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Chaos and Qualia

When I look at a ripe tomato, in good lighting conditions and otherwise normal circumstances, I have a visual experience of redness. When I look at an unripe tomato under similar conditions, I have a visual experience of greenness. If conditions are the same in the two cases except for the ripeness of the fruit, the difference between the two experiences allows me to isolate two qualia, one of redness and another of greenness. Many opine that qualia are troublesome for any physicalist theory of mind because they are difficult (or impossible) to explain in physicalist terms. Daniel Dennett and Leopold Stubenberg are among those who suggest that qualia are impossible to explain in physicalist terms, though they characterize qualia in different ways (Dennett 1988; Stubenberg 1996). I will argue below that if qualia are *emergent properties of chaotic brains*, it is possible to explain them in physicalist terms. While I don't provide logically sufficient conditions for the existence of qualia in physicalist terms, I believe the argument provided here gives good reason to think that it is possible to do so.

In what follows, I will consider eight characteristics that Dennett and Subenberg attribute to qualia. These are characteristics that many or all qualia have in common, not characteristics of any particular quale. In other words, I will discuss what redness and greenness have in common, both with each other and with qualia from other sensory modalities. I will argue that the chaotic emergence theory can explain these shared characteristics, and hence that the theory can explain qualia. This approach is necessary because those who argue against physicalism in the philosophy of mind have got something right about qualia. Any particular quale, I will concede, is impossible to describe completely in a physicalist theory. Consequently, no particular quale can be explained in or reduced to physical terms. However, this does not prevent a physicalist theory from describing and explaining qualia as members of a class distinct from other phenomena. I will begin by assuming that Dennett's and Stubenberg's sets of characteristics plausibly distinguish qualia from other phenomena. I will argue that the chaotic emergence theory can explain each of these characteristics, and that it thus provides an explanation for qualia in general. If qualia are the only things that can have all eight characteristics considered below, then the chaotic emergence theory gives conditions sufficient for the existence of qualia in that chaos theory provides us with conditions for the existence of chaotic systems of the kind required to instantiate the characteristics. While I can't prove and won't argue that qualia are the only things that share the characteristics I will discuss, I believe the case is quite strong since the characteristics provided by Dennett and Stubenberg are part of the reason that qualia are thought to be troublesome in the first place. Accordingly, I contend that the arguments given below suggest that the chaotic emergence theory provides an adequate physicalist theory of qualia.

I will proceed in three steps. The first step is to describe briefly the *chaotic emergence theory*. The second step is to present the main argument described above, by considering a set of characteristics that plausibly are shared only by qualia, and by arguing that if the chaotic emergence theory is instantiated, then something must exist with that set of characteristics. The third step is to examine some implications of this approach.

Step One: The Chaotic Emergence Theory.

The chaotic emergence theory states that mental states are emergent properties of chaotic central nervous systems. This theory is a development of the theory of emergence described by C. D. Broad (1925), and it makes use of the conceptual framework of Chaos Theory (Gleick, 1987; Devaney, 1989; Stewart, 1989; Hilborn, 1994). The basic idea is that mental properties have a special metaphysical status because they are emergent, but that this does not make them metaphysically mysterious since they supervene on physical properties of the central nervous system. I will describe emergence and chaos briefly, and then review the argument that chaotic systems have emergent properties. I will focus on those aspects of the theory necessary for the arguments below; further discussion of the theory can be found elsewhere (Newman, 1996;2001).

According to the chaotic emergence theory, emergent properties are high-level properties that meet three conditions:

(1) they *strongly supervene* on properties of a lower-level theory,
(2) they *cannot be reduced* to properties of the lower level theory,
(3) they *can be explained* in terms of properties of the lower-level theory.

To say that properties at a higher level *supervene* on properties at a lower level is to say that there can be no change in the properties at the higher level without a change of the properties at the lower level. To say that properties at a higher level *strongly supervene* on properties at a lower level is to assert that the supervenience relationship is lawlike rather than simply a matter of coincidence. Kim has investigated the concept of supervenience in detail, including the special characteristics of strong supervenience. I follow Kim's suggestion that any form of supervenience strictly weaker than strong supervenience is not appropriate for theories attempting to relate the mental to the physical (Kim, 1987). To say that properties at a higher level *cannot be reduced* to properties at a lower level is to say that there is no way to define the high level properties using nomic biconditionals and lower level properties (Causey, 1977). On this understanding of emergence, emergent mental properties cannot be defined in terms of low-level physical properties but they are still physical properties themselves since they supervene on low-level physical properties. Finally, in spite of the failure of reduction, emergent properties are explicable in the sense that it is possible to give a causal story about how properties in a class of such properties come about in terms of the lower-level properties on which members of the class supervene. This requires a weaker sort of explanation than Deductive-Nomological explanation (Hempel and Oppenheim, 1948), and is likely to proceed on the basis of properties that large groups of emergent properties share since emergent properties cannot be identified with particular low-level properties. These three characteristics give emergent properties a special status: theories of emergent properties will be autonomous from lower level theories in something like the way that Davidson requires (Davidson, 1970), yet ontological and explanatory unification will be achieved in a way that should satisfy physicalists (Hooker, 1981).

An example of the kind of phenomenon that is emergent on this account is a tornado. Tornados are composed of moving gas molecules, so the properties of a tornado are determined by the properties of gas molecules, and these properties consequently supervene strongly on the properties of the gas molecules. A tornado and its properties are a complex phenomenon that cannot be defined in terms of the properties of the gas molecules in question. This is partly due to the number of molecules in question, but more importantly it is due to the chaotic nature of the weather of which a tornado is a part. Finally, in spite of the epistemic barriers that prevent us from having a complete knowledge of a tornado and its properties, we can explain in general terms why tornados arise and why they have the properties that they do. Consequently the properties of tornados are emergent properties.

According to the chaotic emergence theory, the physical properties on which mental properties supervene are properties of chaotic central nervous systems. A system is chaotic in the sense intended here if it is a dissipative system that meets one or more of the formal definitions of a chaotic system when its dynamical processes are modeled mathematically using the techniques of dynamical systems theory (Devaney, 1989; Hilborn, 1994). There is some disagreement about the proper definition of a chaotic system (Batterman, 1993), but for the purposes of this paper we can understand chaotic systems as systems which have long-term behavior that can be described in terms of a *strange attractor* within the phase space of the system. The phase space of a system is a mathematical coordinate space describing all possible states of the system. The behavior or dynamics of a system can be described as a set of trajectories within its phase space. If all trajectories describing the behavior of a system within a region R of that system's phase space converge on a region S, then S is an attractor, and R is the basin of attraction of S. Strange attractors are distinguished from three other types of attractors: point attractors, limit cycle attractors, and pseudo-periodic attractors (See Figure 1). The distinguishing features of strange attractors are that they are aperiodic curves with fractal dimension (Hilborn, 1994). Because strange attractors are aperiodic, if one chooses a point on the curve and follows the curve away from that point, one will never return to the initially chosen point (unlike a circle or other simple closed curve). Because strange attractors have fractal dimension, they have highly complicated structure that is often repeated at different scales (the well-known Mandelbrot Set is a fractal, though it is not an attractor of a chaotic system). The shape or topology of a chaotic system's strange attractor is thought to characterize the behavior of that system in a qualitative way.

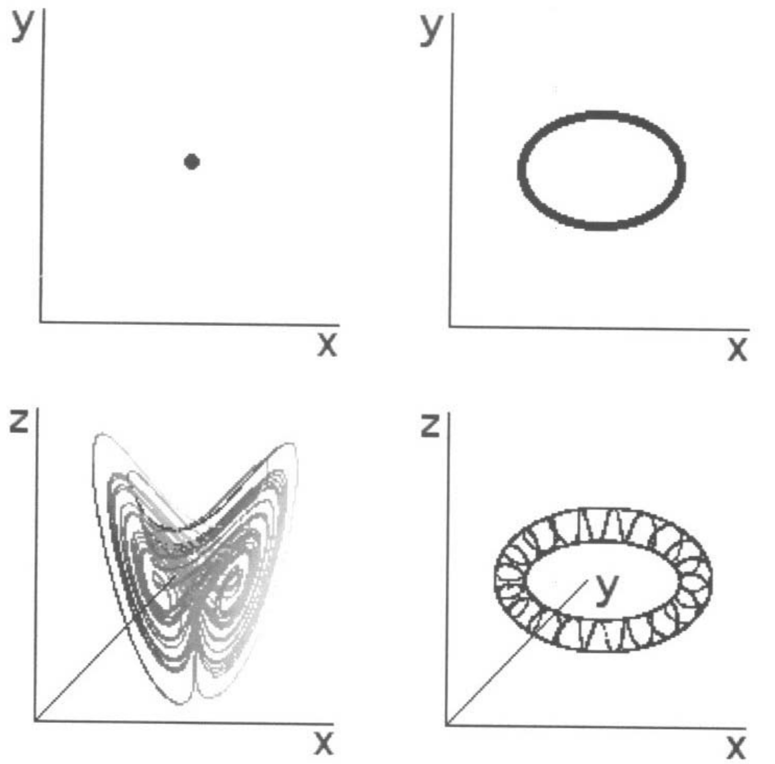

Fig. 1: Clockwise from upper left, a point-attractor, a limit cycle attractor, a quasi-periodic attractor (a torus), and the Lorenz attractor (a strange attractor).

Another property characteristic of chaotic systems is called *sensitive dependence on initial conditions*. Roughly speaking, a system has sensitive dependence on initial conditions if the distance between two nearby points in the phase space describing the system can grow exponentially as the two points evolve according to the dynamical processes of the system. This means that if a chaotic system is in a state similar to a state that it was in at an earlier time, the short-term behavior of the system may be very different from what it was earlier. It is in part because chaotic systems have sensitive dependence on initial conditions that they have strange attractors.

The characteristic features of the four kinds of systems described by the four kinds of attractors can be illustrated by one or more pendulums. The phase space of a pendulum has two dimensions: the position and velocity of the bob. If a pendulum has friction, it will come to a stop at the bottom of its swing, and it will have a point attractor in its phase space where the position is at the center of the swing and the velocity is zero. If a pendulum with friction is driven by a regular impulse, it will eventually come to swing with a regular frequency determined by the frequency of the regular impulse and the length of the pendulum. The attractor of this pendulum will be a limit cycle like the one in the upper right corner of Figure 1. If a system consists of two pendulums swinging with incommensurable frequencies, the joint system consisting of both pendulums will have a quasi-periodic attractor. The phase space of such a system will require four dimensions, and the attractor will be a four-dimensional analog to a torus. If a system consists of a single hinged pendulum driven by a regular impulse, the pendulum will swing wildly and unpredictably, and the attractor of the system will be a strange attractor. Like the two-pendulum system, the attractor for this system will be in a four-dimensional phase space. While the last two examples require four dimensions in the phase space, strange attractors and torii can appear in phase spaces with three or more dimensions, and the two attractors illustrated in the lower half of Figure 1 are in only three dimensions. Chaos is found widely in nature, including in the dynamics of individual neurons (Aihara, 1997).

Because chaotic systems have strange attractors, it is impossible to accurately predict the total future behavior of the system (Kellert, 1993). Prediction of the future state of a physical system proceeds from a measurement of that system and a computation of the predicted future state of the system based on the measurement and the mathematical equations describing the system. This predicted future state can be compared to the actual future state of the system as measured at a later time. It is possible to accurately predict the future state of many non-chaotic systems because the dynamics or behavior of those systems do not separate points near to one another in the phase space of these systems. In other words, the difference between the measured future state of the system and the actual future state of the system is not much greater than the difference between the measured current state of the system and the actual current state of the system. Any measurement errors and any errors of calculation made in predicting the future state of such a system are not exacerbated by the behavior of the system (they are even reduced in some cases). In contrast, because chaotic systems have sensitive dependence on initial conditions, points near to one another in the phase space of a chaotic system are often separated at an exponential rate. Because the dynamics of chaotic systems are particularly complicated (as characterized by the shape of the strange attractor), this separation is maintained or exacerbated. If a chaotic system is within the basin of attraction of a strange attractor, the difference between the predicted future state of the system and the actual future state of the system can be as large as the distance between any two points in the basin of attraction if the time elapsed between the current measured state and the future state is sufficiently long. For this reason, an arbitrarily small error in measurement or calculation can result in a large difference between the predicted behavior of a chaotic system and its actual behavior. To make accurate predictions of chaotic systems over useful time periods would require more computational resources than will ever be available to us.[1] Thus, because we do not have the ability to make, record, and compute with infinite precision measurements, it is impossible to measure or describe the state of a chaotic system with enough precision to guarantee that we can distinguish that state from distinct states that are nearby in the phase space of the system which have dramatically different future behavior. It is thus impossible to accurately predict the behavior of a chaotic system.

Moreover, the properties of chaotic systems prevent us even from approximately predicting their responses to external stimuli. In many cases, the response of a non-chaotic system to external influences can be predicted by noting the various basins of attraction within the system's phase space and by abstracting over the low-level dynamics of the system to obtain generalizations about the possible transitions from one basin of attraction to another. The behavior of a logic circuit, for example, can be predicted in this way. This is possible because a relatively crude measurement of the system is usually sufficient to determine which basin of attraction the system is in, and because the dynamics of the system are simple enough that it responds to external stimuli in relatively simple ways. However, because chaotic systems may have basins of attraction that have fractal boundaries (Grebogi, Ott, et al. 1987), we cannot always measure the state of the system precisely enough to reliably determine which basin of attraction the system is in, and we cannot abstract over the dynamics of the system to formulate useful generalizations about the possible transitions between those basins of attraction. Therefore, it is impossible to measure or calculate precisely enough to predict a chaotic systems' behavior, and consequently there are no finite definitions of the high-level states of a chaotic system in terms of the low-level physical quantities on which they supervene.

In spite of the fact that it is impossible to finitely define some states of a chaotic system, such systems are deterministic physical systems which can be characterized in quantitative terms. There is nothing about chaotic systems which contravenes the laws of physics. All the dimensions of their phase spaces are physical quantities. The basic equations describing the behavior of a chaotic system are written in terms of the physical quantities that make up its phase space, and to the degree that we can measure the system to determine the equations, these equations describe a class of topologically similar strange attractors (Devaney, 1989). One of the attractors in this class precisely describes the behavior of the system, though for the reasons described above we cannot know which one. The topological similarity of the members of the class of strange attractors allows us to make some general assertions about the dynamics of the system. These general assertions can be construed as qualitative descriptions of some properties of the system. Thus, some dynamical properties of a system which is in the basin of attraction of a strange attractor can be explained by reference to the characteristics shared by all members of a class of topologically similar strange attractors.

The point here is similar to one made by Giunti, who suggests that there are physical systems for which there are no good computational explanations (1995). The reason that there are no computational explanations for such systems is that the number of states possible in a computational explanation is strictly smaller than the number of states required for an adequate explanation. Chaotic systems will have this property. There are too many possible states in the phase space of a chaotic system, and the complexity of the dynamics of such systems is too great for computation or communication of the information present in the current state of the system. In contrast, the states of non-chaotic systems are finitely describable since such systems have simpler attractors, simpler internal structure within basins of attraction, and simpler boundaries of attractor basins. This greater simplicity makes it possible to explain the system's behavior computationally by treating the system's basins of attraction as discrete states of the system, and by modeling the system's dynamics with simple state-transitions. Non-chaotic systems also often do not have sensitive dependence on initial conditions, making such computational simulations even easier. Describing the internal state and behavior of non-chaotic systems does not depend on the ability to measure or compute with infinite precision, unlike chaotic systems.

Consequently, being in the basin of attraction of a strange attractor is an emergent property according to the three characteristics of emergent properties discussed above (Newman, 1996). Chaotic systems sometimes have the property of being in the basin of attraction of a strange attractor, and this property can be shown to be an emergent property. Being in the basin of attraction of a strange attractor is a property that strongly supervenes on lower-level physical properties since the location of a system within its phase space is strictly determined by the physical quantities that make up the total state of the system. Being in the basin of attraction of a strange attractor cannot be reduced to a physical property since doing so would involve an infinite precision specification of the state of the system. Moreover, this is a genuine failure of reduction and not just an epistemic limitation since no finite agent could perform the reduction involved. If we assume reduction means that God or Maxwell's Demon could perform the reduction, then even Chaotic systems are reducible, but then the reducible/non-reducible distinction becomes the same as the physical/non-physical distinction, so I choose not to make that assumption. Finally, the property of being in the basin of attraction of a strange attractor can be explained by considering a class of topologically similar strange attractors. It is possible to determine that a system is within the basin of attraction of some member of such a class even though it is impossible to determine exactly which member of the class is involved. Thus, being in the basin of attraction of a strange attractor meets the three conditions defining emergence, and consequently any chaotic system has an emergent property when it is in the basin of attraction of a strange attractor.

Though the matter is not yet settled, there is good evidence that human central nervous systems are chaotic in the sense described above. There are a number of studies of human EEG recordings that suggest that the human brain is chaotic (Basar, 1990). Even some who doubt the validity of these studies suggest that there are reasons to think that the human EEG is chaotic (Jansen, 1991). There are also studies of the rabbit olfaction system which indicate that it is chaotic (Skarda and Freeman, 1990). Since rabbits and humans are both mammals with similar neural systems, chaos in rabbit neural systems is evidence that human neural systems are chaotic as well. These studies show that some things satisfy the conditions for application of the chaotic emergence theory.

Step Two: Characteristics of Qualia.

In this section, I will argue that any chaotic central nervous system must have some characteristics of qualia. On the basis of this argument, I hypothesize that having a particular quale and having a central nervous system that is in the basin of attraction of a strange attractor are token-token identical, though no finite agent can describe this identity precisely. The strength of this argument depends on the set of characteristics that are considered. In order to reduce the possibility that I have chosen characteristics for inclusion in this set in a biased way, I will make use of two lists of characteristics that others have suggested are characteristic of qualia. In each case, a philosopher has proposed a list of characteristics of qualia that each then argues shows why qualia are difficult or impossible for a physicalist theory to explain. The characteristics I will consider are attributed to all or most qualia, or relations that qualia have to other mental and physical properties. Due to the nature of the chaotic emergence theory, however, none can be characteristics of a single quale. Those who originally considered these characteristics chose them with the goal of obtaining a set of characteristics that are unlikely to apply as a set to anything other than qualia.

I do not mean to suggest that any chaotic system is sufficient for the existence of qualia since there are many chaotic systems that presumably do not have qualia. Some additional properties must be attributed to a system with chaotic emergent states to provide a sufficient condition for qualia. I don't yet have an adequate story to tell about these additional properties, but since a chaotic central nervous system seems to be sufficient for the set of characteristics considered below, it seems that the additional properties are to be found in central nervous systems. Thus, while it remains unclear exactly what properties of a chaotic central nervous system are sufficient for qualia, the argument below should make (more) plausible the claim that a physicalist theory of qualia is possible.

The characteristics of qualia I will consider are attributed to qualia by Dennett (1988, p. 47) and

Stubenberg (1996, p. 41). According to Dennett, qualia are

(a) ineffable
(b) intrinsic
(c) private
(d) directly or immediately apprehensible in consciousness.

For Stubenberg, qualia are

(e) monadic
(f) simple
(g) homogeneous
(h) irreducible.

In presenting characteristics a–d, Dennett suggests that these characteristics are impossible to explain in a physicalist way. His goal is to dissolve the problem of qualia — to eliminate them from the set of things that a physicalist must explain. In presenting characteristics e–f, Stubenberg suggests that qualia force us toward Russell's neutral monism and away from both reductive and nonreductive physicalism. Both authors clearly take their set of characteristics to be impossible for a physicalist to explain. It is clearly possible to understand these characteristics in such a way that no physicalist explanation is possible, but such an approach simply assumes that physicalism is false. To avoid begging the question in this way, one must understand these characteristics in a way that is neutral with respect to physicalism, and then ask if there is an explanation of these characteristics in physical terms. This is something like the approach suggested by Seager, who advocates a 'minimalist' interpretation of Dennett's four characteristics (Seager 1999 p. 88). In what follows, I use the chaotic emergence theory to explain each of the eight characteristics mentioned by Dennett and Stubenberg.

Characteristics of Qualia: Dennett.

Consider first the idea that qualia are (a) ineffable. Dennett suggests that qualia are ineffable because one cannot describe exactly the qualitative aspects of one's experience (1988 p. 46). Seager indicates that ineffability of qualia arises because one cannot know a quale without having experienced it (1999 p. 89). The contrast between Dennett's and Seager's ideas of ineffability illustrate why it is so difficult to explain a phenomenon with this property. Dennett says that the ineffable is not describable, while Seager says that is is knowable only by one who has experienced it. How can something be known yet indescribable? First note that the indescribability of qualia is only partial. For example, we can compare different shades of red with respect to how dark or light they are. This is not complete description, only relative description, but it is still a form of description. Second, note that even those who have experienced a quale often do not have complete knowledge of it. Since repeated experience and training are necessary to become a wine-taster, much experience does not give one complete knowledge of the relevant qualia. Thus our knowledge of qualia comes through experience but does not provide complete descriptive ability.

Can we show that chaotic central nervous systems will have something with this kind of ineffability? Since, emergent properties of a chaotic system cannot be finitely and completely described, any description or analysis of these properties will be incomplete. Dennett's condition will be met if such properties play a role in experience. But is experience necessary for knowledge of such properties if they occur in the central nervous system? If an experience is constituted by an emergent chaotic phenomenon in the brain, then experience is necessary and sufficient for first-person knowledge of the experience and any qualia present in it. Given that chaotic phenomena in the brain are particularly difficult to observe from a third-person perspective, there is a kind of practical necessity for experience, but Seager seems to suggest more. Here we must speculate a bit. If first-person experience of the chaotic phenomena in the brain is more complete in some way than any third-person measurement can be, then perhaps first-person experience is necessary for the level of knowledge that we have of qualia. This would explain the frustration we feel when we say that qualia are ineffable: we have much more complete knowledge than we can describe.

More can be said however. If qualia are emergent properties of the chaotic central nervous system, we can explain how it is possible to have informative analogies between qualia. The emergent properties of a chaotic system can be described in a general way since the properties of the class of topological conjugates to which a strange attractor belongs can be described. For example, in Figure 1, the Lorenz Attractor has two lobes. All the members of the class of topologically conjugate attractors will have this general property. Part of the point of discussing systems as chaotic systems is to understand those systems even though no exact quantitative descriptions are possible. Scientists often describe their understanding of chaotic systems as ‘qualitative.’ The use of this word is suggestive. Qualitative descriptions of chaotic systems involve descriptive terms that many different chaotic systems share—terms that do not pick out particular strange attractors, but that are properties of large numbers of strange attractors. This is just the kind of situation we are in with respect to qualia. We can describe some features that all or many qualia share, but we are unable to describe particular qualia.

The analogy drawn here between qualia and the qualitative aspects of the behavior of chaotic systems is awkward in that it seems to break down in delineating what can and what cannot be described. In the chaotic emergence theory, it isn’t possible to describe particular attractors, but it is possible to describe classes of attractors. The analogy drawn above suggests that it is not possible to describe (e.g.) an instance of a shade of color while it is possible to describe the shade itself. This doesn’t accurately represent the ineffability of colors since we seem to be unable to describe either shades of color or their instances. In fact, it is unclear what it means to distinguish between a shade and an instance of that shade. The level at which we begin to be able to describe color qualia is higher—it begins at the level of comparisons across colors, including whether they are light or dark, intense or faded, cool or warm etc. However, the analogy drawn above is a loose one, and it is consistent with a story told at another level of color perception, one in which it is not possible to describe a shade of color while it is possible to describe groups of color shades. This story is consistent with our use of color words, and with the idea that human color vision is not precise enough or reliable enough for us to discern exact color shades.Thus, though the explanation is incomplete pending further investigation of color perception, I regard the disanalogy as insufficient to show that the proposed explanation fails since it operates at the wrong level of description.

A deeper objection to the story of qualia told here is that qualia cannot be explained by basins of attraction of dynamical systems because such basins have precise boundaries, while our experience of qualia suggests that the boundaries between qualia are fuzzy rather than precise.[2] The fuzzy nature of the boundaries between qualia may arise from either or both of two sources.First, our sensory apparatus is neither absolutely precise nor absolutely reliable. A given stimulus may not cause a sensory system to enter the basin of attraction of a strange attractor which constitutes a quale associated with a word. This could occur if the basins of attraction of strange attractors associated with words were not adjacent to one another. An intermediate region would explain those stimuli that we find difficult to characterize as a quale of one kind or another. A given stimulus may also enter the basin of attraction of different strange attractors within a given sensory system on different occasions, depending on the state of the system before presentation of the stimulus. Either of these results will give rise to fuzziness in our color experience in spite of the mathematical precision of the strange attractors that explain our color experience. A third potential source of fuzziness lies in the fact that the strange attractor that explains a quale must be detected by other subsystems of the brain in order for the information represented by the strange attractor to be available for use. If the difference between two different strange attractors is so small that it is at the limit of our abilities to detect it reliably, then fuzziness in our color experience may arise at a relatively high level. I rule this explanation out for reasons that will become clear below. The two remaining sources of fuzziness are consistent with the existence of sharp boundaries on the basins of attraction that explain qualia, and with the remainder of my theory.

Dennett interprets the claim that qualia are (b) intrinsic in two ways, first he suggests that “they are somehow atomic and unanalysable” and then he suggests that “they are ‘simple’ or ‘homogeneous’” (1988 p. 46). Seager suggests, as part of his minimal interpretation of Dennett’s four characteristics, that intrinsic mental properties are those that supervene on physical properties since physical properties are arguably intrinsic (1999 p. 90-91). Seager’s particular suggestion seems to me to beg the question on behalf of physicalism in this context, but he also makes a more general suggestion that is the kind of neutral analysis required here. Seager’s more general claim is that any changes in an intrinsic property must be a result of changes within the property-holder and must not be a result of changes of the property-holder’s relation to other things (Seager 1999 p. 90-91). Dennett’s interpretation and Seager’s more general claim are both consistent with and suggestive of a traditional understanding of intrinsic properties in which an intrinsic property can not be defined or analyzed relationally or functionally. This analysis suggests that an intrinsic property can not be defined or analyzed in terms of anything external to that which holds the property. Mass, for example, is an intrinsic property of physical things, while weight is not. On this understanding of intrinsic properties, they may play a role in a larger functional organization, but their functional role is not their defining character. In order to avoid too much conceptual overlap between Dennett’s proposed characteristics, we must understand Dennett’s claim that qualia are intrinsic to mean that they cannot be defined or analyzed relationally or functionally.

Once again, however, we must be careful. It is clearly possible to consider individual colors to be defined in terms of their position within the color spectrum or in terms of the frequencies of light that cause us to see those colors. Such an approach seems to involve a relational element, and it suggests that the characterization of the intrinsic nature of qualia above is too strong. However, this is to confuse the qualia with their causal antecedents and consequents. The qualia are the qualitative aspects of colors as we experience them, independent of their causal antecedents or consequents, or their relations to one another. This is illustrated by the inverted spectrum argument, which clearly assumes that color qualia can be separated from their causal antecedents and consequents. While those who consider the inverted spectrum argument don’t normally allow the order of color qualia to be jumbled in the color spectrum on the grounds that preservation of the various relations between colors is required to allow someone with an inverted spectrum to remain undiscovered, it isn't clear to me that this is really required. It seems that a jumbled spectrum may involve more complicated adjustments of the causal antecedents and consequents than a merely inverted spectrum, but if the adjustments required for the inverted spectrum are possible then surely those for the jumbled spectrum are too. With the caveat that we must not confuse qualia with their causal relata, I will try to explain how chaotic emergent properties in the brain can explain the claim that qualia can not be defined or analyzed relationally.

Some chaotic systems will have several strange attractors within their phase space. Being in the basin of attraction of a particular strange attractor is a state that is defined by the dynamical properties of a chaotic system. Such states are realized in response to the system's environment, and at any time a change in the environment may move the system from one basin of attraction to another. From a particular basin of attraction some basins of attraction will be more easily reached than others either by virtue of being nearby in the phase space, or by virtue of requiring a smaller environmental change to move the system from one basin of attraction to another.[3] A chaotic system will move among the basins of attraction in its phase space in response to the environment, and each basin of attraction may be interpreted as a functional state of the system since it will correspond to different sets of dispositions to respond to the environment. However, these states are a consequence of the dynamics of the system, and they have their own internal structure that is much finer-grained than the coarse functional structure of which the basins of attraction are a part. Consider once again the characteristic shape of the Lorenz attractor in Figure 1. If that attractor appeared in a system with several basins of attraction, the lobes of that attractor and the fine-grained structure within the lobes would be independent of the functional role that the attractor might play in that system. The high-level functional organization of such a system could be modeled or instantiated in other systems with different attractors and different fine-grained structure, or perhaps without any fine-grained structure at all. The functional organization is a consequence of the dynamics of the system, as is the fine-grained structure, but the functional organization does not define the fine-grained structure; both are consequences of the dynamics of the system as a whole, and either could occur independently of the other. If the brain is a chaotic system, basins of attraction can explain high-level functional states, and the internal structure of such basins can explain the intrinsic aspects of these states, including their qualitative aspects. Thus, because functional structure is multiply realizable, the internal structures of the functional states that is irrelevant to the functional organization amounts to an intrinsic property of the functional states. Consequently, being in the basin of attraction of a strange attractor has a kind of intrinsic character that is distinct from the functional roles that these basins of attractions might play in the system as a whole, and it can explain the intrinsic nature of qualia.

Dennett suggests that qualia are (c) private because "all interpersonal comparisons of [qualia] are (apparently) systematically impossible" (1988 p. 46). Seager suggests that privacy amounts to "incommunicability and privileged access," and notes that once we have dealt with ineffability and intrinsicness, the only thing that privacy contributes that these other properties do not contribute is the idea that two individuals cannot share mental state tokens, and can only share mental state types (1999 p. 92). In other words, Dennett claims that qualia are private because it is impossible for two individuals to jointly compare qualia, and Seager claims that two individuals cannot share a quale (i.e. a mental state token) since qualia are mental, and no two individuals can share a numerically identical mental state. In addition, Seager's claim suggests an explanation for why we might say that two individuals can share the same mental state or quale when speaking casually: the individuals are both in mental states of the same type. Seager is clearly correct to suggest that a kind of incommunicability is captured by (or is a consequence of) ineffability, but while Seager's analysis of the additional import of privacy is clearly a metaphysical truth, it clearly doesn't capture what Dennett means by "privileged access." In addition to the claim that two individuals cannot share particular mental states, it seems to me that Dennett is suggesting that the mental states in question are directly accessible to introspection. After all, the ineffability of qualia wouldn't be so frustrating if we weren't aware of them. After all, we're not worried about the ineffability of subconscious mental states. Thus, privacy suggests that qualia must walk a kind of epistemic tight-rope: they must be accessible to those who have them, yet they must not be so accessible that they can be communicated.

The severe epistemological constraints on our knowledge of the strange attractors in a chaotic system, combined with the hypothesized token-identity of these attractors and our qualia provides just such a balancing act. As discussed above, chaotic systems cannot be described or communicated finitely. At the same time, a chaotic system can know that it is in the basin of attraction of a particular strange attractor if being in that state constitutes the very knowledge in question. In other words, the brain does not need a separate system that monitors the sensory subsystems and represents the results in another fashion; instead the physical state of a sensory subsystem plays two causal roles. The first causal role is to represent the world as it is sensed, and the second is to represent itself. This view of the various causal roles played by brain states is similar to that held by some advocates of the mind-brain identity theory (Lewis, 1980; Armstrong, 1990). It amounts to a rejection of Higher-Order-Thought theories of consciousness (Rosenthal, 1986), and to a kind of first-person point of view, or a kind of subjectivity like that demanded by Nagel (1974). While this kind of self-representation does not depend on chaos, the incommunicability of chaotic systems does. Consequently, there is at the same time a strong epistemic constraint on what can be said about the states of a chaotic system and a kind of accessibility to those states for the system itself that dodges the epistemic constraint since it does not involve the creation of new descriptions or representations of the state of the system. This constraint seems to be as good a model for privileged access and privacy in physical systems as one could hope for, and it is why I rejected the third proposed model of fuzziness considered earlier.

Dennett's fourth suggestion is that qualia are (d) directly or immediately apprehensible in consciousness. Dennett suggests that it is unclear what it amounts to, and I want to suggest that part of the reason he finds this property unclear is that this property is closely related to the notion of privileged access discussed above. I believe that (c) privacy emphasizes the privilege inherent in privileged access while (d) direct or immediate apprehensibility emphasizes the accessibility inherent in privileged access. Seager suggests that qualia are directly or immediately apprehensible just in case they are "non-inferentially accessible," (1999 p. 92) and the fact that Dennett lists this property separately from (c) privacy may be why Seager doesn't consider privacy to include the notion of privileged access. Since I include privileged access in privacy, I have already alluded to how the direct or immediate apprehensibility of qualia is to be explained. According to the chaotic emergence theory, each point in the phase space of a system represents a possible total state of that system. When the system is within the basin of attraction of a strange attractor, the system can know that this is the case because being in the basin of attraction of that strange attractor is part of the total state of the system. If the system includes feedback loops or self-monitoring systems of any kind, the total state of the system will be influenced by them, and will represent itself in some way. Feedback loops are characteristic of chaotic systems. If the system is an evolved intentional system, then the ability to represent its own state may have evolved just as the ability to represent the environment has evolved. Thus, if a chaotic system is in the basin of attraction of a strange attractor, it is possible for that state to represent both the state of the environment and the state of the system itself, depending on how the information is interpreted. Something further should be said regarding higher-order consciousness, but because the total state of any system plays a causal role in the future behavior of that system, there is no question about whether or not the system's future behavior can be influenced by its internal states, and there is no question about whether or not a system can in principle have access to its internal states. Furthermore, this all seems to fit very well with what we know about human beings since we already know that many parts of the human central nervous system include feedback loops and other reciprocal connections, and since various theorists have suggested on evolutionary grounds that representations of one's own states may be beneficial in providing the ability to predict the behavior of others (Byrne 1995). Thus, all four of Dennett's characteristics of qualia seem to be explained by the chaotic emergence theory.

Characteristics of Qualia: Stubenberg

I now turn to the four characteristics mentioned by Stubenberg. The first of these is the (e) monadic character of qualia. For Stubenberg, this means that "the nature of qualia is independent of their relations to other things." (1996 p. 41) This, together with Dennett's use of 'atomic' suggests that Stubenberg's claim that qualia are monadic is identical to Dennett's claim that qualia are intrinsic. I have discussed above how the chaotic emergence theory explains the intrinsic nature of qualia, so I turn without further comment to Stubenberg's remaining three characteristics of qualia.

Stubenberg's second characteristic of qualia is (f) simplicity. In elucidating this, Stubenberg says that qualia "lack inner structure or complexity." (1996 p. 41) First, we must note that this must be interpreted to mean that qualia have no apparent structure or complexity. The flavor of fine wine is famously complex, and yet this complexity is not apparent to one who is untrained in wine tasting. Similarly, those who often work with color are better able to discern the relative weight of different basic colors in a compound color. In these and other similar cases, additional discriminative abilities do not seem to cause the relevant experiences to fragment into a compound of qualia, rather the analytic abilities of the perceiver seem to be increased in a way that allows the perceiver to discern aspects of a quale in ways that were previously impossible. Although Dennett suggests that it is unclear whether or not greater discriminative abilities changes the qualia that one experiences, it doesn't seem that greater discriminative abilities eliminate the apparent simplicity of qualia. Rather, these abilities allow one to more precisely locate a quale in the space of qualia provided by the appropriate sensory modality. Thus, while qualia are apparently simple, the qualia spaces of which they are a part are complex and may be more or less finely divided by different individuals.

This sense in which qualia are apparently simple can be explained by the mild holism the chaotic emergence theory requires of a system that is in the basin of attraction of a strange attractor. Each point in the phase space of a system represents a distinct state of that system completely and unambiguously in the sense that it represents precise values for all the quantities that are relevant to the system.[4] Thus, the claim that a system is at some point in its phase space is a claim about what we may call the total state of that system. This is a kind of trivial holism that chaotic systems share with any system that can be modeled using a phase space. There is, however, a related but stronger sense of holism that applies to chaotic systems. To explain this, it is first necessary to consider what happens in the spaces arrived at by eliminating one or more dimensions from those that make up the original phase space of a chaotic system. Call any of these a reduced space of the system. The phase spaces of some chaotic systems are minimal in the sense that there are no reduced spaces in which the chaotic system exhibits chaotic behavior (Devaney, 1989). The phase space of any chaotic system that is not itself minimal will have a minimal reduced space within it. In contrast, all the subspaces of systems with point attractors, limit cycle attractors, or torus attractors have point attractors or limit cycles. Thus, chaotic systems that are in the basin of attraction of a strange attractor are exhibiting a kind of behavior that cannot be broken down to its components without losing the special characteristic of that behavior. So if having a quale is identified with being in the basin of attraction of a strange attractor, such mental states are simple because they cannot be decomposed into sets of other qualia. On this model of qualia, the simplicity of a quale consists in the fact that a strange attractor is the lowest level of qualitative experience, which is constructed of elements that are not themselves qualitative experiences.

Increased discriminative abilities and the complexity of qualia spaces can also be explained using the chaotic emergence theory. Since strange attractors lie within the phase space of a chaotic system in which other strange attractors may lie, it is possible to explain the qualia space by appeal to the phase space of the chaotic system. Such spaces will be multi-dimensional and can be as complex as required to explain the complexities of a qualia space. Increased discriminative abilities can be explained by regarding learning as movement within the phase space of the relevant system.[5] The system's state would move from a region of the phase space to another similar region of the phase space. The two regions would be similar except that the new phase space region would contain two strange attractors and their basins of attraction where the old phase space region contained only one. This kind of explanation of increased discriminative ability is supported by Freeman and Skarda, who suggest that rabbits learn to identify new scents as a result of the appearance of new strange attractors in the phase space of their olfactory bulb (1987; 1990).

There is thus a sense in which the strange attractors of chaotic systems form wholes that cannot be separated into chaotic parts. This illustrates the fact that emergence is a kind of holism, and I take this to be an explanation for the simplicity that Stubenberg sees in qualia. This holism allows the chaotic emergence theory to suggest that the states of a system are the sums of the states of the subsystems that make up the system, and that such sums are simple in something like the way that the sum of two numbers is another number, not a thing with parts.

Stubenberg's third characteristic of qualia is that they are (g) homogeneous. Stubenberg suggests that homogeneity amounts to 'grainlessness.' To say that different kinds of qualia are homogeneous or uniform is implausible if we mean that they all feel the same. After all, it is the distinct feels of qualia that allow us to distinguish them at all. The suggestion that qualia are grainless also indicates that Stubenberg does not intend homogeneity to suggest a comparison between different qualia. Stubenberg must mean that individual qualia are homogeneous; because this concept seems closely related to simplicity I will attempt to interpret it in a way that makes the two characteristics distinct. Consequently, when Stubenberg writes that qualia are homogeneous, I understand him to mean that when a quale is presented as extended in a sensory field, it is non-discrete, smooth, or continuous.

Colors, for example, do not appear in pixels. A related feature of qualia is that they all fall somewhere in a continuous range or space. The color spectrum, to use the same example, appears to form a continuous range of color from red to violet.[6] I don't think this is what Stubenberg has in mind, but the two concepts are closely related, and I will deal with them both together.

The chaotic emergence theory uses the mathematical formalisms of dynamical systems theory to explain the existence of emergent properties of chaotic systems. In developing the theory, I have assumed that the phase spaces of the systems to be considered are made up of continuously-valued quantities. I have also assumed that the dynamics of these systems will be continuously-valued functions on their phase spaces. These assumptions seem reasonable since the systems in question are physical systems, and science commonly uses continuously-valued quantities and functions to model the behavior of physical systems.[7] Thus, the chaotic emergence theory assumes that the dynamics of systems with chaotic emergent properties are continuous in the mathematical sense. Since the quantities that make up the phase spaces of these systems are continuous, the phase spaces themselves are continuous, and consequently the qualia-spaces that emerge from the behavior of such systems should also be continuous. Special explanation would be required in order to produce discrete qualia spaces from continuously-valued physical systems. Similarly, when such a system represents a qualia as extended in space or time, it will do so using continuously-valued quantities. The represented extension of a quale is no less a part of the phase-space than the quale itself. Again, since special explanation would be required in order to produce a non-continuous representation, I think the fact that the phase spaces of the systems in question are continuous explains the continuity of the qualia and their extensions.

There is a potentially puzzling aspect of qualia that is closely related to the issue of homogeneity: while qualia spaces seem to be continuous, there are limits to our abilities to distinguish different qualities. This is embodied in experiments that measure the smallest noticeable difference in a sensory modality (e.g. the smallest humanly detectable difference in volume or frequency of a sound). This is closely related to our ability to learn to be more sensitive to differences in our experiences (e.g. as people can learn to make finer distinctions about the flavor of wine by taking a wine-tasting class). However, this puzzle is easily resolved using the chaotic emergence theory. The difference between the grain of our experiences and the grain of our ability to report such experiences must be expected. Because we can only make finite descriptions of qualia based on the basins of attraction that constitute them and the internal structure of the attractors within these basins and because these experiences essentially involve infinite information, the descriptions must have a threshold below which they cannot go. While training can move the state of our nervous system into regions of the phase space that allow us to make finer and finer discriminations as discussed above, we can never find a region in which the basins of attraction are so small that qualia can be exactly characterized or communicated. This puzzle is what Raffman calls the 'memory constraint' (1995), and the fact that the chaotic emergence theory can explain it seems to me to be further evidence for the adequacy of the theory.

Stubenberg's final suggestion is that qualia are (h) irreducible. Since the chaotic emergence theory suggests that being in the basin of attraction of a strange attractor is an emergent property and uses a definition of emergence that requires irreducibility, this condition is clearly met by the theory. Moreover, the theory gives an explanation of the irreducibility involving the chaotic dynamics of the brain and the infinite information required to characterize chaotic dynamics exactly. The number of possible states of a central nervous system is so large, and the complexity of the dynamics of that central nervous system is so great, that it is impossible for any finite agent to describe or communicate the relationship between a quale and that central nervous system. Irreducibility is the last of the eight characteristics of qualia that Dennett and Stubenberg mention. Each of the eight characteristics of qualia can be explained in terms of the strange attractors of a chaotic system. Thus, if qualia are plausibly the only things that share the eight characteristics considered here, then the chaotic emergence theory plausibly explains qualia in physical terms.

Step Three: Implications.

It might be suggested that the theory described here doesn't really explain qualia. There is, to some degree, still an explanatory gap since the theory includes the claim that it is impossible to reduce a particular quale to physical properties or processes. However, if the eight characteristics discussed above pick out the set of qualia uniquely, and if the central nervous system is really a chaotic system in which classes of strange attractors can be correlated with classes of qualia, then the width of this gap is much smaller than it has been in the past. The gap will be just large enough to account for the ineffability of qualia, and no larger. In addition, the chaotic emergence theory is an empirical theory subject to experimental confirmation, not an a priori theory subject only to proof. The explanatory gap will be reduced as the theory becomes better and better understood, as the theory is developed further, and as additional empirical confirmation is found. This is what is to be expected given a historical look at other scientific theories. The identification of water with H_2O was probably regarded as insufficiently explanatory by some, but that identification became more and more clear as the atomic theory was developed further and as the identification became more familiar. Therefore, even if the theory doesn't seem sufficiently explanatory at present, this isn't a problem since the explanatory power of the theory will grow or the theory will be discarded due to empirical insufficiency.

Thought I won't argue the point in detail here, I think the argument presented in this paper suggests what is wrong with those arguments that depend crucially on the ineffability of qualia. I put in this class Nagel's argument about what it is like to be a bat (1974), McGinn's argument about the impossibility of understanding an experience without first having had it (1989), Jackson's argument about Mary the color scientist (Jackson, 1986), and Levine's argument for the existence of an explanatory gap (Levine, 1983). If, as I believe, each of these arguments includes the implicit assumption that we can either know everything physical about consciousness, or we cannot know anything physical about consciousness. One consequence of what I have suggested in this paper is that this is false. If the argument above is correct, there is good reason to think that one can have a kind of limited knowledge of the physical basis of consciousness.

I have presented here an argument that having a chaotic central nervous system explains qualia in physical terms and implicitly provides reason to think it is possible to find logically sufficient conditions for the existence of qualia. Chaos alone is not sufficient for qualia since there are chaotic systems that have no qualia, such as the weather. Additional conditions are necessary to obtain a set of logically sufficient conditions for the existence of qualia. My intuition is that the nature of a system's interaction with the environment and the nature of the high-level functional organization of that system determine whether or not the system has qualia. Further, I suspect that evolutionary theory may help us to identify those systems that have the right kinds of interaction with the environment and the right functional organization. Thus, I think that the evolutionary role of a chaotic central nervous system is the right place to look to complete the story begun here.

One advantage of the chaotic emergence theory is that emergence gives qualitative mental states a respectable metaphysical status different from that held by reducible properties. This respects our intuition that the mind is special and distinct from the physical while at the same time respecting our intuition that the physical is all that there is. Another advantage of the chaotic emergence theory is that it explains why qualia are so difficult to explain. Chalmers is correct: consciousness is a very hard problem (1995). Some aspects of the problem are impossible to solve. However, the theory predicts this and explains why it is the case. A third advantage is that the theory is empirically testable. In fact, as suggested above, some experimental evidence suggests that the theory is correct. More investigation is required of course, this is just a beginning. Because it has these advantages, because it can provide reasons to think the physical is sufficient for the mental, and because it can provide reasons to think that some arguments against physicalism are incorrect, the chaotic emergence theory appears to me to provide good reason to think that qualia can be explained in physical terms.

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Notes.

1. Because the rate at which the difference between the predicted future state of a chaotic system and the actual future state of that system grows is typically exponential, the precision with which measurement of the system and computation of its predicted future state must be made is an exponential function of the time interval over which the prediction is to be made. Thus, given typical measurement equipment, the behavior of a chaotic system can be predicted for only a short period of time, which is generally too short to be useful. For predictions of the behavior of chaotic systems sufficiently far in the future, the universe does not contain enough resources to perform the measurements and computations required to obtain a good prediction.

2. I owe this objection to Joseph Goguen.

3. Think of a marble in an egg carton. A shock to the egg carton can move the marble into an adjacent well. If the egg carton was more irregularly shaped, the shock required to move the marble from one well to another would be different depending on the height of the divider between the two wells.

4. Some points in the phase space may not be nomically possible, but this doesn’t show that a point in a phase space is somehow an incomplete representation of the state of the system.

5. There is a second, mathematically equivalent explanation in which we regard learning as a transformation of the system into a new system. The choice of the method of explanation is a largely pragmatic issue involving a decision about which physical quantities are parameters of the system in question and which are variables.

6. There are of course more sophisticated analyses of color in which colors form a three-dimensional color space. One such color space is defined by hue, saturation, and lightness; another is defined by cyan, magenta, and yellow; many others exist. I don’t think that the particular model

of color space makes a difference for the argument given here, so long as the color space is continuously-valued.

7. Quantum mechanics clearly raises difficult questions regarding the validity of the assumption that physical quantities are continuous. For the purposes of this discussion, I assume without argument that physical quantities are continuous.