ABSTRACT. The purpose of this paper is to clarify the concept of marginal consequences of a group moral action. The situations in which a group action is taken are studied and classified. The assumption that the agents of a group action are similarly (or symmetrically) situated is clearly specified and emphasized. Then a probabilistic approach is used to determine the marginal consequences of a group action. It is shown that the refutation of utilitarian generalization by Bart Gruzalski is unjustified because of his misinterpretation of marginal consequences. Finally the delicate situations of maximizing and minimizing conditions are analyzed. It is concluded that if the probability of participation \( p \) is not known, then the contributory consequences approach is the only approach that can be used. If the probability of participation \( p \) is known or can be estimated, then the use of the marginal consequences approach seems to be justified and preferable.

I. INTRODUCTION

There are two different interpretations of the consequences of a group moral action, namely "contributory consequences" and "marginal consequences". Donald Regan describes a simple example of these two interpretations as follows.

"Suppose there are 100 agents, all symmetrically situated so far as the basic description of the problem is concerned. There is a possible benefit which can be secured by the participation of at least sixty agents. Any sixty will do, and the participation of more than sixty produces no extra benefit. Suppose eighty agents are in fact participating. The question is, for purposes of AU, how much credit does each individual agent get for the production of the benefit which results. The 'marginal consequences' approach says that each agent gets credit for nothing . . . The 'contributory consequences' approach says that each agent gets credit for one-eighthieth of the benefit".1
Peter Singer, in his defense of act-utilitarianism, suggests the adoption of the contributory consequences approach. Donald Regan, on the other hand, argues that the contributory consequences approach is not appropriate and, hence, adopts the marginal consequences approach for the interpretation of AU and his PropAU. Bart Gruzalski, in a recent paper, uses the marginal consequences approach to show a divergence between AU and utilitarian generalization. He claims that the prescription by UG is not as plausible as that by AU and, consequently, refutes UG.

In this paper I shall attempt to clarify the concept of the marginal consequences approach, emphasizing the assumption that the agents of a group action are similarly (or symmetrically) situated. With this assumption, a natural inference is that the generally-accepted concept of marginal consequences is not appropriate. I shall use a stochastic approach to determine marginal consequences. In addition, I shall show that the refutation of utilitarian generalization by Gruzalski is unjustified. The main reason why this is so is his misinterpretation of the concept of marginal consequences.

2. GROUP ACTION

A group action is understood to be an action in which each agent participates and also shares the consequences of the action with other agents.

There are different types of group action problems, with different natures. Some of them are not really moral problems. For instance, one type of problem is a pure two-person, zero-sum game problem. The action taken by an agent in such a problem is not a real group action, in that the gain or loss is not shared by the two persons. Instead, a gain of one agent is the loss of the other and the problem is to find the best strategy or the optimal method of making oneself win and the opponent lose. This type of problem is certainly not a moral problem, but rather a purely technical optimization problem.

I shall discuss group action in two situations.

In one situation the agents can communicate, cooperate and optimize the action collectively. For such a problem it is not difficult to establish a principle similar to the basic principle of utility. B.C. Postow proposes an "act-utilitarianism as follows: In any given situation, any group of one or more agents ought to follow a course of action by means of which the group would produce the most good that it can produce in that situation". Such a problem, as I see it, is a technical one rather than a moral one.

In the other situation in which a group action takes place, there is no cooperation and, hence, in general no communication among agents, so that each agent is supposed to know nothing about what courses of action the other agents will take, and to do his/her own part of action independently of the actions of others. Note that even without communication agents can still have well founded subjective probabilities for propositions concerning the others which propositions are true. One may ar-
gue, therefore, that there seems no good reason for saying that even so they cannot know what to expect of one another. However, this knowledge about others in the probabilistic sense will be taken care of by the probability of participation $p$, which I shall introduce later. In fact the probabilities of participation of all members, and the subjective probabilities of one’s participation known by all others, are of statistical nature. In an extreme case the probability of participation of member A may be 1 and member B may precisely know that it is 1. But a prescription for action is supposed to be for the general case, not for such an exceptional case. If an agent has some actual knowledge of some other agent’s decisions for the actions they are to take, this is regarded as an exceptional case and will not be discussed here. Moreover, no ethical theory can give a prescription based on all these probabilities, because it is well nigh impossible to find all these probabilities. What I shall do is to use a probability of participation $p$, which is assumed to be the same for all members.

This type of group action can further be subclassified into two different kinds. One kind is such that each agent does not know which action is optimal because the consequences of the group action depend on the combination of choices of all agents. There are again two different situations. One situation is that different members play different roles and, hence, the members are not similarly situated. Thus there is no general prescription given to all members. This kind of group action is not what I intend to study here, and will be discussed only very briefly later in this section.

Another case is that the members of the group are similarly situated, but some of them have to act differently in order to maximize utility. (This kind of action has been discussed by Marcus G. Singer and David Lyons. Some of the examples given by Donald Regan are also of this nature.) Let us consider a hypothetical example. Suppose that a group plays a game with another group using two dice. The rule of game is that the group wins if the sum of the points on the two dice is an odd number and loses if the sum is an even number. Now let the two members of the group represent two dice. Each member should choose a digit among 1 to 6. If members A and B can communicate with each other and cooperate, then they can decide quite easily by settling on either that A chooses an odd digit and B chooses an even digit or that A chooses an even digit and B chooses an odd digit. As will be discussed later, it will be assumed that there is no communication and cooperation between the members in that neither A nor B knows whether the digit chosen by the other member is odd and even. So A and B do not know whether an odd digit or an even digit is the right digit to choose, and have to depend on a prescription for acting. Now the prescription given to A and B should be different. If the prescription given to A is "choose an odd digit", then the prescription given to B should be "choose an even digit", and vice versa. In fact, this kind of problem is a pure optimization problem at the prescription level and, hence, does not involve morality at the member level. I shall not discuss this kind of problem in this paper, and shall restrict my discussion to the second kind of problem discussed in the following.

The second kind of group action is such that the optimal alternative is common to all agents and each agent knows which alternative is the optimal one. This kind of action can be exemplified by the action of voting. In this case there are only two alternatives, namely voting or not voting. The problem is what prescription
should be given to the agents. One may classify the group action of voting into the first kind and advocate different prescriptions given to different members. If voting involves a small cost, then the situation that the number of members voting is just sufficient to win the voting should be considered the optimal situation, particularly from the utilitarian point of view. For instance, in a meeting of 5 members, an optimal situation of majority voting would occur when exactly 3 agents vote, because 4 or 5 agents voting, while having the same effect of winning, will add slightly to the cost. However, when a prescription is given to an agent, the agent may not necessarily take act in accordance with the prescription, and winning is not ensured when only 3 out of 5 vote. This is in fact the main reason why whether the prescriptions given to all members should be the same or different has been a controversial point. Since the success or failure of a group action has a certain probability, the problem had better be studied as a probabilistic system.

From my utilitarian point of view, the same prescription 'vote' should be given to all members. In the following I shall study this type of group action as a probabilistic system, using a new concept of marginal consequences, and shall defend a uniform prescription for all members of the group.

3. GROUP ACTIONS AND THRESHOLD

The group action under discussion is an action taken jointly by a number of agents, each doing the same thing. The relation between each agent's act and the overall consequences of the action varies widely, depending upon the nature of the threshold. By threshold is meant a critical amount of work, effort or number of members participating, such that when the magnitude of these factors associated with a particular group action is equal to this critical amount, there is a critical step change of the utility of the action, so that at or over this critical point the action usually will be a success and will have a large positive utility. On the other hand, when this magnitude is less than this critical amount, i.e., below the threshold, then the action will usually be a failure with a zero or negative utility. In this paper the term threshold is used in the sense of number of members participating in a group action. 9

For convenience of study, group actions will be classified according to the clarity or sharpness of threshold, as follows. 10

(1) Group Action With No Threshold

A group action with no threshold is one such that no threshold at all can be identified. In such a case, the value or utility of the consequences is usually measured by the work done, which is continuous and does not have a step change. For instance, consider the situation in which a number of workers participate in bricklaying. Assume that all workers have equal ability and efficiency in laying bricks (i.e., each worker lays the same number of bricks in a unit time). Then the work done is proportional to the time and the number of workers participating. Suppose that the bricklaying job is a very large one or is almost endless, so that the concern is the amount of work done at certain time instants, not whether the group action is a suc-
cess or failure. Therefore there is no threshold at all. The share of consequences attributable to each worker is proportional to his time spent in bricklaying, and can readily be determined.

(2) Group Action With Vague or Approximate Threshold

A group action with a vague or approximate threshold is one such that the threshold is so vague that it is difficult, if not impossible, to determine. For instance, consider the bad action of treading on a university lawn. If the amount of treading is small, then there will be only slight, temporary damage to the grass. The growth of the grass will more than compensate for the temporary damage, so that the grass will maintain its flourishing state without permanent damage. If the amount of treading is excessive, then the growth of grass will not be sufficient to compensate for the damage, so that eventually the grass will diminish in amount and die. In this case, it is very difficult to say how much threading is a threshold below which no permanent damage will be caused.

Another example of such action is the strike referred to by Gruzalski. A strike has been called and the participation of 300 workers out of a total of 450 workers is assumed to be just sufficient to make the strike a success. Now this threshold of 300 workers is actually an approximate figure or assumption, in that the success or failure may be a continuous rather than a step function. Moreover, even if there is a clear-cut step change of consequences, nobody can predict exactly where the threshold lies.

(3) Group Action With Sharp Threshold

A group action with a sharp threshold is one such that the threshold is clear-cut, sharp or definitely set. For instance, voting is such an action. Out of 99 members, 50 is a majority but 49 is a minority.

There is another kind of group action in which each member plays a specific role which may be different from those of others. In such a case the group is a team or a tightly-connected system and each agent is a component of the system. In a tightly-connected system each component is essential and indispensable. For example, a mistake on the part of any one member of an espionage team will certainly spoil the whole espionage plot. For such a situation, the participation of every agent is essential to the success of the group action and the threshold is thus equal to the total number of members in the group. For a group of \( n \) members the threshold is also \( n \).

Group actions with threshold effect have been discussed in detail by David Lyons. He calls this condition "non-linearity". Lyons classifies actions with threshold effect into sequential and non-sequential ones and explains them, in a rather complex way, in terms of "relevance of others' behavior".

The group actions to be discussed in this paper are of class (3) or of class (2) with an arbitrarily-assumed threshold. The situation of marginal consequences occurs only for categories (2) and (3), because only for these two categories is the value or
utility of the group action a step function which rises suddenly from zero or a small negative value to a fixed positive value when the number of persons participating in the action attains a threshold. When the number of persons participating is already equal to or greater than the threshold, or is less than the threshold by at least two, the participation or non-participation of a single additional member will not further affect the value or utility of the action. These are what David Lyons calls "maximizing conditions" and "minimizing conditions", respectively. It is only when the number of persons participating is just one below the threshold that an additional participating person will change the state and the value or utility of the action. For instance, in a meeting of 99 members, 49 is only a minority, but 50 becomes a majority.

4. BASIC ASSUMPTIONS IN THE MARGINAL CONSEQUENCES APPROACH

The study of marginal consequences of group actions of categories (2) and (3) discussed above may be done with the aid of probability theory. I thus consider the system a stochastic one. Before proceeding with the analysis, it is essential to make some basic assumptions regarding the conditions under which the consequences are calculated. A basic assumption in the study of group actions of this nature is that the agents or members of the group are "similarly (or symmetrically) situated". The concept of this assumption is not very clear-cut and sometimes is controversial. In order to have a quantitative probabilistic analysis of marginal consequences, it is essential to clarify this concept. In the following, instead of presenting a precise definition of "similarly situated agents", I give three clear-cut assumptions, which, in fact, naturally imply the assumption of similarly situated agents.

(1) There is no communication or cooperation among the agents.

In the case of a small number of agents, it is of course true that communication and cooperation among the agents is possible and optimization can readily be achieved by a common agreement. For instance, in the case of majority voting in a meeting of 5 members, an optimal situation would occur when exactly 3 agents participate. On the other hand, when the number of agents is large, then this kind of communication, cooperation, agreement and optimization is not possible. In the case of a strike, there may be hundreds or even thousands of agents and in the case of voting in election, there be hundreds of thousands or even millions of agents. Thus it is reasonable to assume that communication, cooperation, agreement and collective optimization among agents do not exist.

(2) Each agent does not know the decision on participation of others.

Since there is no communication, each agent does not know the decisions of other agents, i.e., whether or not another agent will participate. Each agent, however, can have a conjecture or prediction about the intentions of other agents, in a probabilistic sense.

Showing the validity of this assumption (2) is equivalent to showing that the assumption of an agent's exact (nonprobabilistic) knowledge of other agents' decision
about participation or nonparticipation is invalid. So I shall have a look at all the different situations in which a certain agent $A$ knows the decisions of a subset of the set of all members of the group. There are altogether four different cases.

Case (1) is that this knowledge of decision is in the probabilistic sense, i.e., it is a kind of conjecture or prediction. Note that this is the general average situation in the statistical sense. Thus this case naturally has to be studied by using a probabilistic and statistical approach, and can well be taken care of by the probability of participation $p$ of each member and the probability of success $P$ of the group action.

Case (2) is that this knowledge of decision is definite, but the number of other agents whose decisions are known by $A$ is very small. For instance, agent $A$ might know that another agent $B$, who is, say, a relative or close friend of $A$, will participate. Statistically, the number of other agents whose decisions are known by a certain agent is a random number. Normally the average of this random number is so small compared with the total number of agents in the group that this kind of knowledge does not affect the general decisions of agents and the result of the group action. Thus this case is pretty normal and is susceptible of a quantitative study using a probabilistic approach as well.

Case (3) is that this knowledge of decision is definite, and the number of other agents whose decisions are known by agent $A$ is abnormally large. Besides, many other members have the same or similar knowledge. Then this situation will naturally affect the decisions of other agents and, in turn, the result of the group action. This means that this situation disturbs the probability of participation $p$ and the probability of success $P$. This contradicts the assumption (3) to be discussed later that there is a given uniform probability of participation $p$ and, hence, this situation is excluded from consideration here.

Case (4) is that this knowledge of decision is definite, and the number of other agents whose decisions are known by agent $A$ is abnormally large, but all other members of the group do not have the same or similar knowledge. This is considered a really exceptional case. Such an exception can either be attached as an exceptional case to a moral rule or be accepted to be up to the agent's own special consideration for decision but not subject to a general moral prescription. In other words, it is in general beyond the moral prescription to cover all these exceptional cases.

Therefore, the assumption of no knowledge of other agent's decision about participation or nonparticipation seems to be fully justified.

(3) There is a given uniform probability of participation $p$.

When the probabilities of the states of affairs are not known, one approach in decision analysis is to simply ignore the probabilities and not to use them at all. For instance, the maximax and maximin methods are based on this approach. However, these nonprobabilistic decision methods are very crude ones and sometimes may lead to absurd conclusions. Therefore, I advocate that, if no objective probabilities are
available, subjective probabilities be used instead, because it seems to be better to use subjective probabilities than to use no probabilities at all.

Hence, in the analysis of marginal consequences, it is assumed that probabilities of participation of the agents are all equal to $p$ and known. This probability $p$ may be an objective one obtained from past experiences or statistics, say a Gallup poll, or, if no objective probability is available, may be a subjective one. Theoretically, the probabilities of participation of the members are not identical, and in the case of subjective probability, the probability not only varies with the member $i$ whose probability of participation is being estimated, but also with the member $j$ who estimates $i$'s probability of participation. That is to say, for each member $i$ and another member $j$, $j$ estimates a subjective probability $p_{ij}$ for the proposition that $i$ will participate. Practically, however, it is impracticable or even impossible to find the probabilities of all agents. Since the agents are assumed to be similarly or symmetrically situated, it seems justified to assume a uniform probability of participation $p$ for all agents.

Now the agents are said to be similarly (or symmetrically) situated in the sense of the above assumptions. That is to say, (1) before the group action takes place there is in general no communication or cooperation among the agents, (2) each agent in general does not know the decisions of others about participation or nonparticipation, and (3) there is a uniform probability of participation $p$. Thus the assumption of similarly (or symmetrically) situated agents may be said to be a combined or simplified statement of the above three assumptions.

5. ANALYSIS OF MARGINAL CONSEQUENCES

Now I shall come to the analysis of marginal consequences proper. While a general analysis will be presented, I shall also use a numerical example for illustration. To demonstrate the assumptions clearly and to make calculations easy, I shall use a very simple hypothetical numerical example. Instead of a large number of agents or members of the group, there are only 5 agents altogether, all similarly situated. The participation of at least 3 agents in a certain course of action is considered a success and will result in a benefit of, say, 100 units.

Now in this example, a member $A$'s participation will have no effect on the success of the group action if, out of the four other agents, 0, 1, 3 or 4 agents participate. For, when 0 or 1 agents participate, $A$'s participation will make a total number of 1 or 2, respectively, which is still not sufficient for the success of the group action and when 3 or 4 agents participate, the number of agents participating, $A$'s participating apart, is already sufficient for the success of the group action and, hence, $A$'s participation adds nothing to this sufficiency. $A$'s participation is significant only when there are precisely 2 agents participating, in which case $A$'s participation changes the number participating from 2 to 3 and turns the group action from failure into success.

With the afore-mentioned assumptions, the expected utility of a group action can be calculated by using the binomial expansion of probabilities as follows: let

\[ n \] be the total number of agents or members in a group,
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i be the number of agents participating,
k be the threshold,

\( C(n, i) \) be the number of combinations of subsets of i members taken from a set of n members,
p be the probability of participation of each agent,

\( q = 1 - p \) be the probability of non participation of each agent,
P be the probability of success of the group action,
U be the expected utility of the group action.

Then we have

\[
P = \sum_{i=k}^{n} C(n, i) q^{n-i} p^i
\]

(5.1)

For the above example, \( n = 5 \), \( k = 3 \). Assume \( p = 0.7 \). Then

\[
P = \sum_{i=k}^{5} C(5, 3) (0.3)^{5-i} (0.7)^i
\]

\[
= \frac{5!}{3!2!} (0.3)^2 (0.7)^3 + \frac{5!}{4!1!} (0.3)^1 (0.7)^4 + \frac{5!}{5!0!} (0.3)^0 (0.7)^5
\]

\[
= 0.309 + 0.360 + 0.168 = 0.837
\]

\[U = 0.837 \times 100 = 83.7 \text{ units}\]

The average utility contributed by each agent is

\[U/n = 83.7/5 = 16.7 \text{ units}\]

The utility of marginal consequences of a member A's participation in a group action is equal to the utility of the successful action, multiplied by the difference between the probability of success when A participates in the group action and that when A does not participate. Or, equivalently, it is equal to the utility of the successful group action, multiplied by the probability that exactly \( k - 1 \) other members participate, where \( k \) is the threshold. In general, the probability of success if agent A does not participate is
\[ P' = \sum_{i=k}^{n-1} C(n-1, i) q^{n-1-i} p^i \]  \hspace{1cm} (5.2)

and the probability of success if A participates is

\[ P''' = \sum_{i=k-1}^{n-1} C(n-1, i) q^{n-1-i} p^i \]  \hspace{1cm} (5.3)

The incremental probability due to A's change from non-participation to participation is

\[ \Delta P = P''' - P' = C(n-1, k-1) q^{n-k} p^{k-1} \]  \hspace{1cm} (5.4)

In the above example, if A has decided not to participate, then the probability of success will be

\[ P' = \sum_{i=3}^{4} C(4, i) (0.3)^{4-i}(0.7)^i \]

\[ = \frac{4!}{3!*1!} (0.3)^1(0.7)^3 + \frac{4!}{4!*0!} (0.3)^0(0.7)^4 \]

\[ = 0.412 + 0.240 \]

\[ = 0.652 \]

The expected utility is \( U' = 0.552 \times 100 = 55.2 \) units.

If A has decided to participate, then the probability of success will be

\[ P''' = \sum_{i=2}^{4} C(4, i) (0.3)^{4-i}(0.7)^i \]

\[ = \frac{4!}{2!*1!*1!} (0.3)^2(0.7)^2 + \frac{4!}{3!*2!*1!} (0.3)^1(0.7)^3 + \frac{4!}{4!*0!*1!} (0.3)^0(0.7)^4 \]

\[ = 0.265 + 0.412 + 0.240 = 0.917 \]

The expected utility is

\[ U'' = 0.917 \times 100 = 91.7 \text{ units.} \]
The expected utility of the marginal consequences of A's participation is thus
\[ \Delta U = U'' - U' = 91.7 - 65.2 = 26.5 \]

The expected utilities \( U, U', U'' \) and \( \Delta U \) are listed at the first row of Table 3.

<table>
<thead>
<tr>
<th>Agent</th>
<th>( U )</th>
<th>( U' )</th>
<th>( U'' )</th>
<th>( \Delta U = U'' - U' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>83.7</td>
<td>65.2</td>
<td>91.7</td>
<td>26.5</td>
</tr>
<tr>
<td>B</td>
<td>91.7</td>
<td>78.4</td>
<td>97.3</td>
<td>18.9</td>
</tr>
<tr>
<td>C</td>
<td>97.3</td>
<td>91.0</td>
<td>100</td>
<td>9.0</td>
</tr>
<tr>
<td>D</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>E</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>0</td>
</tr>
</tbody>
</table>

Note that if A makes a decision but keeps the decision to himself without letting the other agents know, then this is equivalent to keeping the decision unmade until the time of participation. However, if he announces or otherwise lets the other agents know of his decision, then he changes the information set of the other agents. Thus, after A has announced that he will participate, the expected utilities when B (1) has not decided whether or not to participate, (2) has decided not to participate, (3) has decided to participate, and the utility of the marginal consequences of B's participation, will all be different from those before A has decided to participate. These utilities are the entries under the headings \( U, U', U'' \) and \( \Delta U \), respectively, in the second row of Table 3. Similarly, if B has announced his decision to participate, then the expected utilities when C (1) has not decided whether or not to participate, (2) has decided not to participate, (3) has decided to participate, and the utility of the marginal consequences of C's participation are the entries under the headings \( U, U', U'' \) and \( \Delta U \), respectively, in the third row of Table 3.

It is seen that after C has decided to participate, the expected utility of the action is already 100 units because, after A, B and C have decided to participate, the threshold has been attained and the success of the group action is already certain. It is only in this situation that the marginal consequences of D or E's participation is zero. This is shown in the fourth and fifth rows of Table 3.

In a similar manner, the expected utilities after one or more agents have decided not to participate, as well as the utility of the marginal consequences of an agent's participation, are shown in Table 4.
Table 4. Expected Utilities and Utilities of Marginal Consequences with One or More Agents Having Decided Not to Participate

<table>
<thead>
<tr>
<th>Agent</th>
<th>U</th>
<th>U'</th>
<th>U''</th>
<th>8U=U''-U'</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>83.7</td>
<td>65.2</td>
<td>91.7</td>
<td>26.5</td>
</tr>
<tr>
<td>B</td>
<td>65.2</td>
<td>34.3</td>
<td>78.8</td>
<td>44.5</td>
</tr>
<tr>
<td>C</td>
<td>34.3</td>
<td>0</td>
<td>49.0</td>
<td>49.0</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The utility of the marginal consequences of an agent's participation in a group action may now be defined as follows. The difference between the expected utility of a group action when an agent has decided to participate and that when the agent has decided not to participate is called "the utility of the marginal consequences of the agent's participation". We shall designate it by the symbol $U_{mc}$.

In the above example, it is $\Delta U$ of the first row of either Table 3 or Table 4. That is

$$U_{mc} = \Delta U = U'' - U' = 91.7 - 65.2 = 26.5$$

6. CONTRIBUTORY CONSEQUENCES AND CREDIT FOR AN AGENT

From the analysis in the previous section, we see, in the above example, that only after three agents have successively announced their intended participation or after each has decided not to participate will the incremental expected utility of the marginal consequences of an agent's participation decrease to zero. The normal utility of the marginal consequences of an agent's participation is the one when no other agents have announced their decision as to whether to participate or not to participate. Thus, in the above example, $U_{mc}$ is 26.5, rather than 0.

If one or more agents have announced participation, then the utility of the marginal consequences of an agent's participation will be like those shown in the $\Delta U$ column of Table 3. If one or more agents have announced non-participation, then the utility of the marginal consequences of an agent's participation will be like those shown in the $\Delta U$ column of Table 4.

Now I shall discuss two new concepts, namely, $U_{cc}$, the utility of contributory consequences of an agent's participation in a group action and $U_{c}$, the credit for an agent. These two concepts are to be distinguished.
By \( U_{ee} \) is meant the expected utility of the group action ascribed to each member of the group. Since all agents of the group are assumed to be similarly situated, \( U_{ee} \) is simply the total expected utility divided by the number of agents and it is the same for all agents. In the above example, \( U_{ee} = \frac{U}{N} = \frac{83.7}{5} = 16.74 \).

By \( U_{e} \) is meant the utility of the group action attributed to a member of the group after the group action has been taken. Therefore the utility is actual utility, rather than expected utility. Since a member may have either participated or not participated, it is justifiable to distinguish between these two cases, if feasible.

If no agent makes an early decision and announcement to the other agents, then, because of the assumption of uniform probability \( p \) of participation, it is reasonable to assume that each agent gets an equal share of credit for the actual utility. Thus, in the above example, if there were no agents participating, then the group action would have failed, the actual utility would have been zero and each agent would get a credit of nothing. If, on the other hand, all five agents participated, the actual utility would have been 100 and each agent would have received a credit of \( \frac{100}{5} = 20 \) units of utility. If only four agents participated, it would be reasonable to assume that only those agents participating share the credit equally. So each agent would get a credit of \( \frac{100}{4} = 25 \) units of utility. Similarly, if only three agents participated, then each agent would get a credit of \( \frac{100}{3} \approx 33.3 \) units of utility.

There is a slight problem in the case of two agents or one agent participating. In these situations, the group action fails and the actual utility is zero. But there is a difference between two groups of agents, one participating and the other not participating. One arbitrary solution is to assign zero units of utility to each agent. This arbitrary assignment has the disadvantage of being unable to take care of the difference between the two groups of agents, one participating and the other not. However, this disadvantage seems to be unavoidable.

7. INTERPRETATION OF UTILITARIAN GENERALIZATION AND MAXIMIZING CONDITIONS

Having discussed marginal and contributory consequences, I shall proceed to interpret utilitarian generalization and act-utilitarianism in terms of these notions. I shall use Gruzalski's example of group action for illustration.

Gruzalski takes the example of a strike to illustrate the difference between act-utilitarianism and utilitarian generalization. A strike has been called. The strike would still be successful if 300 out of a total of 450 workers were to strike. The success of the strike has a very high utility (4000) compared with the loss of one day's salary (1). Worker Fred knows that far more than enough workers will strike and that the strike will surely be a success. Therefore his participation would have no marginal consequences.

Based on an assumption of zero marginal consequences, Gruzalski's interpretation is that act-utilitarianism would prescribe no participation and utilitarian generalization would prescribe participation. He thus concludes that (1) this is where
act-utilitarianism and utilitarian generalization diverge, (2) act-utilitarianism's prescription is right and (3) utilitarian generalization's prescription is wrong. Hence, he reasons, utilitarian generalization is defeated.

I shall proceed to point out the flaws in the above argument. My analysis is based on my concept of marginal consequences. First, consider the assumption that all agents involved in a group action are similarly situated. It follows naturally that the same prescription should be given to all of them.

In the above example of the strike, the prescription is either to participate or not to participate. Now Fred's case falls into either Case (1) or Case (3) of Section 5. I shall consider Case (3) first. There are two different situations in Case (3). One situation is that many other members have the same or similar knowledge. This violates Assumption (2) of Section 4 and is excluded from consideration. A second situation is that all other members do not have the same or similar knowledge. Then this is a really exceptional case and is not subject to a general moral prescription.

Next let us consider Case (1), in which the knowledge of participation of others is in the probabilistic sense. This interpretation is probably what Gruzeliski meant by "Worker Fred knows that far more than enough workers will strike and that the strike will surely be a success", because it would be impossible for Fred to know definitely that more than 300 workers will strike except as a close prediction based on statistics, past experience, etc. Then this is knowledge in the probabilistic sense no matter how high the probability may be. In that case, naturally, probabilistic method should be considered the most appropriate approach.

Thus, if Fred, on the basis of this knowledge, is entitled to a prescription "not participate", then all other members of the group, being similarly situated, would also have this knowledge and, hence, will be entitled to the same prescription "not participate". Now one may argue that an agent does not necessarily act in conformity with a utilitarian prescription. For instance, Fred can be "entitled to a prescription not to participate" and not "accept" this prescription. He may be entitled to it because act utilitarianism is true and on act utilitarian grounds be ought not to participate, but he may be a general utilitarian or no kind of utilitarian but rather one who "believes in" principles of social responsibility (along with other principles of prima facie duty). However, an ethical theory, although essentially an explanation and justification for what is morally right or wrong, aims at the general acceptance and general conformity to it as one of the main objectives. Prescriptions are hoped to be conformed to by agents. If most agents are indifferent to the prescriptions given by a certain ethical theory, then the effect of the theory is nil. If most agents act contrary to the prescriptions, then the effect of theory will be negative. Moreover, although different ethical theories have different explanations and justifications, for most moral actions the different theories will give identical or similar prescriptions. Therefore I assume that a well-recognized ethical theory, though not universally accepted, has a positive effect in so far as conformity to the theory is concerned, i.e., most agents will act in accordance with prescriptions, or there is a general conformity to the ethical theory. In that case, if the prescription is "not participate", then $p$, the probability of participa-
tion, will decrease greatly and $P$, the probability of success of the group action, will decrease too, thus violating the assumption of sure success.

This phenomenon may be called "the dynamic nature of probabilities". The probability of participation $p$ reflects, in fact, "others' behavior". If the threshold $k$ is low and the probability of participation $p$ is high, then the probability of success $P$ will be very high and the utility of marginal consequences will be negligibly small. Since the agents are similarly situated, this general fact is assumed to be "known", not only by a single member like Fred, but also by all other members of the group.

Now the probability of participation $p$ is dynamic rather than static. The probability of success $P$ is a function of $p$ and $k$, but, as feedback, it in turns affects $p$. Knowing that $k$ is low and $P$ is very high, some members will naturally not participate. This results in a lowering of $p$. A decrease of $p$ will, in turn, results in a decrease of $P$ and an increase of the utility of marginal consequences. This violates the justification for nonparticipation. Therefore, the prescription of no participation for Fred alone by a utilitarian based on the argument that he knows that the strike is sure to succeed, is logically invalid.

If Fred waits until more than 300 workers have decided to participate, then he will not be situated similarly to others. This would violate the basic assumption that all agents are similarly situated. In other words, Fred does not have the privilege to act in this way.

Second, consider the utilities. For utilitarian generalization, the comparisons of utilities is between these two situations, namely (1) all the agents participate and (2) all do not participate. If all 450 workers participate, then the utility will be 4000; if all 450 workers do not participate, then the utility will be 450, comprised of 1 unit of utility for each worker's wage per day.

Act-utilitarianism can be interpreted in terms of either contributory consequences or marginal consequences. In terms of contributory consequences, the utility for each agent is simply the total utility divided by the number of members of the group. In the example of the strike, the comparison is between the utility of participation, namely $40000/450 = 8.89$, on the one hand, and the utility of not participating, namely 1, on the other hand. The ratio of these two utilities is 8.89, which is the same as that in utilitarian generalization, namely 4000 to 450 or 8.89. Thus, if contributory consequences are used to interpret act-utilitarianism, then there will be no divergence between utilitarian generalization and act-utilitarianism.

Next, let us consider the interpretation of act-utilitarianism in terms of marginal consequences. It can be readily verified by numerical examples that, if $p$ is close to $k/n$, then in most cases the utility of marginal consequences is greater than the utility of contributory consequences for a single number. I shall do the calculations for a specific case.

As a probabilistic system, the success of the group action has a certain probability $P$, which is a function of the average probability of participation $p$ and the
threshold k. In the above example of strike, assume $p = 0.67$ and $k = 300$. Then the probability of success is

$$P = \sum_{i=300}^{450} C(450, i)(0.33)^{450-i}(0.67)^i$$

$$= 0.5$$

The utility of the marginal consequences of Fred's participation can be calculated according to the formula $\Delta U = U' - U$, i.e.,

$$U = \left(\sum_{i=299}^{449} C(449, i)(0.33)^{449-i}(0.67)^i\right) - \left(\sum_{i=300}^{449} C(449, i)(0.33)^{449-i}(0.67)^i\right)$$

$$\times 4000$$

$$= (0.517 - 0.473) \times 4000 = 176$$

In this example of strike, $p = 0.67 = \frac{300}{450}$, or $p$ is close to $k/n$. It is thus seen that the utility of marginal consequences, 176, is much greater than the utility of contributory consequences $\frac{4000}{300} = 13.3$. Thus, in this case, act-utilitarianism and utilitarian generalization both give the same prescription, namely, participate in the strike.

There are situations in which the utility of marginal consequences is extremely low, so low, in fact, that it may be less than the utility of no action. However, the utility of marginal consequences will never be exactly zero, although sometimes it may approach zero asymptotically.

If the threshold $k$ is low and the probability of success $P$ is close to 1, then the situation is what David Lyons calls "maximizing conditions". In the above example of a strike, for instance, if $p = 0.9$ and $k = 230$, then the utility of marginal consequences will be extremely small—much less than 1, the utility of one day's wage. It is only in cases like this that it may be argued that utilitarian generalization may diverge from act-utilitarianism.

There is yet another argument to support the claim of no divergence between these two forms of utilitarianism. Let us examine the consideration of "fairness". Since the success of a group action is probabilistic in nature and can never be guaranteed, there is always the possibility of failure. Normally, a group action has a much greater expected utility than the sum of the utilities of no action by all members of the group, for otherwise there would be no justification for the group action. Once the group action is justified and called for, it is more or less a contract or agreement that a member of the group is expected to sacrifice something when the group action fails. In other words, each member has a moral obligation to participate in the group action. This is actually what is called "fairness" by many philosophers and normally con-
sidered in a non-utilitarian argument. However, this non-utilitarian argument can still be given a utilitarian interpretation.

If an agent, as a member of a group, does not participate in a group action, then, from the societal point of view, the agent can be interpreted as causing a negative incremental value or utility to a system, institution and/or practice of society. From the agent's personal point of view, he also receives a negative value or utility from social blame or disapproval for his non-participation. Furthermore, non-participation means acting against the instruction of a union and is similar to acting against a law, which situation could be a very complex problem, as David Lyons points out in his discussion of minimizing conditions.

According to this utilitarian interpretation of fairness, act-utilitarianism should still prescribe participation instead of non-participation and, thus, there is still no divergence between utilitarian generalization and act-utilitarianism.

David Lyons shows the extensional equivalence between utilitarian generalization and act-utilitarianism through the "relevance of others' behavior". I have shown their equivalence through a probabilistic interpretation of marginal consequences, supplemented by a utilitarian interpretation of "fairness". In fact, the probability of participation reflects the "relevance of others' behavior".

8. MINIMIZING CONDITIONS

Now I shall discuss the concept of "minimizing conditions"—so named by David Lyons—and examine the respective moral prescriptions to be given by act-utilitarianism and utilitarian generalization to agents in this kind of situation. I shall explain it by using the following example of Grzalski.

If as many as 2000 people were to revolt jointly against a bad dictator, then the revolution would succeed, but an act of revolt by a single person would certainly result in failure and, consequently, the death of this person.

The prescription given by act-utilitarianism would be "do not revolt". Grzalski presumes that the prescription given by utilitarian generalization would be "revolt". Thus he concludes that utilitarian generalization diverges from act-utilitarianism.

The revolution case may be argued in a manner similar to the strike case mentioned previously, because they are similar in structure—an unsuccessful revolution corresponding to an unsuccessful strike. Using the strike as example, assume that the total number of members is still 450 and that the threshold is still 300, but that the propensity for participation is low, with a consequent probability of individual participation of p = 0.3. Thus, on the average, only 135 members participate in the strike. (Actually, the number of persons participating is a random variable with a certain probability distribution.) Then the probability of success of the strike is
\[
P = \sum_{i=300}^{450} C(450, 9)(0.7)^{450-i}(0.3)^i
\]

and the expected utility of the group action is

\[
U_g = P \times 4000 = 0 \times 4000 = 0
\]

We thus see that the expected utility, \(U_g\), is much less than the utility of no strike, viz., 450. Then there arise questions: "What will be the prescriptions given by utilitarian generalization and by act-utilitarianism?" and "Do these prescriptions diverge?"

These problems are very intricate ones and are particularly perplexing as far as ideal rule-utilitarianism is concerned. David Lyons gives a detailed discussion and concludes that ideal rule-utilitarianism is, here, self-defeating.\(^{23}\)

However, as far as utilitarian generalization is concerned, the problem of prescription can be readily solved. A strike is usually organized by a union. The prescription given by a moral theory to a member of the group is effective after the union has decided whether or not to call a strike. If the union has decided not to have a strike, then, in this case, the prescriptions given to an individual member by both utilitarian generalization and act-utilitarianism will certainly be "do not participate". Therefore, in this case, there is no divergence between prescriptions.

Nevertheless, if, in spite of low probability of success, the union still decides to call a strike, then there could be divergence between the prescriptions of utilitarian generalization and act-utilitarianism. Utilitarian generalization would still prescribe participation because the number of similarly-situated agents counted would be the total number of members, \(n\), of the group, rather than the number, \(m\), of members actually participating. Now if all \(n\) agents did participate, then the group action would succeed. The reason why the group action actually fails is that most members do not follow the prescription to participate, thus resulting in a very low probability of participation. Therefore, at least in theory, it is the members of the group who are responsible for the failure, not the moral theory which gives the prescription.

As to the prescription given by act-utilitarianism, fairness, as an argument, can also be applied here. However, the argument based on the dynamic nature of probabilities is not applicable because, knowing that there is little hope of success, most agents may simply give up and the probability of participation may go down further. Thus the problem reduces to the weighting of two utilities: (1) the loss of one day's wages and (2) the neglect of fairness. The prescription will be participation if fairness is counted more heavily than one day's wages and will be no participation if fairness is counted more lightly than one day's wages.
This is the only situation in which utilitarian generalization could diverge from act-utilitarianism, but this still does not imply that utilitarian generalization is wrong.

Practically speaking, however, knowing that the probability of participation is low and that the probability of success of the group action is extremely low, the union should not decide to go on strike. There are two levels of decision to be made: one, the union level and the other, the individual level. The prescription given by a moral theory to a member of the group as to whether or not to participate is at the individual level. Once the union has made the mistake of deciding to call a strike when the probability of success is extremely low, there is very little that a moral theory can accomplish at the individual level. 

9. CONCLUDING REMARKS

Finally, we come to the question “Which approach, whether the contributory consequences one or the marginal consequences one, is the more appropriate approach to be used”? In the above study, these were studied using the probabilistic method of analysis, but no preference was shown for either approach. I shall now re-examine the analysis with this point in mind.

It can be said that the marginal consequences approach is a refinement of the contributory consequences approach. Normally, the probability of participation p is not known. If p is not known, then there is no way of applying the marginal consequences approach and, consequently, the contributory consequences approach is naturally the one to be used. On the other hand, if the probability of participation p can be estimated or determined roughly, say by statistics obtained from past experience, a Gallup poll or through whatever other sources are available, then the use of the marginal consequences approach is justified and is preferable.

In problems involving minimizing conditions, the probability of success P is extremely low. For such a situation, as pointed out above, the organization in charge of group actions should have decided not to take the action in question. However, once a group action has been decided upon, nothing can be done at the individual level, no matter what the moral theory and its prescription may be. This is the only situation where act-utilitarianism and utilitarian generalization may diverge. However, the divergence occurs only if a relatively low weight is placed on the neglect of fairness in acting against the instruction of the organization.

ENDNOTES


See note 1, 14, 17.


See Note 1, 24, 63, 95.

The utility of a group action with a threshold is a function of the number of members participating. In addition to the large step change of utility at the threshold, there may be other small changes of utility as the number of members participating changes. Therefore, strictly speaking, threshold may be defined to be the number of members participating at which the utility is a maximum. However, for the purpose of classifying group actions based on threshold, the definition of threshold in terms of step change of utility seems adequate, and other refinements of the definition seems unnecessary. Moreover, there is a difficulty in defining threshold in terms of maximal utility. The number of threshold is not necessarily restricted to one, and in the case of two or more thresholds, the utility is the global maximum at only one threshold, whereas the utilities at other thresholds are only local maxima, not real global maxima. Therefore I still define threshold in terms of step change of utility.

See Note 7, 62-91.

See Note 4.

See Note 7, 63, 65, 73, 91-115.


In saying that an optimal situation would occur when exactly 3 agents participate, I take the cost of participating into account. Since the utility of passing a motion is the same whether it is 3-out-of-5, 4-out-of-5, or 5-out-of-5 voting, the cost of 4-out-of-5 voting is higher than that of 3-out-of-5 voting, and the cost of a 5-out-of-5 voting is still higher. This is because the utility of the group action descends as more participate once the threshold is attained, for then some people are wasting their time as far as ends served by the voting are concerned and this may bother them. However, this cost is usually small compared with the utility of the group action. Therefore it is sometimes assumed that the utility of the group action rises suddenly at the threshold and remains fixed as the number of participants still increase.
15 See Note 4.

16 See Note 7, 91-115.

17 See Note 7, 128.

18 The concept of giving "fairness" a utilitarian interpretation is to consider fairness a spiritual value and to compare it with other values, whether material or spiritual, moral or nonmoral, in the weighting process of decision-making and moral judgment. This can be done by assigning a large but finite weight to fairness as a justification for action or no action. In other words, fairness is recognized as something that is realized or exemplified by action to different degrees, and a weight or value assigned does not represent the abstract fairness itself, but represents the degree of realization or exemplification of fairness.

19 See Note 7, 166. David Lyons writes,

"Within legal systems, cases connected with minimizing-conditions and analogous circumstances are extremely complex. Particular laws may fail to satisfy the conditions necessary for an argument for fairness, . . . but, though an argument for observance of particular laws based upon fairness may fail, we may also have a general obligation to obey the law that is based on fairness and the relative utility and justness of the legal system as a whole. Thus it may be wrong to break a bad law in a generally good system".

20 See Note 7, 115. Lyons specifically writes,

"(1) If actions are viewed as completely relatively described, the generalized/simple utility relations is linear. That is to say, non-linearity results from a failure to take all relevant factors into account . . . ."

21 See Note 7, 128-143.

22 See Note 4.

23 See Note 7, 141-2.