

REASONING ABOUT CLOSURE

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ABSTRACT: The specter of epistemic closure haunts current epistemology: some regard the refutation of closure as obvious, while others take its denial to be an epistemic outrage. To some extent, the strong difference of opinion has its source in certain misapprehensions. This paper tries to formulate and clarify the key issues dividing the two sides and contends that, in certain respects, the difference between the friend and the foe of closure may be more a matter of semantics than substance. The paper goes on to argue that once the substantial issues have been properly formulated, there is a limit to how far deductive reasoning can take the parties to the dispute.

KEYWORDS: epistemology, epistemic closure, deductive reasoning

The debate over epistemic closure rages in current epistemology, with fervent opponents of closure confronted by equally ardent advocates. In this paper, we do not take sides in the dispute, but we claim that some of the thrusts and parries may be based on misunderstandings of what it would be for epistemic closure to hold. Based on our clarification of this issue, we address the question of how far reason can take each party in the dispute; we try to determine, in other words, the limitations on reasoning about closure.

The dispute over epistemic closure concerns the question of whether knowledge is closed under logical implication. What exactly is intended by the claim that knowledge is so closed? Several different interpretations are possible.

Normally, the claim that a certain set is closed under a certain operation is understood to mean that the result of applying that operation to an element of the set yields another element of that set. In this sense, for example, the set of natural numbers is closed under the operation of squaring but not under the inverse of that operation. Understood in this strict sense, the idea that knowledge is closed under logical implication would amount to the following principle: where S is a person, and p and q are propositions,

(1) If (i) S knows p and (ii) p implies q , then (iii) S knows q .

Something like this certainly holds when knowledge of a proposition presupposes knowledge of one of its consequences. Perhaps, for example, it is impossible to

know that there is a brown table in the room without knowing there is a table in the room. According to (1), however, one knows every logical consequences of what one knows. Since every truth implies every truth of arithmetic, it follows, according to (1), that anyone who knows any truth knows every truth of arithmetic. If closure in this strict sense were at issue, there would not be much of a dispute: since one does not always recognize that a proposition that one knows implies some other proposition, one does not always know a logical implication of what one knows.

Nor does it help if we amend (1) in the following manner:

(2) If (i) S knows p , (ii) S knows that p implies q , then (iii) S knows q .

This principle is mistaken as well: Someone who knows p and knows that p implies q might not (for whatever reason) believe q . Since belief is a condition for knowledge, it follows that such a person would not know q . Suppose, then, that we add a belief condition to (2):

(3) If (i) S knows p , (ii) S knows that p implies q , and (iii) S believes q , then (iii) S knows q .

The problem here is that while (i)-(iii) ensure that S 's belief q is true, a true belief does not by itself constitute knowledge.

We may be certain that if S holds a belief q that follows from a known premise, then S 's belief is true. A correct principle of closure, however, must ensure not just that q is a true belief but also that it is a matter of knowledge. But even this is not sufficient for a principle of closure. For example,

(4) If (i) S knows p , (ii) p implies q , and (iii) S perceives q , then (iv) S knows q .

is certainly true (given that 'perceives' is taken in its propositional sense, according to which 'perceives p ' implies 'knows p '), but it does not count as a principle of closure. Although (4) is an epistemic truth and guarantees that q is known, it does not specify sufficient conditions for S 's knowing q on the basis of an inference from a known premise. For this reason, (4), though true, is not relevant to the closure debate. The principle does not address the question of whether under certain conditions, knowledge of p is deductively transmitted to q .

Accordingly, we shall understand a principle of epistemic closure to be any principle that attempts to specify (non-circular) conditions in which knowledge of p is extended by deductive inference to knowledge of q . More specifically, it is any principle that claims (without circularity) that if one knows p , p implies q , and

certain other specified conditions hold, then one knows q on the basis of a deductive inference from p . As Williamson remarks, such a principle articulates the intuition that “deduction is a way of extending one’s knowledge.”¹

Adding different sets of conditions to the claim that one knows p and p logically implies q will result in different principles of closure, some more plausible than others. As we understand it, the dispute over epistemic closure concerns the question of whether any of these principles is true. There is, however, no reason to suppose that there is at most one correct principle of closure. Thus, the question is whether there is at least one true principle of closure.

Discussions of closure may focus on one or a few possible principles; nevertheless, many philosophers think of the dispute in the foregoing manner, that is, as the question of whether there is at least one true principle of epistemic closure. Feldman, for example, who is an advocate of closure, characterizes his opponent’s position as follows: “In my mind, the idea that no version of the closure principle is true – that we can fail to know things that we knowingly deduce from other facts we know – is among the least plausible ideas to gain currency in epistemology in recent years.”² On the other hand, Hales, who is an opponent of closure, surveys the various ways in which closure principles could be formulated and comes to the conclusion that it is a mistake to suppose that “knowledge is transmitted or flows down through known implication.” He says: “Not in the offing are non-trivial necessary truths that allow us to conclude what someone ... knows on the basis of other things they... know.”³ De Almeida, another opponent of closure, argues “against every tenable closure claim.”⁴ For these, and other philosophers, the issue is whether there is any true principle of epistemic closure.

How might one argue that there is no true principle of epistemic closure? A common sort of anti-closure argument targets a particular formulation – one taken, perhaps, as the best contender for a correct principle – and attempts to refute closure by showing that the targeted formulation is false.⁵ The selected

¹ Timothy Williamson, *Knowledge and Its Limits* (Oxford: Oxford University Press, 2000), 117.

² Richard Feldman, “In Defense of Closure,” *Philosophical Quarterly* 45 (1995): 487.

³ Stephen Hales, “Epistemic Closure Principles,” *The Southern Journal of Philosophy* 33 (1995): 199.

⁴ Claudio de Almeida, “Closure, Defeasibility and Conclusive Reasons,” *Acta Analytica* 22 (2007): 301.

⁵ See Fred Dretske, “Epistemic Operators,” *Journal of Philosophy* 67 (1970): 1007-23, and “The Case Against Closure,” in *Contemporary Debates in Epistemology*, ed. Matthias Steup and Ernest Sosa (Malden: Blackwell, 2005), 13-26; Colin McGinn, “The Concept of Knowledge,”

formulation is taken as a make-or-break principle: if it, or some close cousin, is not true, then no principle of closure is. Though some of these arguments have considerable plausibility, we shall argue that all such arguments are, in an important sense, self-defeating.

Consider the following formulation of closure: where S is a person, and p and q are propositions:

(PC) If (i) S knows p , (ii) p implies q , and (iii) S believes q because S has competently deduced q from p , then (iv) S knows q .⁶

Among friends of closure, a general consensus seems to be emerging that at least something like (PC) is correct. Many would claim that if (PC) – or at least a close cousin of (PC) – is not correct, then it is hard to imagine what a correct principle of closure would look like.

One argument against closure maintains that a denial of closure is essential to blocking scepticism. But if (PC) is the principle of closure that figures in the classical sceptical argument, then that argument does not make the case for scepticism. The first premise of such an argument will be an instance of (PC); if we use the brain in a vat example (BIV), the premise says:

If (i) you know that you have hands, (ii) the proposition that you have hands implies that you are not a BIV, and (iii) you believe that you are not a BIV because you have competently deduced that you are not a BIV from the fact that you have hands, then (iv) you know that you are not a BIV.

The sceptic maintains, however, that since no one is in a position to know that he or she is not a BIV, you do not know that you have hands. Similarly, the sceptic might argue that since no one is in a position to know that the world did not come into existence five minutes ago, you do not know that you had breakfast a few hours ago.

There is a problem with this line of argument, which is that even if (iv) is false, given (PC), we may infer that (i) is false only if (ii) and (iii) are true. But in the BIV example, (iii) requires that you deduce that you are not a BIV from the fact that you have hands. It strains credulity, however, to suppose that those of us

Midwest Studies in Philosophy 9 (1984): 529-54; and Robert Nozick, *Philosophical Explanations* (Cambridge: Harvard University Press, 1981), 204 ff.

⁶ For similar formulations, see: Williamson, *Knowledge and its Limits*, 117; Dretske, "The Case Against Closure," 13; and John Hawthorne "The Case For Closure," in *Contemporary Debates in Epistemology*, 29.

who believe that we are not BIVs do so because we have competently deduced the latter from the fact that we have hands. In other words, (i) and (iv) are inconsistent, given (PC), only when the other antecedent conditions in (PC) have been satisfied; but it is implausible to suppose, as required by the sceptical argument, that (iii) is satisfied. So the denial of (PC) is not required to block the classical sceptical argument.⁷

Our central thesis, however, concerns any anti-closure argument; we claim that all such arguments are problematic. We shall illustrate the difficulty we have in mind by considering an anti-closure argument directed at (PC), where this is supposed to be a step in showing that no closure principle is true. Our aim is not so much to defend (PC) as to illustrate the problem that all anti-closure arguments face.

(PC) tells us that an argument of a certain kind always ensures knowledge of the conclusion. It will be useful to introduce a term to characterize an argument of this sort. Let us say that a person S has a strong argument for q if there is an argument whose premise is p and conclusion q , where S , p , and q satisfy conditions (i)-(iii) of (PC). Clearly, the premise of a strong argument must be true (because S knows p) and must imply the conclusion. Hence, a strong argument is guaranteed to be a sound argument. If (PC) is a correct epistemic principle, one knows q on the basis of a deductive inference if one has a strong argument for q .

Those who deny closure, at least in the form of (PC), contend that someone who has a strong argument for some proposition does not necessarily know that proposition; that is, they are committed to the idea that there is (or may be) an individual S and a pair of propositions p and q that satisfy conditions (i)-(iii) of (PC) but not condition (iv). Thus, drawing on the concept of a strong argument, we may formulate the denial of (PC) as follows:

(NPC) Having a strong argument for q is not sufficient for knowing q .

How might one argue in support of (NPC)? Suppose that you recognize that Julia knows a certain proposition, p , p logically implies another proposition, q , and Julia believes q because she has competently deduced q from p , but does not know q . You reason as follows:

Julia has a strong argument for q , but she does not know q . Therefore, (NPC) is true.

⁷ A similar point is made by Michael Blome-Tillman, "A Closer Look at Closure Scepticism," *Proceedings of the Aristotelian Society* 106 (2006): 383.

What exactly does your argument show? Does your argument yield knowledge of (NPC)?

Suppose that your argument does ensure that you know (NPC). If you know (NPC), then (NPC) must be true, for one can know only what is true. But if (NPC) is true, then the features required by a strong argument are not sufficient for knowledge of the conclusion. So even if your argument for (NPC) is a strong argument, you do not on that account have knowledge of (NPC). But if an argument yields knowledge of its conclusion, then there must be features of the argument in virtue of which it yields that knowledge. Therefore, if your argument gives you knowledge of (NPC), then you, the premise, and the conclusion must satisfy some other set of conditions – that is, other than the set consisting simply in (i), (ii), and (iii) – in virtue of which the argument yields that knowledge. But if there is such a set of conditions, there is a correct formulation of epistemic closure – one that is true even though (PC) is false. So unless some principle of epistemic closure is correct, your argument cannot give you knowledge that (NPC) is true. It follows that you can come to know that (PC) is false on the basis of a deductive argument only if there is some correct principle of closure. Consequently, your argument, if sound, is a vindication of closure rather than a refutation of closure.

Someone might object that even if closure is false – even if every principle of closure is false – it does not follow that one can never use deductive reasoning to extend knowledge. If (PC), for example, is false, then conditions (i), (ii) and (iii) are not always sufficient for knowledge of the relevant sort. It is possible, nevertheless, that these three conditions are sufficient to ensure knowledge in some circumstances. But then, it is possible that you, the premise and the conclusion of his argument fit those circumstances, in which case your argument ensures knowledge of the conclusion – even though (PC) is false.

Suppose, then, that (PC) is false, but there are further conditions which, in conjunction with (i), (ii) and (iii), suffice to ensure inferential knowledge. Let “*C*” designate those conditions. Of course, to say that conditions (i), (ii) and (iii) are not always sufficient to ensure inferential knowledge is to assert, simply, that (i), (ii) and (iii) are not sufficient for such knowledge. On the other hand, if (i), (ii) and (iii) are sufficient to ensure knowledge of the appropriate sort in the presence of *C*, then conditions (i), (ii), (iii) and *C* are jointly sufficient to ensure that knowledge. So the possibility raised in the objection assumes the following:

- (5) If (i) *S* knows *p*, (ii) *p* implies *q*, (iii) *S* believes *q* because *S* has competently deduced *q* from *p*, and (iv) *C* obtains, then (v) *S* knows *q*.

But if this is correct, then, again, there is a correct principle of closure. Thus, closure must be a viable notion, even though (PC) is false. So the possibility proposed in the objection is incompatible with the idea that every principle of closure is mistaken and, more generally, with the claim that closure has been refuted.

It may be instructive here to take note of Robert Nozick's views on closure. Nozick is usually considered an opponent of closure. He understands closure as principle (2), cited earlier:

(2) If (i) S knows p and (ii) p implies q , then (iii) S knows q .

Nozick takes it that he has refuted this principle based on his account of knowledge. However, he clarifies his position as follows:

... we have not said that knowledge never flows down from known premises to the conclusion known to be implied, merely that knowledge is not always so closed, it does not always flow down. This leaves room for ... situations where because the premises are known and known to logically imply the conclusion, the conclusion is also known. We need to identify and delineate which situations these are.⁸

Nozick goes on to ask, "Under what conditions is knowledge transmitted from the premises of a proof to its conclusion?"⁹

So Nozick appears to recognize that if knowledge is sometime transmitted through deductive inference, then there must be sufficient conditions for its transmission. Indeed, his answer to the foregoing question is:

(6) If (i) S knows p , (ii) q is true and S infers q from p , (iii) if q were false, S would not believe p , and (iv) if q were true, S would believe p , then (v) S knows q .¹⁰

But (6) is another closure principle. It appears, accordingly, that Nozick accepts at least one closure principle as correct.

It may seem incongruous that Nozick, who is widely regarded as an opponent of closure, should endorse what seems to be a principle of closure. But perhaps the best explanation of this perplexing combination of philosophical views is simply this: Nozick, along with others, regards just one principle as a

⁸ Nozick, *Philosophical Explanations*, 230.

⁹ Nozick, *Philosophical Explanations*, 230.

¹⁰ Nozick, *Philosophical Explanations*, 231-3.

principle of closure; in his case, it is our principle (2). On his view, if this principle does not hold, then neither does closure. On the other hand, we, along with others, regard many distinct principles as principles of closure. On our view, if any one of those principles hold, then so does closure. This attitude would seem to be shared by those who, discovering a counterexample to a favoured principle, attempt to specify further conditions in order to rectify the problem; the failure of a particular principle is not seen as the end of the story, as a refutation of closure. In any case, this disparity between the one-principle-of-closure and the many-principles-of-closure viewpoints may be responsible for a certain amount of confusion in discussions of closure. It may be that the difference between certain foes of closure and certain friends of closure is more a matter of semantics than substance.

Thus far, we have explicitly addressed an argument against (PC) and for (NPC). Our claim is that one cannot gain knowledge thereby that closure is mistaken. The reasoning that leads to this conclusion, however, does not hinge on (PC) being the favoured contender for a correct principle of closure; it can be generalized to apply to anti-closure arguments that take other formulations as representative of closure. Thus, one cannot refute closure by trying to discredit particular principles of closure.

Our central argument, moreover, can be generalized still further to apply to any sort of argument against the claim that there is a correct principle of closure. The ultimate conclusion of any such anti-closure argument, whether or not it focuses on a particular formulation, is that closure fails, that is, that no principle of closure is true. Suppose that someone has an anti-closure argument that is not addressed to a particular formulation of closure. Suppose, in addition, that the argument meets the requirements of some closure principle, (PC*). Can one know that closure is false on the basis of such an argument? In order for the conclusion to be known it must be true. But if the conclusion is true, then, since it implies that (PC*) is false, the fact that the argument meets the requirements of (PC*) does not suffice for knowledge of the conclusion. Indeed, since the conclusion must be true if it is to be known, whichever features of the argument one takes to be relevant, the fact that one has an argument with those features cannot suffice for knowledge that closure fails. Thus, an anti-closure argument cannot yield knowledge of the conclusion.

The preceding argument, if sound, shows that when closure is understood as a principle that specifies (non-circular) conditions under which knowledge of p is extended by deductive inference to knowledge of q , one can know on the basis of an argument that a given principle of this sort is false only if some such principle is true. But then, one cannot know on the basis of an argument that no

formulation of closure is true. This suggests that those who, like Hales, conclude that it is a mistake to suppose that “knowledge is transmitted or flows down through known implication” overstep the mark; they certainly do not know this conclusion on the basis of deductive reasoning.

The fact that one cannot come to know that a given principle of closure is false on the basis of deductive reasoning unless some principle of closure is true does not, by itself, show that some principle of closure is true. This raises the question, How, if at all, could one come to know that a principle of closure is true? Won't any argument in favour of closure suffer from an analogous difficulty?

Let P^* be a principle of closure. Since we can know only what is true, let us assume that P^* is true. It seems clear true that one cannot show that P^* is true on the basis of an argument unless some principle of closure is true. It does not follow, however, that we cannot establish, on the basis of a deductive argument, that P^* is true. Indeed, P^* , if true, may itself be sufficient for this purpose. One might object that this would be circular since according to this procedure we must know P^* is true in order to show P^* is true. But this objection is not correct: To infer P^* from some premise, we must of course know that the premise is true. If the truth of that premise were to assume the truth of P^* , then the reasoning would indeed be problematic. Suppose, however, that P^* is a correct epistemic principle that says that when S , p , and q satisfy certain conditions, S knows q . Suppose that we, the proposition O , and P^* satisfy these conditions, and that we can know O without knowing that P^* is true. In that case, given the truth of P^* , our argument must be sufficient to yield knowledge of P^* . Not only is it possible to know, on the basis of deductive reasoning, that a principle of closure is true; it is possible even if there is only one correct principle of closure.

A more serious objection to the above points out that premise circularity is not the only kind of circularity, and, in particular, not the only kind of circularity that is objectionable, that renders an argument ineffective. Circularity is problematic when, in order for the argument to achieve a certain goal, say, knowledge of p , we must already have knowledge of p . That is, the desired output of the argument, φ , requires φ as input. But this is not the case in the argument for P^* . For we may safely assume that the conditions cited in a closure principle do not include knowledge of, or justified belief in, that principle. That is, to satisfy the conditions of P^* , it is not necessary to have prior knowledge of the truth of P^* .

A number of philosophers have heaped scorn on the idea that closure should be rejected. For example, Feldman describes it “as one of the least plausible ideas to come down the philosophical pike in recent years”¹¹ and Bonjour claims

¹¹ Richard Feldman, “Contextualism and Skepticism,” *Philosophical Perspectives* 13 (1999): 95.

that it constitutes a *reductio* of any theory that implies it.¹² We have not explicitly addressed the question of whether the denial of closure is absurd or even mistaken. But our central argument may offer some comfort to advocates of closure: it shows that the denial of closure is not something that we could come to know, at least not on the basis of deductive reasoning. On the other hand, it is possible to know on the basis of deductive argument that a principle of closure is true; it is possible even if there is only one correct principle of closure.

¹² Laurence Bonjour, "Nozick, Externalism, and Skepticism," in *The Possibility of Knowledge*, ed. Stephen Luper-Foy (Totowa: Rowman & Littlefield, 1987), 297-314.