

IS EVERY POSSIBILITY ACTUALIZED
IN AN INFINITE PERIOD OF TIME?

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Abstract:

It has often been thought that the existence of an infinite amount of time implies the realization of all possibilities. However, it can be proved that it is not true that for any T , if T is an infinite period of time, then every possibility is actualized in T . The proof works for any sense of 'possibility' in which there are possibilities that cannot be actualized simultaneously.

It still might be argued that if there is an infinite amount of time, then each possibility is actualized sometime (during some infinite period of time, though not all). In particular it might be claimed that if there is an infinite amount of time, then there is an uninterrupted infinite period of time; and (P^*) for any T , if T is an uninterrupted infinite period of time, then every possibility is actualized in T . However, it can also be shown that (P^*) is not necessarily true. For it to be actually true, some very strong Principle of Universal Random Change must be true.

Is Every Possibility Actualized
in an Infinite Period of Time?

Numerous philosophers have accepted the belief that

(P) Every possibility is actualized in an infinite period of time.

The "principle of plenitude"¹--that no possible kind of being can remain unexemplified--is one well-known version of this belief, which is usually defended on the grounds that it is implied by the perfection of the Creator. However, this paper addresses itself primarily to an interpretation of (P) that is usually accepted for reasons very different from anything like divine goodness. The reasons apparently have something to do with the properties of infinity, as Philo says in Part VIII of Hume's Dialogues Concerning Natural Religion: "A finite number of particles is only susceptible of finite transpositions; and it must happen, in an eternal duration, that every possible order or position must be tried an infinite number of times. This world, therefore, with all its events, even the most minute, has before been produced and destroyed, and will again be produced and destroyed, without any bounds and limitations. No one, who has a conception of the powers of infinite, in comparison of finite, will ever scruple this determination." Nietzsche makes a similar statement in attempting to prove the doctrine of eternal recurrence:

If the world dare be thought of as a determinate magnitude of power, and a determinate number of power centers--and every other idea is indeterminate and hence unusable--it follows that it has run through a calculable number of combinations in the great dice game of its existence. In an infinite time, every possible combination would sometime have been attained: more, each would have been attained an infinity of times. And then, between each combination and its next repetition, all the remaining combinations must then be run through,

¹A history of this principle is presented in Arthur O. Lovejoy, The Great Chain of Being (Cambridge: Harvard University Press, 1957).

and each of these combinations determines the whole sequence of combinations, so that a whole cycle of absolutely identical sequences results. The world is a cycle which has already infinitely repeated itself, and plays its play in infinitum.²

The statements of Hume and Nietzsche (and similar statements by atomists) imply that if there is an infinite amount of time and only finitely many possible states of the universe, then every possible state of the universe must be actualized sometime. However, we shall prove that it is not true that

(P') Necessarily for any T, if T is an infinite period of time, then every possibility is actualized in T,

for any sense of 'possibility' in which it is true that there are possibilities that cannot be actualized simultaneously. Since no two possible states of the universe can occur simultaneously, the proof applies to the possibilities considered by Hume and Nietzsche. However, we will emphasize the generality of the proof's application since there is much disagreement about what conceptions of possibility are acceptable and since we want to be as general as possible in demonstrating the implausibility of (P) as it is interpreted by (P'). Later on we shall also consider other interpretations of (P).

Before the disproof of (P') is presented, it will be helpful to explain some terminology. Suppose that time is represented as a line. Then a period of time would be represented as a line segment unless it is an interrupted period of time. An example of an interrupted period of time would be the following: In the course of a game, the ball passes back and forth between the home team and the visitors several times. The period of time during the game consisting of all the moments in which the home team has possession of the ball would then be an interrupted period of time. It would be represented by a series of line segments. One must be careful not to confuse a period of time with the amount of time in it. The period of time from today on is not the same as the period of time from tomorrow on, but if the future is of infinite duration, these periods occupy the same amount of time. The amount of time in a period corresponds to the length of the line segment representing the period; or, in the case of an interrupted period of time, it corresponds to the sum of

²Friedrich Nietzsche, Unpublished Notes, p. 704 quoted in Arthur C. Danto, Nietzsche as Philosopher (New York: Macmillan, 1965), p. 205.

the lengths of the line segments in the series of line segments representing the period. In adding periods of time, we add the amounts of time in the periods. An infinite period of time is a period with an infinite amount of time in it, and thus must either have no beginning or no end.

We can now present the disproof of (P'):

Let p_1 and p_2 be possibilities that cannot be actualized simultaneously.

Let T be some infinite period of time.

Let t_1 = the period of time in T consisting of all the moments in which p_1 is actualized.

Let $T-t_1$ = the period of time in T apart from t_1 .

1. t_1 is finite or infinite.
2. Suppose t_1 is finite.
3. Then $T-t_1$ is infinite. (If $T-t_1$ were also finite, then T ($=t_1 + (T-t_1)$) would be finite.)
4. p_1 is not actualized in $T-t_1$ (by definition of t_1).
5. So there is a possibility (p_1) that is not actualized in an infinite period of time ($T-t_1$).
6. So (P') is false.
7. So if t_1 is finite, (P') is false.
8. Suppose t_1 is infinite.
9. There is no moment at which both p_1 and p_2 are actualized.
10. So p_2 is not actualized in t_1 (by definition of t_1).
11. So there is a possibility (p_2) that is not actualized in an infinite period of time (t_1).
12. So (P') is false.
13. So if t_1 is infinite, (P') is false.
14. So (P') is false.

There are two assumptions of the proof that deserve mention:

- (A₁) It is possible that there should be an infinite period of time.
- (A₂) There are possibilities that cannot be actualized simultaneously.

If (A₁) is false, then (P') is made trivially true by the failure of any possible period of time to satisfy the antecedent of the necessitated, generalized conditional. (A₂) might be false either because all possibilities are necessities, or because the possibilities being considered

are only the "positive possibilities." If all possibilities are necessities, then (P') is true so long as necessities are always actualized. However, then it would be true as well that every possibility is actualized in every finite period of time. A more interesting challenge to the proof is the claim that 'every possibility' in (P') is to be understood as referring to all positive possibilities only.³ This challenge is successful only if it is possible that any two positive possibilities be actualized simultaneously, and this will not be the case for every construal of 'positive possibility'; though they cannot be contradictory, positive possibilities may be contrary. On some interpretations of 'positive possibility' both Jack's being six feet tall and his being exactly five feet tall are positive possibilities, but of course they cannot now both be the case. Another example: Imagine that the universe consists of three elementary objects o_1 , o_2 , o_3 , and the positive possibilities are the possible objects o_1 , o_2 , o_3 , $(o_1 + o_2)$, $(o_1 + o_3)$, $(o_2 + o_3)$, and $(o_1 + o_2 + o_3)$, where '+' indicates some physical combination. It may well be that, for example, we count $(o_1 + o_2)$ as not existing when $(o_1 + o_2 + o_3)$ exists, and therefore it would not be possible for both of these positive possibilities to be simultaneously realized. No doubt, however, there are construals of 'positive possibility'--perhaps, for example, every possible kind of object--for which it is true that any two positive possibilities can be simultaneously realized.⁴ So to be completely correct, one should consider the proof a refutation of the conjunction of (P'), (A₁) and (A₂).

The following is not being assumed in the above disproof of (P'):

(A₃) There is some time during which p_1 is actualized, and there is some time during which p_2 is actualized.

³I thank John Knox, Jr. for pointing out the importance of this notion of possibility and for other helpful criticism in his reply to an earlier version of this paper that was read at the New Jersey Regional Philosophical Association winter meeting of 1975.

⁴If it is claimed that all (not just any two) positive possibilities are (not just can be) at some time actualized simultaneously, then it would follow that every positive possibility is actualized in a finite period of time. This claim could then be used in an attempt to justify acceptance of (P), but such an attempt would then rest on an intuition very different from the intuition that leads directly to acceptance of (P).

An actuality may be eternal without being a necessity. So, for example, it might be that p_1 is always actualized, and therefore p_2 is never actualized (since we stipulated that p_1 and p_2 cannot be actualized simultaneously); this would not imply that p_2 is not a possibility. However, one qualification must be added to the statement that (A_3) is not being assumed in the disproof of (P') . Sometimes 'necessity' is taken to mean 'possibility that is always actualized' and 'possibility' is taken to mean 'something that is at some time actualized'. In that case, (A_3) follows from the stipulation that p_1 and p_2 are possibilities that cannot be actualized simultaneously. Since p_1 and p_2 cannot be actualized simultaneously, (A_3) is equivalent to (A_2) when 'necessity' and 'possibility' have the above-mentioned interpretations. A virtue of my disproof of (P') is that it works with any interpretation of 'possibility' that makes (A_2) true. There is no need to spell out a detailed interpretation of 'possibility'.

The denial of (P') is equivalent to the statement that it is possible that there should be an infinite period of time in which some possibility is not actualized. In fact, we have proved something stronger. We have shown that necessarily if there is some infinite period of time at all, then there is some infinite period of time in which some possibility is not actualized.⁵ This prevents one from consistently claiming that

(P') For any T , if T is an infinite period of time, then every possibility is actualized in T ,

another reading of (P) , is actually true, although not necessarily true. (The only qualification here is that

⁵An additional assumption of the disproof of (P') , which goes uncontroversially beyond (A_2) , is that every infinite period of time has a possibility actualized in it that cannot be actualized at the same time as some other possibility. Recall that (A_2) might be false either because all possibilities are necessities, or because the possibilities being considered are only the positive possibilities and any two positive possibilities can be simultaneously realized. It seems implausible that one would accept (A_2) and so deny that all possibilities are necessities, and yet maintain that there is an infinite period of time such that all the possibilities actualized in it are necessities. And it seems implausible to deny of some infinite periods of time that all positive possibilities can be simultaneously realized in them, while asserting this of some other infinite periods of time.

(P'') will be true if all actual time is finite.) So rather than (P'') being necessarily true, it is necessarily false relative to our quite weak assumptions.

A third interpretation of (P),

(P''') If there is an infinite amount of time, then each possibility is actualized sometime (during some infinite period of time, though not all),

has not been ruled out. What reason could there be for believing it? If by a possibility one means something that is sometimes actualized, then (P''') is trivially true. But what support is there for a more significant interpretation? The passages from Hume and Nietzsche suggest nothing.⁶ However, consider this: Although it is not true that every possibility is actualized in every infinite period of time, perhaps it is true that every possibility is actualized in every infinite period of time of a certain sort--for example, every uninterrupted infinite period of time.⁷ If, in addition, there must be an infinite period of time of that sort if there is any infinite period of time at all, then (P''') must be true. Of these two conditions together sufficient for the truth of (P'''), the latter is certainly true, if we continue to use uninterrupted periods of time as our example of a "certain sort" of infinite period of time; that is, there must be an uninterrupted infinite period of time if there is any infinite period of time. Using the same example, the former condition,

(P*) For any T, if T is an uninterrupted infinite period of time, then every possibility is actualized in T.

is definitely not necessarily true. We can imagine an uninterrupted infinite period of time T, where t_1 and $T-t_1$ (defined as previously) are also uninterrupted, and

⁶Danto has attempted to spell out the assumptions underlying Nietzsche's attempted proof of eternal recurrence. However, Danto only lists assumptions that would show that no state of the universe could occur only a finite number of times; the assumptions do not show that every possible state of the universe must occur. See Danto, Nietzsche as Philosopher, pp. 204-209.

⁷I owe this suggestion and other valuable discussion to Fred Katz.

then we can use our proof to show that there is some uninterrupted infinite period of time (t_1 or $T-t_1$) during which some possibility (p_2 or p_1) is not actualized. (The only additional assumption is that it is possible that t_1 and $T-t_1$ should be uninterrupted.)

However, (P*) may be actually true. This requires that t_1 and $T-t_1$ are never actually uninterrupted; that is, that no contingent possibility for which there are incompatibles is actualized during all of the infinite period of time up to some point in time or during all of the infinite period of time past some point in time. This requirement might be put in the form of some cosmic Principle of Universal Change. However, even if such a questionable principle be granted, it is not sufficient to make (P*) true. Suppose there are three possible states of the universe, A, B, and C. Couldn't there be an uninterrupted infinite period of time consisting of infinitely many finite periods of time in which A and B are alternately actualized while C never gets actualized? The defender of (P*) might appeal at this point to some Principle of Universal Randomness, but it is hard to see how this could be sufficient if formulated in a non-question-begging way. Why couldn't the change between A and B be random, so that even with continual random change, C would not get actualized? So besides being highly questionable these two principles appear not to be sufficient to establish (P*), and it is difficult to see what plausible further principles might be so employed.

The intuition that (P) is true might be expressed by the thought that, with an infinite amount of time ahead of us, we need only wait around long enough for whatever possibility we are looking for to turn up; and, with an infinite amount of time behind us, we need only go back far enough to find the possibility sought for. Since these are uninterrupted periods of time, (P*) is the natural way of supporting (P'''). Having disproved (P') and (P''), and having seen the difficulty of defending (P''') (that is, having seen the strong implications of (P*)), we may have at least diminished the intuitive push to accept (P).

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