

be mechanically applied to the solution of concrete problems, as a lever may be pulled to start a machine which then does its work without requiring further human guidance. Nor does this conclusion drive us to "temperament and tradition" searching for a "standard."

Needless confusion might be avoided if we would cease to press the possible analogy between the activities of nature and the decisions of men. Study of law by the case method does not incline one to liken the volitional judgments of the agents of society to the phenomena of the physical realm. Both furnish material from which men may generalize, and so make or discover what are called laws. But until the apple falls in one direction in Kansas and in the opposite direction in Nebraska, the parallel between the decrees of men and the observed uniformities of nature will be incomplete.

But there is a scientific *method* for law as well as for physics. The merits of the case system have been amply established by long service. So much is common to the nature of legal and of ethical judgments that the method which has proven valuable for the study of law may safely be applied to ethics. By this contribution of a scientific method to the study of ethics, the lawyer may render a signal and possibly unexpected service to the moralist who has often chosen so lightly to look to him for bad example rather than for precept.

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#### A NEO-REALISTIC THEORY OF ANALYSIS

IT is perhaps dangerous to criticize the "new realism"; you are likely to be overwhelmed by a score of six to one. But though a sympathizer with the aims of the movement, I should like to play the part of "devil's advocate" for the moment. I have recently been interested to discover what may be the nature of analysis. But when I turned expectantly to Professor Spaulding's article on the subject,<sup>1</sup> I met with some disappointment.

M. Bergson bears the brunt of the attack in the article in question; an attack so interspersed with symbols that it has overawed several of the reviewers. Moreover, any criticism of the paper would not be a criticism of Professor Spaulding alone. When the realists give battle to the idealists on the field of logic, they remind me of an argument I once overheard between two youngsters of the age of four. One said to the other: "I can lick you." The other squared

<sup>1</sup>"The New Realism." Pages 153-247.

away: "You can not." "Well," said the first, "if I can't, I got a big brother what can." Now, in logical quarrels, the big brother of all the realists is Mr. Bertrand Russell. Professor Spaulding recurs to him for confirmation at all the difficult points, till it would seem the real protagonists in the article are Mr. Russell and M. Bergson. The real aim of the paper is, therefore, perhaps best stated as a vindication of the powers of modern symbolic logic to handle all questions concerning spatial and temporal continuity. But I seem to find the result unsatisfactory on three different scores: first, as a critique of Bergson; secondly, as an account of analysis; and thirdly, as meeting the difficulties involved in applying the mathematics of the continuum to the spatial-temporal world.

First, as to Bergson. I fully agree that M. Bergson ought to know more about symbolic logic. But we have only to read the criticism offered by Mr. Russell himself<sup>2</sup> to have awakened in us a desire that the realist critics might in turn perceive the strong as well as the weak points of Bergson. Bergson does not, as I understand him, deny either the possibility of analysis or the value of concepts, but he does deny the validity of most "analysis by concepts." Concepts, for him, are convenient artificial handles, by which reality may be grasped; and there is no contradiction in using them, any more than in using language, providing you remember that neither words nor concepts are identical with the entities symbolized. The most fundamental proposition of his philosophy is perhaps the statement that change is more simple than permanence. But this is not the denial of analysis; on the contrary, it is the affirmation of a new sort of analysis. All apparently permanent things are complex; analysis will reveal that they are due to a combination of changes. A motion may have natural limits, it may be complex,—such, for instance, is the case with the motion of walking, which is naturally broken into steps,—but it is not *per se* broken into an infinite number of points passed through. When, later, you put down your finger and say the moving body passed through *this* permanent point, the point thus designated as permanent is really the complex product of two motions, the first, continuous motion, and the second, motion of cutting across it. Professor Spaulding well insists that this possibility of cutting at any point is our best proof that the motion was not at that point discontinuous. But for Bergson the continuity was prior, and is not constituted by the cutting. Furthermore, I am not convinced that Bergson is logically compelled to an indiscriminate monism because he denies the adequacy of Professor Spaulding's type of "analysis." There is complexity in Bergson's world,

<sup>2</sup> *Monist*, Vol. XXII., pages 321 ff.

but he denies that the most "simple" existent elements are the mathematical points and instants. And so, I do not think Professor Spaulding's main purpose, that of refuting Bergson, has been accomplished.

But secondly, I am doubtful about Professor Spaulding's notion of analysis. Is it analysis at all, this theory of the continuum? Professor Spaulding speaks frequently of "modern analysis." I confess "modern analysis" looks to me very much like synthesis. The modern doctrine of the continuum appears, at first sight at least, to be an attempt to produce a synthesis of elements which shall correspond, as nearly as may be, to what the ordinary man, doubtless vaguely, calls continuity. There is a sort of irony in the language by which we designate our most thoroughly synthetic sciences, when we name them, for instance, "mathematical analysis," or "analytic mechanics." Of course it may be denied that mathematics is synthetic: it is a quarrel of long standing. Nevertheless, I, for one, can not agree that all of mathematics is contained within the first few postulates, and all the mathematician ever does is to draw out the endless complexity concealed in these, as the conjurer draws a succession of rabbits and brickbats out of a hat. The mathematician seems to me to be a builder of structures and systems. Therefore again I was disappointed when I turned to Professor Spaulding's article: its title promised "analysis," but the burden of the text was synthesis.

And then, thirdly, as to what is actually the theme of the paper, even here was I disappointed that Professor Spaulding did not deal more pointedly with those doubts which the theory of the continuum, taken as a whole, still leaves lurking in the minds of some of us. Let me run over the theory itself as expounded by Professor Spaulding, laying stress on certain points he passes by. Professor Spaulding begins with the series of whole numbers. At this stage in his paper he is not discussing the constitutive theory of relations. This is unfortunate. Instead he discusses the analysis of wholes into parts,—and his conclusion seems to be that there is no prior whole nor any analysis. Therefore the case is not illuminating for the problem of analysis. So it might have been better to discuss internality of relations at this point. For had I been called upon to name the most perfect example I could find of entities which are constituted by their relations, I think I should have mentioned first of all the whole numbers. The numbers seem to consist of nothing but their relation to other numbers. Conceive, if you can, of a number two which is neither following one nor before three. Indeed, the numbers are "separate" and "individual," but this is just that to which the de-

fenders of the internality theory "point with pride," as a beautiful example of the capabilities of their theory. The paper seems, therefore, to reveal here also a certain misapprehension of the real "bite" of the opposing arguments.

But recurring to the development of the continuum, we may remark that the rational numbers are really a series separate from the whole numbers, though built up of pairs of the latter. Logically they are not interpolated into the whole number series, though I fear Professor Spaulding's formulation might suggest it. They consist of all those pairs of whole numbers which do not contain a common factor, arranged according to a rule of "natural" ordering. Three-over-one may *thereafter* be correlated with the whole number three, but until that is done there are no natural divisions in the monotony of the rational series. The real numbers, in like manner, are the series of all the possible limits in the natural order of the rationals, and this series, in turn, is divided into stretches by being correlated with, but not interpolated into, the rationals. This real-number series is the "numerical continuum."

Now what happens when we apply this continuum to space? For one thing, it is obvious that the series in question contains no units of length, indeed in itself no suggestion of spatial extension whatsoever. The series which is contained in a long line and that contained in a short line are mutually indistinguishable. So it arises that some of us have a doubt, perhaps unjustified, whether what we commonly think of as continuous extension, as existential spatial continuity, is completely defined by equating it with a mathematical continuum. Or, to put it otherwise, in the space-series considered as a point-manifold, the points are "non-extended." Extension must, therefore, enter by way of the "organizing relations." But so long as these relations are those of the mathematical continuum merely, there is no distance; each point is an infinity of points distant from any other; nor is there any natural process of correlation with whole numbers, for since the points are indistinguishable from one another, there are no three-over-one's which can be conveniently correlated with three. Of course we can superpose upon the continuum a concept of distance, or its equivalent, defined in ordinal terms, but in the perceptual world there is something almost qualitative about spatial extension which still escapes us. Our mathematics has produced an imitation of space, but not space itself. It will be remembered that M. Bergson distinguishes between (mathematical) "space" and (real) extension, and Professor Spaulding's true task was to attack the validity of this distinction. But has he done it? Let us grant he has shown the identity of the mathematical and the

existential spatial continua, though Bergson would not grant this. Even then the task lies before him of proving that continuous extension can be analyzed into continuity plus a somewhat,—which is either extension, or else something other than extension. But in the former case, the validity of Bergson's distinction between mathematical space and extension is not impugned but justified. So, if Professor Spaulding is to give us an adequate analysis of space, it therefore remains that he tell us what that is, which is not extension, but which, added to mathematical continuity, will give us continuous extension.<sup>3</sup> Then and then only can he claim to have analyzed space,—by giving us the inverse synthesis.

The same considerations as to the relation of mathematics to experience apply, in double measure, to time and the other topics of Professor Spaulding's paper. In short, as I said before, the article left me with many doubts and queries: has he refuted Bergson; has he given us analysis, or is it synthesis; has he given us an exhaustive account, not merely of the mathematical properties of mathematical space, but of real space as well?

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#### REVIEWS AND ABSTRACTS OF LITERATURE

*Instinct and Experience.* C. LLOYD MORGAN. New York: The Macmillan Company. 1912. Pp. 298.

Readers of *The British Journal of Psychology* will recall the articles on Instinct by Lloyd Morgan, Stout, Myers, McDougall, and Wildon Carr which appeared in that magazine,<sup>1</sup> and will remember that the papers were written for a joint symposium on the subject which was held by the Aristotelian Society, the British Psychological Association, and the *Mind* Association, in the summer of 1910. Professor Morgan, in the present book, taking as a kind of text his own views and those of some of the others who shared in the discussion, elaborates a statement of what we may call his philosophy of science. The earlier chapters, on "Instinctive Behavior and Experience," "The Relation of Instinct to Experience," "Reflex Action and Instinct," and "Hereditary Dispositions and Innate Tendencies," serve as a fuller expression of the author's views on the immediate subject of the symposium. Instinct is defined as "that which is on its first occurrence independent of prior experience; which tends to the well-being and preservation of the race, which is similarly performed by all

<sup>3</sup>Let it be borne in mind that I am here inquiring about the relation of mathematical to empirical space, and not of metrical to ordinal geometry, except as the latter relationship exactly represents the former, which itself is part of the question at issue.

<sup>1</sup>Vol. III.