

PROVING NECESSITY

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Abstract:

It is thought that a valid inference to a logically necessary conclusion must proceed from entirely necessary premises. Counter-examples show this is false. Perhaps while the truth of a necessary proposition may follow from non-necessary premises, its necessity cannot so follow. Counter-examples show this to be mistaken. Must anyone who comes to know the non-necessary premises employed in the various counter-examples have prior knowledge of the necessity of the conclusions of the counter-examples? I argue against this. It is true that, for any necessary proposition, there must be necessary premises from which it may validly be inferred; but no one need use these, or know these, or know how to use them, in order to know the necessity of any proposition.

Proving Necessity

It is part of our standard lore about necessary statements that they do not, cannot, imply any non-necessary statements. It is also sometimes urged that the converse of this is true: that necessary statements cannot be implied or entailed by non-necessary statements; that a valid inference to a logically necessary conclusion must proceed from entirely necessary premises. I have no quarrel to be aired here with the former of these two views, but as I hope to show, the latter is false.

Here are two statements, from two philosophers, which employ the general view I think false. In Scientific Explanation, R. B. Braithwaite objects to the view that scientific laws are logically necessary with the claim that: ". . . since all the premisses in a valid inference to a logically necessary conclusion must be logically necessary propositions, to treat scientific laws as being logically necessary propositions removes all possibility of basing them on empirical data."¹ Ernest Nagel makes a similar point in The Structure of Science: ". . . if laws of nature are logically necessary, the positive sciences are engaged in an incongruous performance whenever they seek experimental and observational evidence for a supposed law. The procedure appropriate for establishing a statement as logically necessary is that of constructing a demonstrative proof in the manner of mathematics, and not that of experimentation."²

Now I certainly do not want here to defend the view that some or all of the laws of physics, or of any other science, are logically necessary. Nor am I sympathetic with the view that "nomic necessity" (as Braithwaite calls it) may be identified with logical necessity. But to infer that this cannot be so, on the grounds that scientific laws are sometimes supported by non-necessary premises, is mistaken. It employs just the principle I want to show false, viz.,

¹R. B. Braithwaite, Scientific Explanation, New York, Harper & Brothers, Harper Torchbook edition (1960), p. 294.

²Ernest Nagel, The Structure of Science, New York, Harcourt, Brace & World, Inc. (1961), p. 54.

that necessary conclusions follow only from entirely necessary premises. Not only may a person come to know the truth of a necessary proposition via non-necessary premises (which ought not surprise anyone who has witnessed the verification of relatively transparent necessities in some social sciences), but it is an easy matter to construct arguments in which non-necessary premises entail necessary conclusions. Consider the following example:

- (A) 1. Every person in Room 328B Hoyt Hall is unmarried.
2. Every bachelor is in Room 328B Hoyt Hall.
Therefore,
3. Every bachelor is unmarried.

The argument above is clearly valid; and I hope it will be agreed that the conclusion is logically necessary.³ But furthermore, it is obvious that not every premise of (A) is a logically necessary premise; indeed all of the premises are non-necessary. Thus Braithwaite's view is wrong; clearly, necessary propositions can follow validly from non-necessary propositions.⁴ And insofar as Nagel's claim involves the same general principle used by Braithwaite, it stands refuted also. It seems, in fact, quite likely that scientists are primarily interested in the truth of their prospective laws, and not in their necessity (if they have any). Consequently, in seeking experimental evidence for proposed laws they are doing something which is not at all inappropriate or incongruous. It may be too difficult, or inconvenient, to proceed as does the mathematician; besides,

³Given the qualms many philosophers feel about analyticity, I may be rash to hope for such agreement. But as I think will be obvious, we needn't quibble about putative examples of necessity; if there are any necessary propositions at all, arguments similar to (A) may be manufactured with minimal ingenuity. And philosophers, if any there be, whose scruples permit no necessity at all will hardly espouse the view I am concerned to refute.

⁴It has been argued before that non-necessary premises may entail necessary conclusions. Keith Lehrer, in "A Note on the Impossibility of Any Future Metaphysics," Philosophical Studies, Volume XIII (1962), pp. 49-51, provides the following example: (i) In Spain the rain falls mainly on the plain and all Spaniards are Spanish; therefore, (ii) All Spaniards are Spanish. I find argument (A) more interesting since it, but not Lehrer's, contains within its premises no component that is a necessary proposition.

as (A) shows, for the truth of a necessary proposition, empirical evidence may do very nicely.

Argument (A) is a simple, deductively valid argument; the truth of its premises would make it certain that the conclusion is true. But it is worth noticing that we may also infer inductively from non-necessary propositions to necessary conclusions. In fact, as far as I can see, whatever sort of arguments there may be, whatever sorts of support can be provided, we can construct examples of that sort with non-necessary premises and necessary conclusions. Imagine a group of investigators who are good observers but mediocre logicians. They investigate a large number (n) of 'time-places' with an eye toward discovering whether or not it is raining at those time-places. As luck would have it, in some cases it is raining quite hard, while in the others the sky is clear. The results of their observation, and their reasoning, is represented by the following inductive argument:

- (B) 1. At time-place 1, it is raining.
2. At time-place 2, it is raining.
3. At time-place 3, it is not raining.
4. At time-place 4, it is not raining.
.
.
.
n. At time-place n, it is raining.
n+1. These are all the places examined. Therefore,
probably
n+2. At every time-place, it is either raining or
not raining.

The conclusion of (B) is simple logical truth; it is a necessary proposition. Each of the premises is non-necessary.

It is clear that the view under consideration, that non-necessary premises cannot support necessary conclusions, is radically false. Indeed, the falsity of such a claim ought to be evident since, on the standard view about logical necessity and entailment, necessary truths are entailed by any statement whatever. (At least this ought to be evident to anyone who accepts the standard view.)⁵ Still, there

⁵Thus, on the standard view, the entailment holds even in those cases where the content of the entailing proposition is clearly irrelevant to the content of the entailed, necessary proposition. In view of this fact, the bit of standard

seem to be important things that remain to be said about my examples. Something like this is what we want to say, not only about (A) and (B), but all such examples: We may use non-necessary propositions to prove the truth of necessary propositions, but if we want to prove their necessity, we must confine ourselves to necessary premises; non-necessary propositions cannot entail that a given proposition is necessary.

There is something correct here, but not as the statement stands. If we take this sentiment to imply that no valid argument from non-necessary premisses can have as its conclusion the claim that a given proposition is necessary, then again it is simply mistaken, false. Another example:

- (C) 1. Every proposition expressed on Page N is a necessary proposition.
- 2. That all bachelors are unmarried is a proposition expressed on Page N. Therefore
- 3. That all bachelors are unmarried is a necessary proposition.

Once more, the premises are non-necessary; but this time they entail the conclusion that a given proposition is necessary. And it requires little wit to see that any expressible necessary proposition could be made the focus of the argument.⁶ So necessity may, in this sense, follow from non-necessary premises. All the same, it may seem a rather fishy sense in which necessity follows from non-necessity. In order to see one thing that may be fishy about it, consider another sort of example.

lore might be amended to read something like this: a necessary proposition cannot be entailed in a 'content relevant' way, by any other than necessary propositions. Or, one might reject the standard view of entailment, and define it in terms of something like content relevance. Either line would require, among other things, a theory of 'content', 'relevance', or analogous terms. However, it is obvious that since none of my examples exploit this interesting feature of the standard view of entailment and necessity, such amendments and theories would be of no help against my argument.

⁶In this paper, I confine my discussion of necessary propositions to ones that are necessarily true; but it should be clear that my remarks could easily be extended to cover necessarily false propositions as well.

- (D) 1. If Susie streaks her hair, then it is a necessary truth that either it is, or it is not, raining here and now.
 2. Susie streaks her hair. Therefore
 3. It is a necessary truth that either it is or it is not raining, here and now.

Again we have validly argued to the claim that a given proposition is necessary; one of the premises (at least) is non-necessary. In fact, let us grant, both premises are true. There is nothing wrong here in terms of truth and validity. But we may well ask how the premises are known to be true. After all, if we claim to have proven that a certain proposition is necessary, then surely we must know that our premises are true. But now it may be argued that in order to know the first premise to be true, we must already know that the disjunction about rain is necessarily true. If I'm going to use modus ponens as I did, then the antecedent of the first premise must be true (assuming that the argument is a proof); and if that's so, then for the premise as a whole to be true, the consequent must be true. So, I must already know that the disjunction about rain is necessarily true. And since my knowledge of the premise requires my knowing that the conclusion is true (given the truth functional analysis of 'if...then...' employed here, the truth of the first premise is entailed by the truth of the conclusion), I certainly have not proved, by (D), that the rain sentence is a necessary truth; it is no counter-example.

Such a reply may be tempting, but it just won't do. I may have learned the truth of the first premise by verifying its consequent, but then again I may have learned it in quite some other way. For example, I may have learned it on the basis of some reliable person's testimony. Suppose that I've been assured--sincerely as far as I can tell--by an expert logician and hairdresser, of the truth of the first premise. Still, we might insist, he must know 'directly' that the rain proposition is necessary. But might he not have learned of the first premise in much the same way as I did? There is no reason, as far as I can see, why he might not have done just that. It seems obvious that such a chain of knowledge by testimony must end somewhere; and that it must terminate in what we might call a 'direct' or 'proper' proof of necessity. But arguing for this would be very hard. For example, arguing that the chain must terminate, is rather reminiscent of St. Thomas' attempt to demonstrate the impossibility of an infinite sequence of causes. Such arguments have not been widely convincing. One might content oneself with pointing out

that, since there haven't been infinitely many persons, no one now can have learned via an infinitely long chain of testimony of the truth of any proposition--including ones which impute necessity to other propositions. But we should not be content with this position; it yields the conclusion that as a matter of fact, proofs of necessity do (eventually) rely on necessary premises. What we would like to believe, however, is that this is somehow necessarily so; that somehow it must be that if P has been proved to be necessary, then somewhere in the proof--if not in the premises themselves, then somewhere in the proof of those premises--there is to be found a real or 'proper' proof of the necessity of P.

I think we may arrive at the same point with argument (C). Suppose we know the premises of (C) on the basis of some reliable person's testimony. How will we argue that the basis for this argument is not, cannot be, testimony 'all the way down'? But perhaps we can side-step this puzzling problem by another example. Suppose that we have been verifying the propositions expressed on Page N, for truth; and also checking them for necessity. We are not clever logicians and philosophers, so we go slowly. So far, every proposition checked has been true and necessary. When we get part way down the page, we find the assertion that every proposition of the page is true; and that all 'except this one' are necessary. (Because we are not very clever--or perhaps in spite of the fact--self-reference is not thought by us to be problematic.) Here, then, the chain of testimony ends. No one knows, in the 'direct' or 'proper' way, (except perhaps Page N) of the necessity of any of the propositions. We, on the basis of its past record, trust, and believe the Page--as it seems to me we have a right to do. Thus, we have adequate backing for the premise that all the propositions expressed on Page N are necessary; which backing does not include proofs of the necessity of every proposition on Page N. So, to prove a later proposition on the page necessary, we do not employ premises whose truth can be known only if we've already established the necessity of that later proposition.

Of course one might simply reject this odd example, and the others with the claim that what I regard as 'knowledge', is just true belief on the basis of testimony--it is not knowledge at all. This way out seems unnecessarily heroic, affording at best a distinctly pyrrhic (indeed Pyrrhonic) victory, considering how much of what we know, we know on the basis of testimony. But I think we can avoid delving into whatever problems beset the acceptance of testimony as a legitimate avenue to knowledge. For whatever special

difficulties (if any) accompany the notion of knowledge via testimony may be side-stepped by appealing to principles of knowledge that are less problematic. Suppose that instead of the Page, we have a machine that produces sentences. We may, after slowly checking the sentences so far produced, and finding as many as we check to be necessary, infer inductively that every sentence the machine produces is (will be) necessary. That is, we may produce for some of these sentences, 'proper' or 'direct' proofs of necessity; and may then, persisting in the peculiar logical blindness typical of the people in these examples, infer inductively that the machine produces necessary propositions only. Of course someone might want to argue that in such a case, the premises wouldn't really be non-necessary, since they are evidently claims that certain propositions are necessary. I suppose this raises the question of whether claims that propositions are necessary are themselves necessary propositions. But this is another avoidable puzzle. In the first place, it must be recognized that my job here is not to expunge every necessary premise from an alleged proof of necessity, even to removing them from proofs of the premises. If an argument by which it becomes known that a certain proposition is necessary, really employs even one non-necessary premise, that is enough. As noted before, an objection might be made if knowledge of the non-necessary premises rested on prior knowledge of the necessity of the very proposition that is the focus of the conclusion. But that is not the case here. It might also be objected that though we could come to know the non-necessary proposition about the production of the machine without already knowing about the particular necessary proposition in question, still we had to employ some real proofs of necessity to establish that universal about the machine, some necessary premises (the bases of our induction). But again, even in this case we still have a crucial non-necessary premise in one proof of necessity; and it is non-question begging. If some necessary premises are somehow involved, so what? We may notice, for one thing, that whatever necessary propositions are involved in the premises, they may not be just the ones that would be required (given whatever rules of inference are available) to demonstrate in the standard, proper way the necessity of any of the subsequent, as yet unexamined, propositions. It is also worth noting that since the supposedly necessary premises are used to infer the conclusion that the machine in question produces only necessary propositions--itself a non-necessary proposition--pursuing this line too vigorously would lead us into conflict with the other part of the standard lore; viz., the doctrine that necessary propositions do not imply non-necessary propositions. (The inference here is only

inductive, but it seems peculiar anyway.) But finally we may imagine that the people in the example do not infer the proposition about what the machine will produce from just the premises about the necessity of the examined propositions. They may--wisely I think--refuse to draw the inference without certain non-necessary premises--for example, the fact that given the physical structure of the machine, certain sentential forms cannot be produced. Indeed, they may regard induction by simple enumeration to be inadequate in such a case, and insist that only because they know the physical characteristics of the machine--and are hence able to know that it can produce only certain forms, not others--does it seem proper to them to infer, in the light of the necessity of the examined propositions, that it will produce only necessary propositions.

In such a case, we avoid exploiting any peculiar features of knowledge via testimony. Still, the knowledge of the people in question rests on at least some obviously non-necessary premises (about the structure of the machine). And so their proof of necessity of the unexamined propositions will involve non-necessary premises: from facts about the structure of the machine, to the claim that the machine can produce only necessary propositions; and from this non-necessary fact, and the non-necessary fact that a given proposition is produced, to the conclusion that the given proposition is necessary. To deny knowledge in this case, either that the machine can produce only necessary propositions, or that the given proposition is necessary, would require that we reject virtually every case of knowledge via induction; a less palatable move than in the case of testimony. For though the example as sketched employs still pretty simple inductive procedures, I think the facts and the inferences could be made as 'theoretical' as one might like. In short, it seems highly unreasonable to deny at this stage that both knowledge of the truth of necessary propositions, and knowledge of their necessity, can be obtained via premises that contain at least some non-necessary propositions.

This conclusion may seem somewhat less peculiar if we consider one final attempt to block it. One might say this: A proper proof of necessity is not an argument with true, known premises, to the conclusion that a certain proposition is necessary. It is, rather, a proof that a proposition is true, that proceeds from none other than necessary premises. This is true, I think. But of course if such an argument is to prove to us that the conclusion is necessary, then we shall have to know both that the premises are true, and that they are necessary. And we might know this on the

basis of induction, testimony, and so on. So we are in an important sense still caught by our peculiar conclusion. Here I think we must further require of a proper proof of necessity that the premises be arrived at in some similar way; i.e., we shall have to provide a sort of recursive definition of 'proper proof'. I see no reason why such a project might not be successfully carried out. We can, in other words, explain what a proper proof is. But I do not think that we can explain why such proofs are necessary for us to have knowledge of the necessity of any given necessary proposition. I don't think we can explain this, because I don't think it is true that we must have such a proof of a given proposition in order for us to know that it is a necessary proposition. Nor do I think we need have such proofs for any propositions employed or implied in the 'improper' proofs we may have of the necessity of any proposition. We may, it could be argued, require proper proofs of the necessity of some necessary propositions, in order to have a real grasp of the concept of necessity. But these propositions may play no role in our proofs of the necessity of other propositions.

In short, I don't think this new suggestion concerning proper proofs, shows at all that we cannot have knowledge of the necessity of a given proposition on the basis of empirical premises; we may employ non-necessary premises at every stage of our knowledge of the necessity of the given proposition. But this much is correct about the new suggestion; in this sense only, I think, are entirely necessary premises required: if a proposition is necessary, then there are necessary premises from which it may (given suitable rules of inference) be validly deduced. This, perhaps, is part of the concept of necessity. But I believe my argument shows that it is not required by someone's knowledge that a proposition is necessary, that anyone know these premises, know that they are necessary, have actually performed the appropriate deduction, or know how to do so.

The difference between logically necessary propositions and non-necessary propositions is fundamentally a logical one. In the light of my argument, it is clear that that difference is not that necessary propositions follow only from necessary premises; it is that, unlike non-necessary propositions, they can or do so follow. This logical point has certain epistemological consequences which, like it, can be--and sometimes are--mis-stated. Necessary propositions can be known a priori; and this distinguishes them from non-necessary ones, for these latter cannot be so known (at least not in just the same way--by deduction from necessary premises). But this is not to say that necessary

propositions must be, or can only be, known a priori. For
it seems that they can be known in any way you please.

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