

duced in the process of recall the original material comprises three classes of data—first, words correctly reported; second, those not recalled at all; and third, words not correctly reproduced but represented by substitutes resembling those originally given, as ‘sot’ for ‘sit,’ ‘fad’ for ‘pad,’ ‘hug’ for ‘mug,’ and the like. The proportion of such errors nearly equals the difference between the percentages of words correctly reproduced through voluntary recall and words recognized upon re-presentation. The fact that the total number of words set down, including both perfect and approximate reproduction, thus roughly equals the proportion which the observer is able to identify amid a larger group suggests the inference that of the whole series of impressions received under the given conditions—some 25 per cent.—are already ‘dead,’ in the sense that they can neither be voluntarily reproduced, nor even identified upon re-presentation, while of the remainder a certain proportion, though beyond the reach of direct associative recall, are still ‘viable,’ capable of arousing the supplementary processes of recognition on the renewal of the sensory impression. If this be so, the proportion of words correctly and incorrectly recalled and of those recognized would be a function of the temporal phase presented at the moment by these disappearing after-effects, and not a true phenomenon of memory. It would, therefore, be impossible to apply the results, without verification, to subsequent processes of reproduction proper in their relation to recognition. There remains the question of the period during which the excitability due to the original impression persists in such cases. The rapid initial decline in the memory content described in the classic investigations of Ebbinghaus is perhaps a reflection of this process of fading, and in its curve indicates its temporal progress. As the bulk of recorded experiments on memory and association depend upon reports made within the probable limits of this dissolution process, it is to be considered whether the formulæ which they have afforded—the laws of recency and primacy, for example—do not need revision before they can safely be applied to the more permanent content of associative memory.

ROBERT MACDOUGALL.

NEW YORK UNIVERSITY.

DEFINITIONS OF INTENSITY

IN this paper I investigate the meaning of intensity (intensive quantity or magnitude) as a scientific category; that is, intensity as used in psychology and physical sciences. Most of the definitions hitherto given seem obscure, even contradictory. For instance, one authority says intensities are in no sense measurable; another that

they are in principle measurable. When authorities can thus differ in interpreting this term, it must be that we have not yet brought to light its essential nature. I therefore undertake another analysis of the term in the hope of suggesting what this essential nature is.

In some subjects it is enough to build up a definition deductively; thus, we define cardinal numbers by combining the concepts class and one-to-one correspondence. This method seems to hold good in a so-called ideal realm like that of mathematics. But there are regions in which a definition will be of little use unless it be also to some extent obtained by analysis of given facts. The category of intensity, we shall see, belongs to such a region. For example, a conceptual or mathematical definition of intensity like the following,¹ intensity implies greater, less or equal, but without the whole-part relation, must be regarded as insufficient, even though correct; for we can not easily tell what facts are intrinsically incapable of such relation. As to whether sensations are in any sense wholes, the point is in dispute. Velocities and temperatures seem to be *treated* as though they were capable of division, even if we call them intensive. So purely conceptual a definition as the above is, therefore, not easily applied to the facts. Is there not some mark by which we can tell what kind of a fact will have these properties, and what kind will not? Such a mark, if it exists, must be discovered by analysis of the facts in question. In a purely mathematical realm, the definition given above would suffice, but intensity is used in applied science, and unless its characteristics can be detected in facts by a simple inspection, our definition has not told us all that it should. We must then not only state the general characteristics of intensity, but also by analysis of facts ascertain that property which makes them have these characteristics. The task of defining intensity then falls into parts: first, enumeration of the characteristics, telling us the general nature of our subject-matter, and second, study of the facts in question, telling us the real essence of intensity which gives rise to the above characteristics.

The present paper will try to bring out the general characteristics of intensity; to this end we shall consider briefly some of the recent definitions.

The facts called intensive are of two kinds, psychical and physical. Examples of the former are, intensities of sensations, of images, of pleasure and pain, of affective states; of the latter, velocity, force, electric potential. As these facts seem to differ so widely, it is certainly not easy to pick out the properties, on account of which we label them all by the one name, intensity. Accordingly we find that the definitions given by psychologists differ more or less from

¹ Substantially that of Mr. Russell, 'Principles of Math,' Vol. I., pp. 182-3.

those of physicists, and also that the former do not all agree with one another. I give now some of these definitions in order that we may see where the obscurity and contradiction lie.

Every definition I have seen admits that intensities have the property of being greater, less or equal. Now for the points of difference.

The Century Dictionary gives a definition covering both fields. An intensity is a kind of quantity apprehended all at once, not by a successive synthesis; its parts are not separately identifiable. Professor Külpe² gives a psychological definition: an intensity has no parts; a louder tone-sensation does not contain softer ones, but is just one sensation, different from any softer one. The Dictionary of Philosophy and Psychology says that 'Intensive Quantity' does contain parts, although we can not distinguish them; the parts must really be there if it is to be a kind of quantity at all. In other words, there is no empirical evidence of parts, but we can not help believing that there are parts. Also under the heading 'Intensity' (which must mean intensive quantity since we are told intensity is a kind of quantity) we are told there is no whole-part relation. Now this seems a contradiction. And certainly the above conflicts with the view of Professor Külpe, and also with that of Professor Münsterberg³ that sensations are not complexes of parts nor even measurable. I may add also the view of Mr. Bradley,⁴ that psychical intensities are in principle quite measurable.

Here is a conflict of psychological views, then. Do greater and less imply that there are parts or not? Obviously we need to know what sort of greatness or smallness is in question. This ambiguity is revealed by the words of Mr. Stout,⁵ who says that intensive magnitudes have, properly speaking, no mathematical difference, though there is something analogous to mathematical difference. That is what causes the difficulty in defining intensity: we need to know how far the analogy reaches. Does intensive quantity reduce, after all, to extensive quantity? This is the problem raised by the psychological definitions. Can there be a kind of quantity which is logically incapable of addition or division, or other properties usually associated with measurement? And if we can without contradiction define such a kind, do we have a case of it in sensations, images, etc.? We shall now find the same problem confronting us in the field of physical intensities.

Here Professor Ostwald⁶ has given the most careful definitions.

² 'Grundriss der Psychologie,' p. 47.

³ 'Grundzüge der Psychologie,' Vol. I., p. 263.

⁴ *Mind*, N. S., 4, 1.

⁵ 'Manual of Psychology,' p. 206.

⁶ 'Naturphilosophie,' Lecture IX.

He uses the term *stärke* to denote intensity, and mentions two distinctions between *stärken* and extensive quantities: (1) *stärken* can not be superposed, while extensive quantities can, and (2) they can not be added. As to the first: you can not superpose one sensation on another, for it is a distinct individual event in time. You can, however, superpose one body on another; bodies therefore have extensive quantity, sensations are *stärken*. You can not superpose one interval of time on another, for then it would lose its temporal individuality; therefore time is a *stärke*. As to the second: you can not bring together two moving bodies and get twice the velocity by combining them. You can not join two bodies of equal temperature and get double that temperature. Therefore velocity and temperature are *stärken*.

Now addition is a condition of such measurement as we apply to extensive quantities, and superposition seems to be at least a criterion of our ability to measure such quantities. Where these are impossible, then, it would seem that measurement, of the kind applied to bodies in space, is in principle impossible. And yet the *stärken* undoubtedly have quantity, and are measured. But if they can not be added (and of course, consequently, have no whole-part relations) how can they be measured, and how can they be quantities at all? In short, we have the same problem as above: can there be a kind of quantity (for quantity it must be since it is measured) which is logically incapable of addition or division?

The special problem in defining intensity is, then: How can there be quantity without the whole-part relation? Or: how can there be measurability without addition? For physical and psychical intensities are certainly measured—the former more, the latter less accurately. Fortunately the recent development of the theory of order enables us easily to answer this question, and thus to solve the first part of our problem by enumerating the properties of intensity. We know now that it is possible to define a series each member of which may be greater than the one before it, such even that the difference between two next members is always the same, but that there will be no addition and no whole-part relation between any two members unless you assume the commutative and associative laws.⁷ Intensities might be ordered in such a series, at equal distances apart (*e. g.*, as in the just perceptible differences of Weber's law) and a kind of measurement which would correlate them to the number-series is then perfectly possible. Only we must remember that the larger ones are not *n* times the smaller ones in the sense that they contain them.

⁷ Cf. Russell, 'Principles of Mathematics,' Vol. I., Ch. XXI., especially p. 183.

Thus by considering the properties stated in the definitions of intensity we find it possible to frame a conceptual definition which seems to resolve their conflicts: intensities are characterized by greater, less or equal, and by absence of the whole-part relation, because they are describable in terms of order only. This confirms the above definitions except where they conflict with one another; it tells how far Mr. Stout's 'analogy to mathematical difference' can be carried, namely, up to the commutative and associative laws, but not including them. And it by no means reduces to extensive quantity, since it is a case of order.

As was stated at the outset, if we were giving a mathematical definition our work would be accomplished. But we are dealing with a category whose presence we wish to verify in experience, and our interest is more than conceptual. The above definition does not enable us to tell what facts are intensive with any great certainty. For are we sure that there is not a whole-part relation in some of these facts? Psychical states are often complex, apparently. Time, velocity, temperatures,—can they not in some sense be added? If time is a *stärke* as Professor Ostwald says, how comes it that a day seems to *contain* 24 hours? Does not a temperature of 90° include one of 50° ? The difficulties of adding velocities and temperatures seem to be material ones, rather than logical ones, to a first inspection. If they are due to the nature of the facts and not to our defective methods (as Mr. Bradley seems to think, in the article above mentioned, is the case with sensations) can we not find some one criterion common to all these facts, which will show why in the nature of things no method could possibly turn them into extensive quantities? We know that anything spatial, except points, has extensive quantity, and perhaps it would not be going too far to say that *all* the extensive quantities of science are in the last analysis measured in spatial terms. Now is there not some equally simple criterion for intensive quantity? In short, we come now to the second part of our task. We must take up in turn the various facts called intensive, and ask what is that property in each one that makes possible the greater, less or equal relation but prevents addition and the whole-part relation.

W. H. SHELDON.