level. We will only outline an intuitive understanding and characterization of the terms occurring in ($L^k$) which do not belong to ($L$) without taking into consideration, seperately, the *token-type* distinction. We can find a formal presentation of the theory ***TCG*** in Wybraniec-Skardowska (1989, 1991).

The set $S$ of *wfes* is defined by means of two *sets*: $E_1^c$ (*of all complex expressions*) and $E_1^s$ (*of all simple expressions*). It is important that in distinguishing these sets from $W_1$ we use *categorial indices,* which play a principal role in approach (1). Categorial indices are some objects of the universe $U_L$ (on the *token*-level they are *tokens* and on the *type*-level they are *types*) but they do not belong to the set $W_1$ of words of a given language $L$, but are rather words of the metalanguage of that language. They are so-called *auxiliary words* of $L$ *belonging* to the *set $W_2$ of all auxiliary words* of $L$. The set $W_2$ is generated from the non-empty, *auxiliary vocabulary $V_2$* of $L$ (a further primitive notion of ***TCG***) by means of concatenation relation $c$, similarly to the set $W_1$ of words from the vocabulary $V_1$. The vocabularies $V_1$ and $V_2$ are, of course, disjoint sets. The *auxiliary vocabulary* $V_2$ includes *basic indices* and auxiliary symbols, e.g. brackets, commas, fraction lines, etc. The *set of all auxiliary words* $W_2$ includes two nonemty and disjoint sets: the *set* $E_2^s$ *of all basic indices* from $V_2$ and the *set* $E_2^c$ *of all functoral ones*. The sum $E_2^s$ and $E_2^c$ gives the *set* $E_2$ *of all categorial indices*. *Functoral indices* are formed from basic ones by the *relation* $r_2$ *of the formation of functoral indices* (another primitive notion of ***TCG***), and the set $E_2^c$ is defined as its counter-domain $D_2(r_2)$. In the theory ***TCG*** the *relation* $r_2$ *of the formation of functoral indices* is characterized by axioms; formally it is a binary relation but it can be regarded as any finite, at least ternary, relation defined on subsets of $W_2$ which are sets of indices of words of $L$. It can be treated as a substitute for any rule for the formation of functoral indices of $E_2^c$ independently of their specific notation, in particular: fractional, quasi-fractional, parenthesis (see *Example 2*).

***Example 2***. Let us assume that for a given language $L^1$ its auxiliary vocabulary $V_2^1$ is equal to 1) { *s*, *n*, /, ' } or to 2) {*s*, *n*, /, \, (,)} or to 3) {*s*, *n*, ' , (,)}and the set $E_2^{s1}$ of all basic indices of $L^1$ equals {*s*, *n*}, where *s* is the index of sentences and *n* is the index of names of $L^1$ and concatenation consists in right-sided linear juxtaposition. The relation $r_2$ replaces any rules which allow us to form from two indices a new, functoral index as well as any rules which make it possible to create a new functor from three or more indices. Certain of them can serve to form indices 1) by means of the auxiliary sign "/" — slash (see Ajdukiewicz 1935) and others 2) with both "/" and "\" — backslash (see Lambek, 1958) or in the case 3) by means of coma "," and parentheses "(" and ")" (see van Benthem, 1986). For instance, the following different indices of a quasi-fractional or parenthesis form assigned to the same kind of functors can be created by means of suitable rules corresponding to the relation $r_2$:

- indices of a sentence-forming functor of one name argument

1) *s/n*, 2) *s/n* or *n\s* , 3) (*s*, *n*);

- indices of a sentence-forming functor of one argument which is a sentence-forming functor of one name argument

1) *s*//s/*n* , 2) *s*/(*n*\s) or (*s*/*n*)*s* , 3) (*s*, (*s*, *n*));

- indices of a sentence-forming functor of two name arguments

1) *s/nn* , 2) *n*\s/*n* , 3) (*s*, *nn*);

- indices of a functor forming a sentence-forming functor of one name argument and whose arguments are two functors of the same category

1) *s/n//s/n*, *s/n* , 2) *s/n*\\s/*n//s/n* , 3) ((*s*, n), (*s*, *n*)(*s*, *n*)).

For a fixed language $L$ a concrete application of the rule $r_2$ is its use as a quasi-fractional notation:

$$a \,/\, b_1\, b_2 \ldots b_n$$

for the functoral index of any functor forming any expression with the categorial index *a* and of *n* arguments ($n \geq 1$) which are, in turn, expressions with indices:

$b_1, b_2, ..., b_n$. We will apply this non-formal notation later instead of the functoral index $b$, which satisfies the following formula of the theory ***TCG***:

$$r_2(a, b_1, b_2, ... b_n; b).$$

The unique categorization (see a)) of defined words of $L$ is obtained by the relation ι of the *indication of indices to words*, which is a new primitive notion of ***TCG***. The relation ι assigns to every word of a subset of $W_1$ one (with exactitude to indiscernibility) categorial index from $E_2$. It is only from the set of words possessing categorial indices that we separate the *set* $E_1$ of *all expressions* of categorial language $L$ as the sum of two disjoint sets of expressions: the set $E_1^s$ of words of the vocabulary $V_1$ possessing indices ($E_1^s$ is the intersection of $V_1$ and the domain $D_1$ (ι ) of the relation ι , similar as the set $E_2^s$ is the intersection of $V_2$ and the domain $D_2$ (ι ) of ι ), called the *proper vocabulary* of language $L$ or the *set of all simple expressions of* $L$, and the *set* $E_1^c$ *of all its complex*, *functor-argument expressions* which is the counterdomain $D_2(r_1)$ of the *relation* $r_1$ *of the formation of complex*, *functor-argument expressions*. While for any language $L$ these complex expressions are determined by the specific syntactic rules of $L$, in theoretical considerations these rules are replaced by the relation $r_1$. The relation $r_1$ is a primitive notion of the theory ***TCG***. It is a binary relation but it can be regarded as any finite, at least ternary relation defined on sets of words possessing indices, *i.e.* defined on any finite number > 2 of domains $D_1$(ι). And what is very important is that relation $r_1$ can be treated as a substitute of any rule of forming functor-argument expressions of $L$. The relation $r_1$ refers to syntactic rules of any categorial language $L$ without regard for the notations, symbolism, or calligraphic system used to form their complex expressions (see *Example 3*).

***Example 3.*** Let us assume that we formalized the classical logic in such different ways that functors of implication and conjunction, and quantifiers have, respectively, the symbols from the proper vocabulary $E_1^s$:

1) ⇒, ∧, ∀, ∃ ,     or   2) ⊃, &, $\Lambda$, $\mathrm{V}$,     or   3) C, K, ∏, ∑

and concatenation consists in right-sided linear or non-linear juxtaposition. The relation $r_1$ replaces various rules allowing us to form from at least two expressions a new one. These rules can be rules of the formation of terms or sentential expressions of classical logic. Certain of them can serve to form sentential functor-argument expressions, which are different but synonymous expressions in various formalized languages of the classical logic. For instance, we can form the following three synonymous implication expressions with three kinds of connectives and quantifier symbols 1) or 2) or 3):

1) $\forall x\ (P(x) \wedge Q(x)) \Rightarrow \exists x\ P(x)$,   2) $\underset{x}{\Lambda}(P(x)\ \&\ Q(x)) \supset \underset{x}{\mathrm{V}}\ P(x)$,   3) $\mathrm{C}\prod \mathrm{xKPxQx}\sum \mathrm{xPx}$.

We see that the above implications recorded by means of different symbols have the same meaning and can be created by means of different suitable syntactic rules corresponding to the relation $r_2$ ( the third of them is written down in Łukasiewicz's parenthesis-free notation).

If the relation $r_1$ holds among the system of expressions ($f, e_1, e_2, ..., e_n$) and the expression $e$, where $f$ is the main functor of $e$ and $e_1, e_2, ... , e_n$ are its $n$ ($n$ >0) arguments, and the expression $e$ is formed by means of this functor and its arguments, then on the ground of theory ***TCG*** we record this fact as follows:

$$r_1(f, e_1, e_2, ..., e_n; e).$$

For a fixed language $L$ instead of this formula we can use the following non-formal notation (on the *type*-level):

(e) $$e = f(e_1, e_2, ..., e_n)$$

and if $e$ is a *wfe* and has the index $a$, then the index $b$ of its main functor $f$ is formed from the index $a$ and indices $b_1, b_2, ..., b_n$ of arguments of this functor, respectively, and has to satisfy the principle ($r$) given below, and so the following formula of the theory ***TCG***:

$$r_2(a, b_1, b_2, ..., b_n; b).$$

We can use for it (on the *type*-level) the mentioned nonformal notation:

(fi) $$b = a / b_1 b_2 ... b_n.$$

The set $S$ of all *wfes* of $L$ can be generated by the ordered system

($CG_L$) $$< U_L, c, E_1^s, E_2^s, i, r_1, r_2, (r) >,$$

in which ($r$) is a rule establishing relationships between the index of any functor-argument expression and the index of its main functor and indices of its arguments. We call it the *principle of the syntactic connection of compound expressions.* For any expression $e$ in the form ($e$) it has the following verbal formulation:

> ($r$) *The categorial index b of the main functor f of a functor-argument expression e is obtained from the index a of the expression, which that functor forms together with its arguments, and the indices* $b_1, b_2, ..., b_n$ *of all successive arguments of that functor.*

The system ($CG_L$) may be regarded in ***TCG*** as a general reconstruction of any classical categorial grammar whose idea goes back to Ajdukiewicz (1935), and in the formal-logical characteristics of language one can also find some ideas of other formal grammars, e.g. generative grammars (Chomsky 1957). The system ($CG_L$) generates any language $L$ described by the theory ***TCG***.

The most important notion of the theory ***TCG*** is the *set S of all wfes* of $L$. It is defined as the smallest set containing all simple expressions from $E_1^s$ and every such functor-argument expression of $E_1^c$ which has the property that both it and any compound expression being its constituent satisfy the rule ($r$). This definition allows us (see d) and c)) to describe an algorithm for testing the syntactic connection of expressions of $L$ (if $L$ is a categorial language without variables and operators that bind them).

Let us know that the notion of the *set S of all wfes* of $L$ could be also formally introduced in a little modified way if $L$ is any categorial language that include operators and variables bound by them. A narrow space of this paper does not allow to concentrate on this issue; thus we refer the reader to the books of the first author (1985, 1991).

The categorial approach to language $L$ is connected with the unique categorization (see a)) of each of its expressions, *i.e.* with assigning it to a defined *syntactic category* which in ***TCG*** is defined as a class of expressions possessing indiscernible indices. The syntactic category of expressions possessing the index $\xi \in E_2$ is denoted by $CAT\xi$ and $Ct(S)$ is the family of all syntactic categories of expressions of $S$. It simultaneously is a logical partition of the set $S$. The family $Ct(S)$ is divided into two families: the family of all basic categories, *i.e.* categories with basic indices from $E_2^s$, and the family of all functoral categories, *i.e.* categories with functoral indices from $E_2^c$. The sum of all basic categories of the first family gives the *set B of all basic wfes* of *L* and the sum of all functoral categories — the *set F of all functors* of *L*. The sum of sets *B* and *F* is equal to the set *S*, and *B* and *F* are disjoint sets. So, symbolically:

$$(S\bigcup) \qquad S = \bigcup \{CAT\xi \mid \xi \in E_2\},$$

$$(S\cup) \qquad S = B \cup F \text{ and } B \cap F = \varnothing.$$

We mention in this place that the traditional definition of syntactic categories drawing upon the notions of replaceability (see e)) and a well-formed expression, in particular a sentence, requires a prior definition of these notions in order to avoid the risk of a vicious circle, so the notion of a relation of replaceability should be first defined. It is possible on the base of the theory ***TCG*** and leads to theorems which are important for the theory of syntactic categories (see Wybraniec-Skardowska 1989, 1991). The scope and non-formal character of this paper justify omitting these issues.

Let us also observe that the unique categorization and unique functor-argument structure of linguistic expressions is not idealization if we treat them functionally and use them in contexts in accordance with pragmatic aims. Syntactically or semantically ambiguous expressions can, in particular, be understood as schemas of *wfes* with one functor-argument structure and with one (with exactitude to indescernibility) categorial index.

**2. Two opposite ontological approaches to logical syntax**

As we know, language generated by the system ($L^k$) ($k$ = 1, 2) should be characterized on two levels which refer to the distinction *token-type* made by Peirce (1931-1935) and used by Carnap as *sign-event* and *sign-design* (1942). The twofold ontological character of linguistic objects understood as *tokens* (material objects) or *types* (abstract objects) should be emphasized in the formalization of theory ***TCG***. The choice as the first level of formalization of either *tokens* or *types* is in relation with two fundamental strains in philosophy: *nominalistic* (*materialism*) and *Platonistic* (*idealism*), which formed in the controversy over universals.

If we begin our formalization of the theory ***TCG*** (as a theory ***T1***) from the *token*-level and we first characterize language $L^1$ and the notions of the system ($L^1$), *i.e.* sets of *tokens* and relations defined on such sets, and then we formally describe language $L^2$ expanding the theory ***T1*** by introducing the concepts of the system ($L^2$), *i.e.* the sets of *types* and relations defined on such sets, then we present a *nominalistic* approach to the categorial syntax (see Wybraniec-Skardowska 1985, 1991). At the *type*-level, all notions of system ($L^2$) in such extended theory are derived constructs defined by means of the dual notions at the *token*-level. *Types* are obviously defined as equivalence classes of *tokens* by means of the relation $=^1$ of indiscernibility (the relation ~). Every set of *types*, which is a dual counterpart of a set of system ($L^1$), is defined by means of the dual set of *tokens* and the relation $=^1$ of indiscernibility of the system ($L^1$). Similarly, every relation between *types* characterizing language $L^2$, on the *type*-level, is defined by means of the dual relation between *tokens* and the relation $=^1$.

It is possible to present another biaspectual formalization of the theory ***TCG*** (as a theory ***T2***) that embraces a different, opposite and categorial approach to language syntax — the *Platonistic* approach. It assumes that the first level of such formalization is the *type*-level and the second one is the *token*-level. First, on the *type*-level, the theory ***T2*** is constructed as a theory characterizing language $L^2$ and the notions of the system ($L^2$), *i.e. types*, appropriate sets of *types* and relations between them. The second level of formalization of ***T2*** concerns the *token*-level and is considered in the dual theory as the theory describing the notions of the system ($L^1$) and, in this way, language $L^1$. The axioms and definitions of ***T2*** at the *type*-level are either dual analogous counterparts of the axioms and definitions of ***T1*** or expressions equivalent to the latter. In ***T2*** we assume axiomatically that *types* are nonempty sets and two *types* are equal (are indiscernible) if some element belongs to both of them. Two of the basic notions at the *token*-level, *i.e.* a *token* and the *relation of indiscernibility*, are characterized as follows: a *token* is an element of a *type*, and two *tokens are indiscernible* if and only if they both are the elements of a certain *type*. All remaining notions of the system ($L^1$) are defined by appropriate dual concepts from the *type*-level.

It has been proved that both dual approaches to language syntax, *nominalistic* and *Platonistic*, are theoretically equivalent because both theories ***T1*** and ***T2***, formalized on the two different above-mentioned levels and on the ontologically opposite base of these levels, are inferentially equivalent (see Wybraniec-Skardowska 1988, 1989).

In the next section we will introduce some basic semantic notions without regard for the two above-mentioned manners of formalization of the theory ***TCG*** as the theory ***T1*** or as the theory ***T2***.

## 3. Categorial semantics

The biaspectual formalization of language generated by categorial grammar and thus the formalization of theory ***TCG*** on the two levels is important but not sufficient. This is because the ability to use language requires the same knowledge of the users of language about *interpretation* of its well-formed expressions, in particular sentences, and when these sentences are true. It is not a trivial task to provide a general definition of the truthfulness of any sentence of any language generated by the grammar ($CG_L$). In this section we try to outline a certain basis for accomplishing this task and to introduce some theoretical, formal-logical background for categorial semantics. For this aim we have to expand the theory ***TCG*** (cf. Wybraniec-Skardowska, 1998). It should be developed on the *type*-level for language $L^2$ characterizing an extended system ($L^2$) regardless of its formalization as ***T1*** or ***T2***. Speaking about semantic questions of language $L^2$ we will omit all superscripts 2 relevant to it.

### 3.1. On the adequacy of syntax with respect to semantics

In what way do we want to understand *interpretation* of well-formed expressions of any categorial language $L$? Let us proceed to try to outline some base for categorial semantics by describing an intuitive side of theoretical consideration.

We know what an *interpretation* of a *wfe*, in particular a sentence, is if we can assign to it — and earlier, where present, to each of its well formed constituents — an object to which the expression refers or which this sentence represents. This object belongs to an *ontological category* and is determined by a set-theoretical *function of denotation* which assigns an object of reality described by $L$ to every its *wfe*, in particular to each sentence of $L$. This object is called the *reference* or *denotation of this expression*. So, semantic *interpretation* consists in defining for *wfes* of $L$ a *denotation function*, which can also be called a *reference function* or an *extension function.* The choice of a given *denotation function* for *wfes* of $L$ is limited to the *ontology* for $L$. Only when we know what kind of ontology we are dealing with can we assign ontological categories to objects, which are described by $L$ and are the values of the *denotation function*. In our categorial approach to semantics we take into consideration the referential aspects.

So, the theory ***TCG*** needs to be enriched by semantic and ontological notions. It is connected with a certain expansion of the system ($CG_L$) characterizing in a general way any categorial grammar generating any language $L$. This expansion consists in adding to ($CG_L$) the notion of the *denotation operation*, which will replace any denotation function defined for any language $L$. We develop ***TCG*** according to some innovative ideas of Frege, which are visible in the syntactic and semantic categorial agreement of language expressions, *i.e*. in *the principle* (*CA*) *of categorial agreement* (see Suszko, 1958, 1960; Stanosz and Nowaczyk, 1976, Buszkowski 1989) based on

(*CA*) the *agreement of the syntactic category of each language expression with the ontological category assigned to the reference of this expression.*

This principle is a general rule of interpretation in the ontology of any categorial language generated by the grammar ($CG_L$). Keeping this principle we keep a correspondence between categorial syntax and referential, categorial semantics: every two expressions belonging to the same syntactic category have *references* (*denotations*) belonging to the same *ontological category*, and conversely. This correspondence can be called the a*dequacy of syntax with respect to semantics*. It allows us to identify syntactic categories with *semantic categories*, where these latter are understood as sets of expressions whose denotations belong to suitable ontological categories.

Let us note that categorial indices, used first by Ajdukiewicz (1935) to characterize the syntactic role of expressions in sentences, were used first by Suszko (1958, 1960) as a tool for the coordination of expressions and extralinguistic objects. The authors will use them in the same way in the next subsection, which will be presented more formally than the preceeding.

### 3.2. On typical ontology and denotation operation

The semantic description of any categorial language $L$ requires defining an appropriate *ontology*. Speaking about *an ontology* we will take into account the family of sets determined by categorial indices of $E_2$ and called *ontological categories*. The latter correspond to syntactic categories with the same indices, which serve as a tool for the coordination of expressions of language $L$ and their references, which belong to the reality described by language $L$. The *ontological categories* create a branched hierarchy determined by indices from $E_2$, like the syntactic categories. The *ontological category* with the *index* $\xi$ will be denoted by $ONT\xi$. It is a set (not necessarily a nonempty set) consisting of set-theoretical objects constructed by means of *universes* $U^b_R$ *of the reality described by L and determined by a basic index b* ($b \in E^s_2$). *Universes* $U^b_R$ are primitive notions of the extended theory ***TCG***. We assume axiomatically that for each $b \in E^s_2$ the *universe* $U^b_R$ is a nonempty set, *i.e.*

A0. $U^b_R \neq \varnothing$ for any $b \in E^s_2$

and we introduce the following definitions:

D1. *Typical ontology* we will call the family $\{ONT_b\}_{b \in E2}$, where

$1^0$ $ONTb = U^b_R$ for any $b \in E^s_2$,

$2^0$ $ONTb = ONTa / b_1 b_2 \ldots b_n = \{ g \mid g: ONTb_1 \times ONTb_2 \times ONTb_n \to ONTa \}$

for any $b = a / b_1 b_2 \ldots b_n \in E^c_2$;

*i.e.* the *ontological category* with (see notation (*fi*)) *the functoral index* $a / b_1 b_2 \ldots b_n$ is the set of all set-theoretical functions from the Cartesian product $ONTb_1 \times ONTb_2 \times ONTb_n$ of ontological categories with indices, in turn, $b_1, b_2, \ldots , b_n$ , into the ontological category *ONTa*.

D2. By *reality corresponding to L* we will understand the set $ONT_L$ defined as the sum of all ontological categories from typical ontology which have indices from $E_2$ and, at the same time, are assigned to the *wfes* of *S.* So, symbolically (cf. ($S\bigcup$) and ($S\cup$)):

($ONT_L\bigcup$) $ONT_L = \bigcup \{ONT\xi \mid \xi \in E_2 \cap \iota(S)\}$.

It is easy to see that

($ONT_L\cup$) $ONT_L = ONT_B \cup ONT_F$ and $ONT_B \cap ONT_F = \varnothing$,

*i.e.* the reality $ONT_L$ is the sum of two disjoint sets: the *set* $ONT_B$ *of all ontological categories with basic indices corresponding to basic wfes of* the set *B* and the *set* $ONT_F$ *of all ontological categories with functoral indices assigned to functors* of the set *F.*

As we mentioned, in theoretical considerations the counterpart of every denotation function is the *denotation operation* $\delta$. It is understood as a substitute for any concrete denotation function for fixed language *L*. It is a new component for the semantical formal characteristics of categorial grammar as the following system:

$<(CG_L); \delta>$ $< U_L, c, E^s_1, E^s_2, \iota, r_2, r_2, (r); \delta >$.

In this way the set *S* of all *wfes* of any interpreted language *L* will be generated by the categorial grammar $<(CG_L); \delta>$.

According to the ideas of Frege's semantics, the mutual dependence of syntactic and semantic characteristics of *L* should be considered by keeping the *principle* (*CA*) *of categorial agreement*. *Denotation operation* $\delta$ is a new primitive notion of the extended theory ***TCG***. It is characterized by the following axioms:

A1. $\delta : S \to ONT_L$

($\delta$ maps the set *S* of all well-formed formulas into the reality $ONT_L$ corresponding to *L* ),

A2. $e \in CAT_\xi$ if and only if $\delta(e) \in ONT_\xi$ for any $e \in S$, for any $\xi \in E_2$

(the axiom A2 corresponds to the principle (*CA*) of categorial agreement: *any well-formed expression belongs to the category with the index* $\xi$ *if and only if the reference* $\delta(e)$ *of this expression belongs to the ontological category with the same index* $\xi$ ).

A3. For any functor-argument expression $e \in E_1^c$ in the form $(e)$, i.e. such that

$$e = f(e_1, e_2, \ldots, e_n),$$

where $f \in F$ is the main functor of $e$ and $e_1, e_2, \ldots, e_n$ are its arguments, the following condition of a homomorphism holds:

$$(h) \qquad \delta(e) = \delta(f(e_1, e_2, \ldots, e_n)) = \delta(f)(\delta(e_1), \delta(e_2), \ldots, \delta(e_n))$$

(the reference of the expression $e$ is the value of denotation of its main functor defined on references of successive arguments of this functor).

So, on the basis of ***TCG*** from the condition $2^0$ of the definition of the ontological category with the functoral index (see D1, $2^0$) and the above axiom A2 we have, according to the Frege's idea, that the reference of the main functor $f \in F$ of the compound expression $e$ of the set $S$ is a function belonging to the set $ONT_F$ and it is defined on the denotations (references) of successive arguments of this functor. Thus the condition ($h$) is a correct formulation of the condition of homomorphism because we treat functors as partial set-theoretical functions defined on expressions of $L$, values of which are expressions of $L$, too. So, the denotation operation $\delta$ should be understood as a mapping that is a homomorphism of a specific algebraic language structure into an algebraic ontological structure. Then we can formally define the concept of a model of categorial language $L$ and the notion of the truthfulness of its sentences. We will do this in the next subsection.

**3.3. Models of categorial language**

Let us consider the subset $F^s$ of the set $S$ of all *wfes* of categorial language $L$ such that

D3. $$F^s = F \cap E_1^s ,$$

*i.e.* $F^s$ is the *set of all simple functors*. The functors of this set belong to a syntactic category with the functoral indices in the form (*fi*), i.e. $a\,/\,b_1\, b_2 \ldots b_n \in E_2^c$ and we can regard them as some set-theoretical functions of the following set:

$$\{f \mid f\colon CATb_1 \times CATb_2 \times CATb_n \to CATa\}$$

while their extralinguistic counterparts in ontology are the functions of the set (see D1, $2^0$):

$$\{g \mid g\colon ONTb_1 \times ONTb_2 \times ONTb_n \to ONTa\}.$$

D4. The algebraic *structure of language* $L$ is understood as the system:

$$\boldsymbol{L} = \langle S, F^s; E_1^s \rangle ,$$

where the proper vocabulary $E_1^s$ is the *set of generators* of the partial algebra $\boldsymbol{L}$, and

D5. The algebraic *ontological structure* of reality is understood as the following partial algebra:

$$\boldsymbol{R_L} = \langle ONT_L, ONT_{F^s}; ONT_{E_1^s} \rangle ,$$

where the set $ONT_{F^s}$ is composed of all functions of ontological categories determined by indices of simple functors of $F^S$ and the set $ONT_{E_1^s}$ is the *set of generators* of the partial algebra $\boldsymbol{R_L}$ and consisting of all ontological categories determined by indices assigned to all the simple expressions of the vocabulary $E_1^s$.

So, really, the denotation operation $\delta$ as satisfying the condition (*h*) of the axiom A3 is a homomorphism from the algebra $\boldsymbol{L}$ into the algebra $\boldsymbol{R_L}$.

D6. A *model of categorial language L* we call the substructure

$$\boldsymbol{M_L} = \langle \delta(S), \delta(F^s); \delta(E_1^s) \rangle ,$$

consisting of the homomorphic images of the sets of the algebra $\boldsymbol{L}$ with respect to the denotation operation $\delta$. The operation $\delta$ is an epimorphism from $\boldsymbol{L}$ onto $\boldsymbol{M_L}$ .

D7. The model $\boldsymbol{M_L}$ is called a *proper model of language L* if the denotation operation $\delta$ is a one-to-one operation. Then it is an isomorphism from $\boldsymbol{L}$ onto $\boldsymbol{M_L}$.

Let us observe that the notion used here of a model of language is a referential model and differs from the standard notion (cf. Hodges, 1993).

### 3.4. On the notion of truth

Introducing the notion of true sentences of language *L* requires us to mark out the category of sentences. For this reason in the set $E_2^s$ of basic indices we have to distinguish a *sentence index sen*, assuming that

A4. $$sen \in E_2^s.$$

Then accepting the convention that

D8. $CAT_{sen}$ is the *syntactic category of sentences of L*

we can axiomatically assume that

A5. $$B \cap CAT_{sen} \neq \varnothing,$$

*i.e.* that there exist basic *wfes* of *L* which are its sentences.

Let

D9. $$Sen = B \cap CAT_{sen}$$

The set *Sen* we call the *set of all sentences of language L*.

By means of the *denotation operation* $\delta$ we can define the notion of *a true sentence of language L*. For this aim we distinguish as the *chief ontological category* $ONT_{sen} = U^{sen}_{R}$ (see D1, $1^0$, A4) as a universe which satisfies the following axioms:

A6. $$ONT_{sen} = T \cup F \subseteq \delta(Sen),$$

A7. $$T \cap F = \varnothing,$$

where $T$ and $F$ are new primitive notions of the theory ***TCG*** which we can intuitively understand as truth and falsity, or the set of all states of things that are and the set of all state of things that are not, respectively.

Because $B \subseteq S$ ( see ($S\cup$)), from D9, A6 and A2, (*CA*) immediately follows that

$$\delta(Sen) = T \cup F \subseteq \delta(S).$$

So, we see that by means of the *denotation operation* $\delta$ we can introduce the definition of the notion of a *true sentence* in the following way:

D10. If $e \in Sen$ and $\boldsymbol{M_L}$ is a model of $L$ then

$$e \textit{ is true in } \boldsymbol{M_L} \text{ iff } \delta(e) \in T \text{ in } \boldsymbol{M_L}. \qquad (L = L^2).$$

The definition D10 says that *e is a true sentence in a model* $\boldsymbol{M_L}$ iff its reference in this model corresponds to truth.

As well as saying that *e* is true in $\boldsymbol{M_L}$ we can also say that $\boldsymbol{M_L}$ is *a model of the sentence e*.

For formalized languages of the zero and first order and for a denotation function satisfying the axioms of the theory ***TCG*** (see Wybraniec-Skardowska, 1998), the question of whether the notion of *satisfaction* in Tarski's sense can be replaced by the notion of *denotation* appears. This problem is being solved by the second author of this paper.

The reader might have a question whether the biaspectual formalization of the theory ***TCG*** described in Section 1 would also be useful to semantic problems. The answer must be positive because, apart of the semantic definition D10 of the notion of truth, we can also introduce a syntactic definition of this notion. Such a definition can be given as the following counterpart of the classical definition of truth:

D11. If *e* is a *type*-sentence of $L^2$ ($e \in Sen$) and $\varepsilon \in e$ ($\varepsilon$ is a *token*-sentence of $L^1$ ) then

(T) $$e \textit{ is a true sentence of } L^2 \text{ iff } \varepsilon.$$

It states that:

if *e* is a *type*-sentence with the representative ε , which is a *token*-sentence, then

*e is a true sentence* if and only if ε ;

the definition D11 can be interpreted in the classical way as follows:

*e is a true sentence* if and only if the state of things is as the representative ε of the sentence *e* says that is.

Let us observe that the *type*-sentence *e* in D11 can be understood as the equivalence class [ε ]~ and simultaneously as the quotation name 'ε ', and in this way, this definition corresponds to Tarski's famous convention (T) (cf. Grzegorczyk, 1997).

Let us note that if we introduce the above-given syntactic definition of a true sentence, then the denotation operation should satisfy the additional condition:

$D10^{-1}$. If *e* is a *type*-sentence of *L* ($e \in Sen$, $L = L^2$ ) then for any model $\boldsymbol{M_L}$ of L

$$\delta(e) \in T \text{ in } \boldsymbol{M_L} \text{ iff } e \text{ is a true sentence of } L.$$

In other words, on the base of D11 and keeping the conditions of $D10^{-1}$ the above equivalence can be replaced by the following:

$$\delta(e) \in T \text{ in } \boldsymbol{M_L} \text{ iff } \varepsilon\ ,$$

*i.e.* the reference of the *type*-sentence *e* corresponds to truth if and only if the state of things is as the representative ε of sentence *e* says that is.

On the base of the definition $D10^{-1}$ it can clearly be seen that both definitions of a true sentence, the semantic and the syntactic, are equivalent on the ground of the theory ***TCG***, *i.e.* the following theorem holds:

*If* $e \in Sen$*, then e is a true sentence of L iff e is true in any model* $\boldsymbol{M_L}$ *of L.*

The mutual relationships between the notions of truth, denotation and satisfaction for formalized languages of systems of knowledge are the subject matter of the authors' further researches.

**4. Final remarks**

In accordance to the purposes of this paper, we have presented the main ideas connected with the formalization of classical categorial grammars and, thus, with the languages generated by such grammars. In the formal logical and categorial approach, we have taken into consideration both semantic and syntactic, as well as ontological aspects. The generality of the approaches given to such a formalization allows us to understand the discussed theory ***TCG*** as a theory of universal grammar. It seems to the authors that languages built according to other principles than those formulated in this paper, may be formal models to which the basic notions of this paper — considered in formal systems of categorial languages characterized both syntactically and semantically — may be applied. In the categorial approach to the syntax of language, the biaspectual character of language is important, and depends on the ontological status of its objects: firstly as language of *token* expressions, which are physical representations of *types* of expressions (at the *token*-level); and secondly as language of *type*-expressions (at the *type*-level). The bi-level nature of language makes it possible to reconstruct a syntactic, classical definition of truth. Categorial semantics is referential, and departs from the classical semantic description orginated by Tarski (1933). Thus, here in this paper, the basic semantic notion is the notion of denotation, and not of satisfaction. By means of the notion of denotation we can give a semantical definition of true sentences.

In this categorial approach, every linguistic expression has a reference. In particular, in formalized languages, names and individual variables should correspond to other references belonging to other ontological categories, and the same is also true in the case of sentences and sentential functions. Quantifiers can be treated as certain functions whose denotations are functions in the reality described by language. The assigning to the quantifiers and their denotations of suitable syntactic or, respectively, ontological categories depends on the number of variables, to which these quantifiers bind (see Wybraniec-Skardowska1998).

However, the categorial approach proposed in the theory ***TCG*** omits the issue, which is frequent in practice, of the ambiguous assigning to linguistic expressions of constituents of reality corresponding to them; in the formal approach the authors do not consider the situational context. This approach also does not deal with the assigning of objects to *token*-expressions of language (at the *token*-level). It is possible to develop this theory, taking into consideration these issues, as has been proposed by Dr. Edward Bryniarski, with whom the authors of this paper are conducting their further research.

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