16. THE LOGIC OF LEARNING

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ABSTRACT. A constructive analysis of reasoning as a self-corrective process of learning in which a dialectic between inquiry and anomaly, between intuition and inference, between analysis and synthesis, between induction and deduction, gradually produces a virtually unconditioned but always corrigible solution to a problem. The argument is both a synthesis of contributions from classical and modern philosophers to the interpretation of learning and an attempt to bridge the gap between critical thinking and formal logic in the analysis of reasoning. The aim is to show that learning as well as demonstration has a logic susceptible to philosophical analysis.

Learning and logic are often opposed to each other. Learning, the process of acquiring knowledge of a subject, is sometimes taken to be a spontaneous, intuitive, and informal operation, analyzable, if at all, only in cognitive psychology. Logic, by contrast, is usually regarded as a deliberate, rational, and formal procedure, justifiable by philosophical analysis, for articulating knowledge already acquired about a subject. The consequence is for learning to be considered an irrational but serendipitous pursuit, while the study of logic becomes a vain but challenging quest for the formal justification of the rationality of argument. In the process, the possibility is discounted that learning might be rational since it is what generates knowledge (justifiable true belief) and logic might be implicit in the acquisition of knowledge if it becomes explicit in its justification.

This possibility, that there may be a logic to learning which may be the origin of logic, is what I want to explore. I shall ask if learning cannot be conceived of as a self-corrective process in which the format of answers corresponds to the structure of questioning, the formality of inference to the patterning of intuition, the comprehension of synthesis to the progression of analysis, and the conclusiveness of deduction to the tentativeness of induction. This is a conception that would place the efficacy of learning in the process itself rather than in its components. It is an attempt, therefore, to construct what learning may be in fact from an analysis of the actual process, in contrast to the attempt to reduce the theoretical possibility of learning to the effectuality of observation and reasoning.
Learning begins in inquiry, and inquiry begins with the perception of a problem. Such a perception has a subjective as well as an objective dimension. The objective dimension is obvious. It derives from what presents a problem, an anomaly of some kind. Frequently the anomaly will be a sudden, unusual, or regular irregularity—a departure from the ordinary, a break in routine, some deviation from the norm, a discontinuity in the course of nature, the violation of a civil or a scientific law—for which there is no obvious explanation or evident justification. But an anomaly can just as easily be a remarkable, unexpected, or irregular regularity—the appearance of order, the emergence of a routine, the recognition of a norm, a seemingly natural course of events, the authority of a civil or a scientific law—for which there is no conventional explanation or satisfactory justification. To know what is happening, we try to understand why it is happening. We seek a solution to the problem, and inquiry begins.

For if inquiry begins because of the perception of an anomaly, the anomaly is perceptible only because we ask what is the matter. The subjective dimension of wonder is just as important as the objective dimension of anomaly for the perception of a problem. Although the object of a problem is what we pay attention to when we are trying to solve the problem, we recognize upon reflection that an anomaly can present a problem to us only because we can ask about the meaning of any departure from a regular or an irregular set of events. And the kinds of question we ask about it anticipate the kinds of solution we can give to it.

How this happens is something Aristotle demonstrated by showing how the wonder at the origin of knowledge ramifies into a structure of questions to which there are specific kinds of answers. Since solutions are the answers to questions, and the kind of questions we can ask are four in number, he argued, there are four kinds of solutions to problems. To get to know a fact, we ask whether a phenomenon happens to another thing, and to learn the cause, we ask why a phenomenon happens at all. At the same time, we must ask if a phenomenon exists to discover its existence and also what it really is if we want to grasp its nature. Knowledge consists, Aristotle concluded, in getting these answers to these questions: they give us the solutions to our problems.

Aristotle illustrated his point, in part, with the example of a solar eclipse (Table 1).

<table>
<thead>
<tr>
<th>Questions to be Asked</th>
<th>Things to Know</th>
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</thead>
<tbody>
<tr>
<td>Is x happening to y (e.g., Is there an eclipse of the sun?)?</td>
<td>Fact: The sun is being eclipsed.</td>
</tr>
</tbody>
</table>
THE LOGIC OF LEARNING

Why is x (e.g., this eclipse) happening?  

Cause: The moon is coming between the sun and the earth.

What is x (e.g., solar eclipse)?

Nature: A solar eclipse is the obscuring of the sun or darkening of the earth because of the interposition of the moon.

Is there such a thing as x (e.g., a solar eclipse)?

Existence: Everything must have a natural explanation.

Clearly, questions of fact and of cause complement each other, as do those of existence and of nature, and these two sets of questions complement each other as well. That questions of fact and of cause are complementary is evident from the fact that if we want to know whether x is happening to y, we also want to know why x is doing it, and only if we know why x is happening do we understand what x is doing to y. The same complementarity is evident between questions of existence and of nature, for we want to know what x really is if we wonder whether it exists at all, and we have to know what x is if we are to grasp the meaning of its existence. And the complementarity within each set of questions is compounded by the complementarity between the two sets themselves. The immediate questions of fact and existence interact upon each other, for we have to discover if x exists before we can ask whether x is happening to y, just as we have to find out whether x is doing something before we can ask if x exists. And the interaction between the mediate questions of cause and nature is just as evident, for we gain an insight into the nature of x from discerning why x acts as it does, while we hypothesize about the effects of x from a grasp of its nature. Hence, the questions from which knowledge originates form an interlocking dynamic structure.

Applied to the perception of a problem, this means that the knowledge of x is a gradually emerging product of the immediate interaction between the empirical data which reveal an anomaly in the action of x and the wonder that makes such a perception possible. But this interaction is mediated by an interaction between inquiry into why x has such an effect and reflection upon what the efficacy of x manifests about its nature. In the ordinary course of events--in science, under conditions of normal scientific inquiry--questions of nature and existence are mooted by the authority of a conventional worldview and reigning theory, leaving us free to concentrate upon the direct questions of fact and cause. But if the answers to these questions cannot be accommodated within prevailing presuppositions, we have to raise explicitly the reflective questions of nature and existence, with the possibility of precipitating a revolution in our universe of discourse. Yet if we have a structure of questioning capacious and nuanced enough to comprise the conditions for inquiry under both normal and revolutionary conditions, we can find the answer to any question, the solution to any problem, that might arise.

The acid test is that this structure can be used to solve the problem Hume raised about matters of fact. The problem with such knowledge, Hume contended, is that a grasp of causality is necessary to establish a fact, and since a belief in causality is irrational, the product of an instinctive feeling or an ingrown habit, we can never therefore
really get to know a matter of fact. Why must this belief be irrational? Because causality, Hume claimed, is concretely nothing but the constant conjunction of one fact with another, and the contingency of such a phenomenon logically prohibits attributing to it the necessity implicit in the cause-effect relationship. In the absence, therefore, of any objective basis for attributing a necessary relationship to a contingent phenomenon, Hume concluded, the motive for attributing causality to matters of fact must be instinct or habit.⁶

Though Hume was correct to deny that a contingent phenomenon could be a necessary relationship, he was wrong to suppose the phenomenon of constant conjunction was as such the relationship of causality. Constant conjunction (or an unusual conjunction, as well) is, as we have seen, evidence not of causality but of an anomaly. Instead of being the solution to a problem, it is the sign of a problem. It is what makes us ask a question of fact (e.g., Is that an eclipse of the sun? or, Does the sun rise and set over the earth?), and the question of fact leads to a question of a cause (e.g., We have to ask, Why is the sun being eclipsed? or, Why does the sun rise and set over the earth?). For only if we know why something is happening can we be sure we know what it is that is happening (e.g., We can know the sun is being eclipsed only if we can establish that the moon is coming between the earth and the sun, but we cannot pretend to know that the sun rises and sets over the earth if we cannot discover an adequate mechanism for explaining how that could occur.). We can tell what anything that appears to be anomaly really is by explaining why it is really not an anomaly at all.⁷

Before we tried to do that we would have to presume, of course, that an anomaly is ultimately intelligible (e.g., We must presume there can be a natural explanation for anything before we look for a natural cause for the darkening of the sun at noon.). And we would also have to be able to incorporate any explanation we might give for it into a theoretical framework before we could be sure the explanation was anything but a likely story (e.g., We must be able to incorporate our explanation of sunrises into the same framework as our explanation of solar eclipses--indeed into the same framework as our explanation of every other astronomical, and ultimately every other natural, event--before we could be sure our account of it was justified true belief). But if we are able to solve the mediate question of causality (either in a straightforward and direct way or else by a revision of background theory or even by conversion to a new set of basic beliefs), we can then answer the immediate question of fact (e.g., We can say that the darkness at noon is a solar eclipse caused by the interposition of the moon between the sun and the earth.). Thus we become able to explain a fact that presents a problem--an anomaly--by finding the cause of it.

Therefore Hume's conception of the problem of factual knowledge need not interdict the search for a solution to the problem. Constant or unusual conjunction is not causality but an anomaly to be solved by finding the cause. The framework of questioning suggested by Aristotle does not guarantee that a solution will be found for the problem presented by any anomaly; it does not even indicate how to discover the cause that would give a solution to the problem. But it does elucidate
the distinction between the phenomenon of anomaly and the nature of causality. That shows that a search for the cause of a fact is not futile in principle. The next step is to analyze how the discovery of the solution to a problem actually occurs.

2. THE PURSUIT OF LEARNING

If the genesis of a problem derives from a dialectic between anomaly and wonder, the search for a solution proceeds through a dialectic between intuition and inference. In the strong sense of either term, intuition and inference are contraries by definition. For intuition in the strong sense is the kind of unjustifiable immediate apprehension that is to be found in either a hunch or a tautology, and inference in the strong sense is the mediately justified knowledge of a formal argument. But in the weak sense intuition can be the kind of justifiable immediate apprehension that both Peirce and Lonergan have analyzed as insight into the meaning emergent from patterns in empirical data. And inference in the weak sense can include the kind of informal inference Lonergan has analyzed as implicit in a rational assertion for which the grounds remain tacit until questioning prompts an explication of the assertion. In the weak sense, therefore, intuition and inference can be not only compatible but complementary. For an insight is justifiable, and a rational assertion is immediate, and without inference the justifiability of an insight would remain inarticulate, while an assertion would be baseless without an insight into the meaning of a pattern of events.

The best illustration of the dialectic between insight and inference is to be found in what might at first seem to be an unlikely spot, the locus classicus in the Meno for Plato's attempt to demonstrate the necessity of anamnesis. Remember, the problem that prompted this demonstration was the dilemma Meno presented to Socrates that learning seemed to be either unnecessary or impossible: either we already know something and do not need to inquire into it, or else we do not know it and could not recognize if we discovered it. Socrates' response was to claim that acquiring knowledge was not learning but recollection—the recollection of knowledge gained by the soul apart from the body in another form of existence. The proof he offered was to show how a slave boy who showed no sign of innate knowledge nor had ever been instructed in geometry could nevertheless come to know the Pythagorean theorem without being given a formal proof. By process of elimination, Socrates argued, the only explanation for the boy's acquisition of this knowledge must be anamnesis.

What Socrates' proof actually shows, however, is that discovery is a function of a dialectic between intuition and inference. For Socrates demonstrated the theorem to the boy by posing the problem of how to find the root of a square double the size of a square of four (that is, one with a root of two). And he led the boy to discover the solution by sketching for him a series of squares, each with a root closer to the right one, until he drew for him a square whose root was the diagonal of the original square.
Since the square (DBKL) is obviously twice the size of the original square of four (ABCD) and half the size of the circumscripive square of sixteen (AEFG), it is obviously the square of eight, for the root of which the boy had been looking. The boy could see from the figure that the root of this square (√8), even though it had no determinate number, was the solution to his problem. At this point, Plato cuts off the story, without allowing Socrates to help the boy to infer the significance of his insight—that the square of a diagonal is equal to the sum of the squares of the other two sides. And contrary to the evidence he has provided himself, he has Socrates impose the conclusion that the demonstration proves the possibility of anamnesis.

What the passage does illustrate, though, is that to discover the solution to a problem we must use our imaginations. The questions raised by the problem provide criteria for drawing from the data of the problem the lineaments for a model in which the solution to it can become evident. No matter how plausible an initial model may be, though, the specifications for it may have to be reinterpreted and the features varied before it manifests a solution. But then the consequence will be an insight into a pattern evident in the data which incorporates as a normal component the anomaly causing the problem. We will see either that the anomaly fits into the pattern in a way we had previously failed to realize or else that the pattern needs to take the anomaly into account before it incorporates all of the data pertinent to the subject under investigation. This insight becomes the basis for an assertion whose rationality becomes evident from the presuppositions and consequences we can infer by further reflection upon the model. As Hanson has commented, "Perceiving the pattern in phenomena is central to their being 'explicable as a matter of course.' ... This is what philosophers and natural philosophers were groping for when they spoke of discerning the nature of a phenomenon, its essence; this will always be the trigger of physical inquiry."12

Hanson himself has shown how insight is as necessary for discovering the solution to an empirical as to a mathematical problem. He made an elegant and compelling analysis of how Kepler finally discovered the curvature of the orbit of Mars by converting the data of Brahe's observations of the planet into a series of mathematical models, each a closer...
fit to the data, until he finally saw that only a perfect ellipse could provide an exact match. Only this insight into the unique aptness of the ellipse to the data convinced Kepler that his model sufficed not only just "to save the appearances" but to yield a genuine physical theory of the orbit of Mars. "Finally [Kepler's] perplexities dissolved", Hanson says, "before an insight which transformed his data and all subsequent astronomy and physics." For Kepler's approach set a precedent for the way physicists would gather the data and develop the methods Newton subsequently capitalized upon to reduce celestial and terrestrial physics to the single law of universal gravitation. And Lonergan has argued that this dependence upon insight into appropriate models for the discovery of the meaning of things is as congenial to contemporary as to classical physics, and to historical and social studies as to the physical sciences.

What is important to recognize, too, is that not only can there be a positive demonstration of the function of insight in intellectual discovery, but the veritable rationality of insight as a justifiable immediate apprehension can also be distinguished from the spurious rationality of the unjustifiable immediate apprehension to be found in a hunch. The distinction can be most aptly illustrated by an example Lonergan has drawn from Euclidean geometry. Euclid gives no formal proof for either his parallel postulate or for his theorem about the construction of an equilateral triangle in the intersection of two circles. But for the theorem it is possible to draw a diagram in which a triangle inscribed in the intersection of two circles can virtually be seen to be equilateral. The product of an insight, this theorem remains unchallenged even today. For the parallel postulate, however, it is impossible to draw a diagram in which it can be seen that two lines bisected at right angles by another line remain parallel into infinity. The product of a hunch, Euclid's parallel postulate, obvious as it may seem, has become the catalyst for rejecting the objectivity of Euclidean geometry and founding non-Euclidean geometries based on other parallel postulates. For an insight, therefore, there must model in which the implications of a theory can be immediately apprehended, and when no such model can be found, any intuition, no matter how sound it may eventually prove to be, remains, for the moment at least, merely a hunch.

Given, therefore, that intuition in the sense of insight is a justifiable immediate apprehension, there is no contradiction in principle between intuition in this weak sense and inference in the correspondingly weak sense of a spontaneous rational assertion. For a spontaneous rational assertion is an argument in a nutshell. Lonergan has described it well: "It appears a fact that spontaneous thinking sees at once the conclusion, B, in apprehending the antecedents, A. Most frequently the expression of this inference will be simply the assertion of B. Only when questioned do men add that the 'reason for B' is A; and only when a debate ensues does there emerge a distinction between the two elements in the 'reason for B', namely, the antecedent fact or facts, A, and the implication of B in A (if A, then B)."

This structure, implicit in informal inference, becomes explicit in formal inference. The statement in informal inference of the reason for the assertion becomes in formal inference the minor premise of an argument. And the formulation in informal inference of the condition for the reason becomes in formal inference the major premise of the argument. But in formal inference, once analysis reveals the elements of the argu-
ment, the process of reasoning is reversed, producing a synthesis of the elements in the form of a logical demonstration. "Thus the transition from informal to formal inference". Lonergan argues, "is a process of analysis; it makes explicit, at once in consciousness and in language, the different elements of thought that were present from the first moment. For when B simply is asserted, it is asserted not as an experience but as a conclusion; else a question would not elicit the answer, B because of A. Again, when this answer is given, there would be no meaning to the 'because' if all that was meant was a further assertion, A. On the contrary, the causal sentence (because A, therefore B) compresses into one the three sentences of the formal analysis (if A, then B; A, ∴ B)." Thus a spontaneous rational assertion, informal inference, and the syntax of formal inference are three stages in the process of explicating the rationality of an insight.

The logical form cognate to this process, as Lonergan describes it, is the simple hypothetical argument (table 2).

TABLE 2
The Form of Inference: First Approximation

<table>
<thead>
<tr>
<th>If A, then B</th>
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<tbody>
<tr>
<td>But A</td>
</tr>
<tr>
<td>∴ B</td>
</tr>
</tbody>
</table>

This form is flexible enough for the units, A or B, to stand for any number or any type of propositions and yet so simple every inference demonstrates the implication of the conclusion in the premises.

Yet this form is too simple if inference is supposed to represent an articulation of the rationale grasped by an insight into the meaning of an object as it is represented in a model drawn from empirical data. The major premise must be a biconditional, If and only if A, then B (or, If A, then B and if not A, then not B.), for it is supposed to represent the grasp of the necessary and sufficient conditions for the intelligibility of the object. The minor premise, correspondingly, must be a disjunction, And A or And not A, to represent the possibility of the fulfillment (in an insight) or non-fulfillment (in the lack of an insight) of the conditions necessary and sufficient for the intelligibility of the object, with the consequence that the denial in the minor of the antecedent in the major (or, the affirmation of the negative alternative of the disjunction) does not lead to the fallacy of denying the antecedent. And the conclusion, which follows by strict implication from the subsumption of the disjunction in the minor under the biconditional in the major, must also be a disjunction, B or Not B, depending upon which of the alternatives in the minor is affirmed. This form of inference precludes the so-called paradoxes of material implication—a false antecedent implying any consequent or a true consequent any antecedent—since it frames an argument about the concrete and specific relationship between a certain set of conditions and the rationality of a given assertion. Therefore, the form of inference operative in a substantive argument—one articulating
the rationale implicit in an insight—is a particular model of the possibilities suggested by formal logic for the structure of a conditional argument (table 3).

**TABLE 3**

The Form of Inference: Second Approximation

<table>
<thead>
<tr>
<th>If and only if A, then B</th>
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<tbody>
<tr>
<td>(If A, then B, and if not A, then not B</td>
</tr>
<tr>
<td>And A Or And not A</td>
</tr>
<tr>
<td>:: B</td>
</tr>
<tr>
<td>:: Not B</td>
</tr>
</tbody>
</table>

On this analysis, intuition and inference complement each other in the search for a solution to a problem. This analysis is not, indeed, a theoretical explanation of the form of either intuition or inference; it is simply a hypothesis about the functions of either operation. Drawn from a putative insight into the complementarity of these operations in two historical examples, this hypothesis would have to be fitted into a theory of formal inference in all its ramifications before it could be evident that it supplied an adequate theory of the logic of factual knowledge. For the nonce, though, this hypothesis has the value of suggesting how informal inference can interact with insight to suggest a solution to a problem.

3. THE ATTAINMENT OF LEARNING

To discover a solution for a problem, though, it is not enough to infer whatever hypothesis is suggested by an insight into a model of the data for the problem. We have to test the insight by evaluating the relation between the logical implications and the empirical consequences of the hypothesis until we can reach a reflective equilibrium between theory and fact. For what we propose as a solution to a problem must fit within conventional theory unless we are prepared to revise the theory, just as it must cohere with current explanations for the rest of the facts unless we are prepared to defend it as an exception to the rule. To enable the attainment of reflective equilibrium, therefore, the use of the form of inference must be a dialectic between analysis and synthesis. This dialectic is a protraction of the dialectic already evident in the process by which the synthesis of a conditional argument in formal inference complements the analysis in informal inference of the elements of a spontaneous rational assertion.

The biconditional in the major premise represents the term of analysis and the starting point of synthesis. As the term of analysis, it articulates the logical presuppositions for the validity of the insight from which the hypothesis in question derives. As the starting point of synthesis, it states the necessary and sufficient conditions for the rationale in the insight to be adequate and relevant to the solution of the
problem under investigation. It represents, therefore, the framework within which a problem is investigated and a solution devised for it.

Outside this framework—and therefore beyond the scope of the dialectic between analysis and synthesis in any given case—is the background of what is regarded as axiomatic about the possibilities of existence. This background may be expressed in the form of statements (e.g., "God has created the world" or "There is a natural explanation for everything") or simply as names for the kinds of things there are supposed to be (e.g., "God" and "creatures" or "nature" and "bodies"). The statements may govern everything to be known (e.g., "Every effect has a cause") or they may simply be domain assumptions (e.g., "Society is an entity sui generis"). The names may refer to historical processes (e.g., "time", "evolution", "civilization"), to basic structures (e.g., "mass" and "energy", "elements", "molecules", "animals", "individual" and "state", "person" and "society"), or to fundamental notions (e.g., "life", "freedom", "justice", "religion"). In the context of learning, it does not matter whether they are (or are regarded to be) primitive concepts and self-evident first principles, or regulative ideals, or cultural epistemes, or simply the limits of linguistic conventions. By establishing the background against which something is studied, they function as the indemonstrable but (in this context) unquestionable standards for reaching a reflective equilibrium between theory and fact.

But within the framework of inference, given the major premise states the conditions for the occurrence of something to be intelligible, the minor premise has both a logical and an existential function. In analysis, it has the logical function of representing the reasons for either of the alternatives in the conclusion, while in synthesis it has the existential function of positing or negating the fulfillment of the (bi)condition in the antecedent of the major. For in analysis the minor represents the significance grasped by an insight into a pattern emergent in a model and articulated in the rational assertion which, when inference is complete, will become the conclusion of an argument. Under these circumstances, the disjunction in the minor represents the process of determining the appropriate features to be included or excluded as independent variables in the formation of a prototype of the event or object under investigation and specified in the major as necessary and sufficient conditions for the event or object to occur. In synthesis, by contrast, with the major already specifying the conditions (the independent variables) for the relevant insight, the minor represents the determination of whether or not these conditions are fulfilled; that is, whether or not the independent variables represented in the prototype occur in actuality. The oscillation between analysis and synthesis necessary for achieving a reflective equilibrium between a hypothesis and empirical data requires the disjunction in the minor to represent the alternative possibilities of accommodating the premises to the data or of assimilating the data to the premises. Hence, the minor is the axis of the interaction between analysis and synthesis, the point for assigning a truth-value, hypothetical or actual, to the antecedent of the biconditional in the major and thus for affirming or denying a reason for the event or object under investigation to occur.

As the starting-point of analysis and the term of synthesis, the conclusion has an equally complex function. In analysis, the disjunction in the conclusion represents the assertion or denial of a possible solution to a problem. If analysis succeeds in articulating an insight implicit
in this disjunction, and synthesis produces the premises of an argument, the conclusion represents a reassertion of the soundness of the original insight. An affirmation in the minor of the fulfillment of the rationale for the insight leads to a positive conclusion: a reassertion of the original insight, now as an affirmation of the necessity for the object to occur, given the fulfillment of the conditions predicated in the major as necessary and sufficient for it. A denial in the minor of the fulfillment of the rationale for the insight leads, by contrast, to a negative conclusion: once again, a reassertion of the original insight, but now in the form of a denial of the occurrence of the object in the absence of the conditions predicated as necessary and sufficient for it. Thus the immediate function of the conclusion in synthesis is to represent the logical consequences of affirming or denying in the minor the actual fulfillment of the conditions postulated in the major as necessary and sufficient for the occurrence of the event or object in question.

But the conclusion has an existential function as well. This function is most obvious in the synthetic phase of inference. For in this phase the conclusion represents a statement about whether or not empirical data confirm either the positive or the negative formulation of the solution implied by the premises. And to tell this, it is necessary to investigate the data themselves. Therefore, the form of inference must be expanded to include the process of an empirical investigation of the conclusion of an argument (Table 4).

**TABLE 4**

The General Form of Inference

If and only if A, then B

(If A, then B, and if not A, then not B)

And A OR And not A

\[ \therefore B \text{ (logically)} \quad \therefore \text{not } B \text{ (logically)} \]

Verification

Positive: Identity OR Negative: Correlation

That B

\[ \therefore B \text{ (factually)} \quad \therefore \text{not } B \text{ (factually)} \]

OR

Falsification

Positive: Negation OR Negative: Reciprocation

That not B

\[ \therefore \text{not } B \text{ (factually)} \quad \therefore B \text{ (factually)} \]
Since the results of an investigation can be positive or negative, a distinction has to be made between these two eventualities in the confirmation of either the positive or the negative conclusion of an argument. This leads, therefore, to the possibility of four simple sets of actual results—Identity, Negation, Correlation, and Reciprocation—which are state descriptions of the logical world implied in the conclusion. They correspond both to the four possibilities in the truth table for the conditional and to the quaternary group Jean Piaget devised to interpret formal operations.22

But it is one thing to state alternative possibilities for the results of investigation, another to determine whether the possibilities have actually been fulfilled, and yet another to assess the significance of various combinations of fulfillment. Within each set of possible results, therefore, a distinction must then be made between a judgment about the actual occurrence of each purported instance of a predicted consequence and a decision about whether in fact it confirms (Verification: *modus ponens*) or contradicts (Falsification: *modus tollens*) the prediction. And since evidence may be discovered for none, any, some, or all four of the sets of alternative results, an assessment must be made of the significance for the conclusion of the occurrence of any of these sixteen possible combinations.23 To the formal complexity of making such an assessment must be added any substantive complications that might arise from having to assign diverse weights to the four sets of simple results. No wonder, therefore, paradoxes and dilemmas dog the attempt to give a formal analysis of confirmation as either a qualitative or a quantitative process.24

In the analytic phase of inference, the existential function of the conclusion is just as complex as in the synthetic phase and even less susceptible to formal analysis. For the empirical associations in this phase lack the guidance of a specific hypothesis, whether because one is lacking altogether for the subject in question or because the guidance of a hypothesis is confined to the synthetic phase of inference. In the absence of any explicit hypothesis, the four sets of simple results represent random associations about the events and the objects making up the world of our experience. This is the quarry from which we draw the information for our hunches and observations. But, more importantly, it is also the source of the grounds for spontaneous assertions whose rationality remains tacit until the pressure of questioning brings it out. Even when analysis follows upon synthesis, however, the function of empirical investigation in this phase of inference is to turn up data not predicted by the hypothesis whose conditions are stated in the major premise of the argument. For any hypothesis, no matter how speculative it may be, is an extrapolation from a limited range of data and needs to be corrected, by refinement or by expansion, through confrontation with more and better data. The reflective equilibrium sought in inference is the product not just of the coordination synthesis brings through an explicit hypothesis to the data of experience but of the objectivity analysis brings through a compilation of the data to any hypothesis.

Yet this reflective equilibrium does not eliminate the probability endemic to the learning process. No matter how sophisticated or how protracted the dialectic between analysis and synthesis, it cannot eliminate the variability of the data nor overcome the fallibility of percep-
tion, it cannot compensate for the complexity of interpretation nor an-
ticipate the polyvalence of application. To adjust to this endemic proba-
bility, the dialectic of learning must extend even to the framework of
inquiry itself. It must include a dialectic between induction and deduc-
tion through which expansion and refinement of experience can counter-
balance clarification and precision of thought.

4. THE DEVELOPMENT OF LEARNING

Within the framework of the learning process induction and de-
duction appear to be two phases of reasoning with an identical struc-
ture but complementary functions. The structure they share is the gen-
eral form of inference. But while the function of induction is to generate
and formulate hypotheses, that of deduction is to test the hypotheses
and integrate them into a framework of theory. Thus the dialectic of in-
quiry operative in inference appears to include a reciprocation between
induction and deduction as well the complementarity of analysis and
synthesis.

This constructive (a posteriori) analysis of induction and deduc-
tion in terms of their functions in the learning process is recommended
not least by the failure of the attempt to analyze these procedures on a
reductive (a priori) basis. The presupposition that knowledge is infer-
ential and inference deductive has led to the conclusion that induction
and deduction may be necessary in practice but are indefensible in the-
ory. For induction must be necessary for knowledge if deduction has no
content but what induction supplies. But if the form of inference must
be deductive, induction can be nothing but an inferior form of deduc-
tion--statistical and probabilistic, as opposed to normative and neces-
sary--or else an informal process with no logical validity at all. And if
knowledge has no form but deductive inference, deduction must be nec-
essary for knowledge. But no instance of deduction can be known to be
sound if it must depend upon the discredited procedure of induction for
its content. Nor can the forms of deduction be known to be valid if a
formal proof of the validity would lead to an infinite regress and the
use of the form to gain knowledge entails a fallacy: begging the ques-
tion in the case of a syllogism; positing the consequent in the case of
a conditional argument. Therefore the reductive analysis of induction
and deduction leads to the dilemma of having to use these procedures
without any justification in order to reason at all or else to refrain from
using them at the price of giving up reasoning.

No such dilemma follows, though, from a constructive analysis of
the respective functions induction and deduction must serve for infer-
ence to be an effective learning process. For induction performs the
function of "saving the appearances", but deduction is necessary to
arrive at a genuine physical explanation of the facts. Therefore, while
induction does indeed supply the content of deduction, it has a logical
validity of its own, one which does not, however, supplant the function
of deduction to test and ground hypotheses. And deduction, without be-
ing able to dispense with induction for the genesis of hypotheses, has
nevertheless the potentiality for increasing knowledge in the course of
determining the empirical objectivity and the theoretical significance of
hypotheses.
4.1 THE FUNCTION OF INDUCTION

In induction we use experience to predict the future. We act on the presumption that an infinitesimal set of actual events (the short run) is typical of a potentially infinite course of events (the long run) insofar as it suggests a cause-effect relationship entitling us to expect the same kind of events to occur whenever the same relationship obtains. Objectively, this relationship is supposed to be neither merely coincidental nor yet metaphysically necessary, but something factually (physically or historically) necessary, given the actual nature of this world. Subjectively, it is assumed to be neither empirically obvious nor self-evidently true, but something probably justifiable, and not just in terms of its survival value, its inevitability, or its apparent success. Interpreting induction as a form of inference with the function of grasping the meaning of a phenomenon from the evidence of its signs would provide such a justification (Table 5).

**TABLE 5**

The Form of Inference: Inductive Phase

When/where and only when/where these signs occur, this object may be present

<table>
<thead>
<tr>
<th>And the signs do occur.</th>
<th>OR</th>
<th>And the signs do not occur.</th>
</tr>
</thead>
<tbody>
<tr>
<td>∴ The object may be present</td>
<td></td>
<td>∴ The object may not be present.</td>
</tr>
</tbody>
</table>

**Verification**

**Positive:** Confirmation

This is the object.

∴ The object is here.

∴ These are (sometimes) signs of the object

**Negative:** Corroboration

This is not the object.

∴ The object is not here.

∴ These are (the only) signs of the object

**Confirmation plus Corroboration:**

If the object is present, these will be signs of it.

**OR**

**Falsification**

**Positive:** Exception

But this is not the object.

∴ The object is not here.

**Negative:** Addition

But this is the object.

∴ The object is here.
These are not (always) signs of the object.

These are not (the only) signs of the object.

**Exception plus Addition:**

If the object is present, there will be other signs of it.

For induction, on this interpretation, has the function of restoring an understanding of an object against its background. It originates from a question about an apparent anomaly in the operations of the object and terminates only when it has "saved the appearances" by modifying the idea of the object or reinterpreting the significance of the background so that the two seem to fit together once again. Thus the function of induction is to generate a plausible hypothesis, the realism of which can be tested in deduction.

To generate the hypothesis, though, both insight and inference have to come into play. In induction, the first phase of inference is analysis: the formulation of a hypothesis from an insight into empirical data. This is the procedure to which Bacon, without acknowledging the need for insight, gave the name simply of induction. Peirce recognized, however, the necessity for integrating insight with inference in this procedure and of distinguishing it, as abduction or retroduction, from the synthetic phase of the process, which he then called induction. Abduction (as we shall call it) is, therefore, the analytical movement by which induction leads from the confines of our comparatively infinitesimal experience, through an insight into the possibilities suggested by a model of the information at our disposal, to the conception of a hypothetical general law for a potential infinity of similar cases.

The necessity for insight is as obvious as the necessity for inference. While experience may supply a manifold of associations about the object under investigation, empirical data do not of themselves coagulate into a plausible hypothesis. Hume was right to argue that a constant (or, I would add, an unusual) conjunction of facts is a contingent and indeed an imperceptible phenomenon without imagination to spot a pattern in the data. And insight at this stage of the learning process is simply the grasp in the pattern of a possible rationale for the anomaly precipitating the investigation. Without benefit of a hypothesis, insight has to rely upon analogies and homologies, upon traces, symptoms, and fragments, to construct a model for an intelligible interpretation of the anomaly. The task of abduction is accomplished if we can infer from an insight into the model a hypothesis about the conditions necessary and sufficient for the apparent anomaly to be a component of the object under investigation.

This hypothesis we formulate as the major premise of a conditional argument. We postulate that probably only whenever and wherever certain signs appear may the object in question be present. The statistical character of this law is evident from the fact that while the major is indeed a positive biconditional, the verb in the consequent is in the subjunctive mood and expresses contingency. The law represents a belief that under the circumstances, all things considered, the object or event can best be expected to be present when and where certain conditions occur and not otherwise. The degree of probability in this belief
is a function of the supposition that the actual experience we have gained of the object is representative of the way it appears always and everywhere. The precise implications of this hypothesis are what remain to be understood in the synthetic phase of induction which is the complement to abduction.

In this movement we draw out all and only the logical implications of the hypothesis and subject them to factual confirmation. This is the procedure some methodologists now refer to as informal reasoning or as critical thinking. More generally, though, it is what contemporary logicians, without any allusion whatsoever to insight, commonly call induction, a kind of concrete (or non-demonstrative) inference they contrast to the formal (or demonstrative) inference of deduction. But in recognition of the fact that this movement is the counterpart of abduction, we shall call it adduction (a designation for which there is also some precedent).

In adduction, therefore, the function of insight is twofold. The logical function is to derive from the hypothesis in the major premise the lineaments of a prototype capable of suggesting all and only the conclusions implicit in it. The existential function is to fit this prototype to samples, examples, or paradigm cases of the object under investigation. Since the point of adduction is to develop clear and precise criteria for distinguishing between any and all signs of the presence as opposed to the absence of the object, the need is for a model in which random or coincidental or marginal aspects of the object are discounted or rounded off to leave its salient features in high relief. Insight is supposed to establish the basis for asserting in the minor of the argument whether or not the conditions postulated in the major for the presence of the object are intelligible and extant.

Adduction, then, is the phase of induction which moves from a grasp in the major of the probability of an object to be present when the conditions putatively necessary and sufficient for it occur, through an insight into a prototype of the object in which these conditions are (logically or existentially) fulfilled, to a conclusion about the consequent presence of the object.

For the conclusion to be objective as well as logical, though, it must be confirmed by events. The object must turn out to be present when the hypothesized signs appear and absent when none of these signs are apparent before the postulated nexus of these signs with the object can be deemed to be objective. Should that occur, the necessity for using the signs to catch sight of the object will appear to derive from the necessity for the object to be present for these signs to be produced. In that case, there will be grounds to investigate in the deductive phase of reasoning the hypothesis that the object in question is the cause and the signs for detecting it are its effects.

The obvious employment of confirmation is, therefore, in adduction, the synthetic phase of induction, when the implications of a precise and specific hypothesis are to be tested. But just as it is a terminus of each movement of abduction, confirmation is also the origin of each movement of abduction. Clearly, this is the case in the oscillation between analysis and synthesis necessary to establish a reflective equilibrium between the logical specifications of a hypothesis and the details of its concrete implications. Yet it is from the random and causal observations of ev-
everyday experience that may arise the first intimations of the anomaly from which an investigation begins. Unless we glimpse some regular irregularity or some irregular regularity we will not be inclined to investigate an object. Hence, while methodical and deliberate confirmation is a feature of adduction, confirmation in the sense of an active concern for the objectivity of ideas is a component of both phases of induction, of abduction as well as of adduction.

In both of these phases there is a bias toward verification. Since the function of induction is to generate hypotheses to be tested in deduction, confirmation (in the strict sense) and corroboration are the anticipated results of the process. Due regard must be paid, of course, to the possibility of falsification, whether it produces exceptions or additions to the rule, in order to eliminate altogether worthless hypotheses. But since deduction has, in any case, the critical function of testing the realism of every hypothesis, in induction we can afford to be open-minded about any hypothesis with some genuine likelihood. Too critical an attitude in induction would deprive us of enough viable hypotheses to subject to the rigors of deduction.

At any rate, confirmation is as complex for induction in particular as it is for inference in general. To mitigate this complexity, many philosophers have suggested sets of rules, methods, or canons for establishing definitively whether purported signs of an object or event are actually the effects it produces. Helpful as they may be, these formulas amount to intuitive assessments of the various combinations of the sets of simple results. They cannot substitute logically for ringing the changes on all the possible combinations before arriving at a considered judgment. More importantly, they cannot make the infinitesimal evidence gathered from any short run of actual events approximate, except asymptotically in theory and pragmatically in practice, the infinite possibilities of the long run of all events. Induction remains in the end induction.

4.2 THE FUNCTION OF DEDUCTION

To determine the realism of a hypothesis, therefore, and to integrate it into the framework of theory, it is necessary to complement induction with deduction. For deduction enables us to develop and use theory both for speculating about the necessary implications of a hypothesis and for integrating a hypothesis into a universal framework. It represents an attempt to convert a grasp of historical probability into a conception of rational necessity.

In deduction, we act on the presumption that by defining our terms more precisely and reducing the definitions to a system we can at once expand and certify our knowledge of the objects making up our world. If the definition of an object does not lead to the discovery of hitherto unknown and otherwise unforeseen properties and functions, we presume it must be nominal, not real—merely the summation of available information about an object, but not an authentic interpretation of its nature. We also assume that if the definition of an object cannot be incorporated into a unique and formal system for the field of study of which it is a part—and, ultimately, into a unique and formal system for all fields of study—either the definition may be merely an ad hoc rationalization or the field itself may lack a rational foundation. Hence,
deduction expresses an supposition that if knowledge cannot be a reproduction of nature, it must be more than just a saving of the appearances; it must amount--ultimately and ideally, though perhaps not immediately or pragmatically--to a rational explanation of the world.34

Interpreting deduction as a phase in the self-corrective process of learning, a phase with the same form as induction but with a different function, corresponds to this conception of its nature (Table 6).

TABLE 6

THE FORM OF INFERENCE: DEDUCTIVE PHASE

If and only if such were the nature of the object, these would have to be its effects

<table>
<thead>
<tr>
<th>And such is its nature.</th>
<th>OR</th>
<th>And such is not its nature.</th>
</tr>
</thead>
<tbody>
<tr>
<td>.: These effects must follow.</td>
<td>.: These effects cannot follow.</td>
<td></td>
</tr>
</tbody>
</table>

Verification

Positive: Demonstration

These are the effects.

.: The effects do follow.

.: This object must be a cause of these effects.

Negative: Exclusion

These are not the effects.

.: The effects do not follow.

.: This object must be the (only) cause of these effects.

Demonstration plus Exclusion:

If these effects occur, this object must be the cause.

OR

Falsification

Positive: Refutation

But these are not the effects.

.: The effects do not follow.

.: The object cannot be a (principal) cause of these effects.

Negative: Alternation

But these are the effects.

.: The effects do follow.

.: This object cannot be the (only) cause of these effects.

Refutation plus Alternation:

If these effects occur, this object need not be the cause.
For deduction, on this interpretation, has an ontological function. The structure of a deductive argument is to explicate the meaning and the truth of a counterfactual conditional. Whereas the major of an inductive argument frames a statistical probability, the major of a deductive argument advances a theoretical necessity. The form of inference does indeed endow induction with a logical necessity, but the substance of induction remains nevertheless probabilistic. In deduction, the substantive necessity of a putatively unexceptional theory is added to the logical necessity it gets from the form of inference.

Thus the biconditional in the major of a deductive argument implies not only that we cannot help thinking of an object in a certain way but that the object itself cannot exist except as we conceive of it. To substantiate this supposition, we must first infer and then test its most radical implications until no valid doubt remains about its correspondence to the facts. And then we must also reflect upon its presuppositions until we are satisfied they fit without gap or remainder into the structure of a grand unified theory for the field of which the object is a part (and, if we could fulfill our mind's ambition, into a grand unified theory of the world as a whole).

Hence, in deductive inference the sequence of analysis and synthesis is the reverse of what it is in inductive inference. With hypotheses supplied by induction, the first phase of deduction is synthetic, an explication of all and only the implications of assuming a hypothesis represents a genuine theory. The synthesis begins with a conversion of the hypothesis from a probabilistic to a counterfactual conditional. It turns from being a supposition that if an object is present the effects taken to be its signs may well occur to an assumption that unless the nature of the object were as it is supposed to be, these signs would not be its effects (or they would not otherwise occur). For, whereas induction follows the sequence by which we learn about something from its manifestations, deduction is an attempt to reconstruct the order by which something produces what we perceive as manifestations of itself. Deduction has the task, therefore, of substantiating not just the conditions for an object to be conceivable, but the conditions for it to be possible. Hence, its function is to formulate theories capable of explaining what are supposed to be otherwise contrary-to-fact events and to eliminate any theory contradicted by an event otherwise supposed to be inexplicable.

In this synthetic procedure, which we may call eduction, the function of insight is to devise the basis for an operational definition of the object to serve as the minor of the argument. Before there can be any test of the counterfactual conditional in the major, there must be a precise and adequate statement of the concrete significance of the postulated condition in the antecedent of the premise. And for this to occur there must be some icon of the object in which the conditions can be perceived. In the physical sciences, this is supplied by a test case or a crucial experiment; in the social sciences, by the parameters for a double-blind test, a random sample, and/or a control group. Mathematics requires an geometric reproduction of an equation. Philosophy needs a figure (like those in Plato's Meno, Norwood Hanson's Patterns of Discovery, or Herbert Simon's Models of Discovery), a form (like Carl Hempel's "covering-law" model or Bernard Lonergan's "form of inference"), a table (like the ones in this paper), or a metaphor (like Kuhn's use of revolution and paradigm) to provide a graphic representation of a theory.
Whatever the icon may be, its function is to enable someone to "see" what a theory really means.

By explicating the full scope and precise significance of a counterfactual conditional, eduction is supposed to enable a definitive test of whether or not the reason for an alleged association between the object in question and a set of data is that the object is indeed the cause of the data. The conclusion of the eductive phase of deduction is, therefore, an explicit statement of the test implications of the major premise, in light of the operational definition of the object in the minor, together with an actual test of these implications. Since the minor spells out the conditions for discriminating between the presence and the absence of the object, the conclusion likewise comprises a statement of both the positive and the negative implications of the major. The test must, therefore, determine whether or not both of these sets of implications are borne out by the facts.

The logical complexity and the concrete imponderability of confirmation are, of course, as great in deduction as in induction. But while induction's function of generating likely hypotheses tilts confirmation in that phase of reasoning toward verification, the bias of confirmation in deduction, is, by contrast, toward falsification, in conformity with deduction's function of vetting the realism of hypotheses. Tests are devised to check for the possibility of any refutation of a hypothesis by the evidence or of any alternate hypothesis for the evidence. Due allowance must be made even here, of course, for the benefit to be gained from the verification of a hypothesis. Positive demonstration of a cause-effect relationship is what a researcher always seeks, and an apparent proof of exclusive causality is a researcher's dream. But since induction can always provide new information and suggest innovative hypotheses, there is nothing to be lost and everything to be gained by making deduction strict in its critical task of approving as theories only the hypotheses capable of surviving the most exacting of trials.

At this point, deduction doubles back upon itself, the synthetic movement of eduction yielding to the analytic movement of reduction. Whatever has been the outcome of the test implications of a hypothesis, it is necessary to compare the results to the hypothesis itself. Even in the absence of complete refutation, experimentation normally requires a revision, perhaps a series of revisions, in a hypothesis before it corresponds precisely to the test results. But experimentation can also compel a radical reinterpretation of the terms of the hypothesis; at the extreme it can entail a transformation of the definition of an object into the terms of another framework altogether. Hence, reduction couples with eduction to establish a reflective equilibrium between the virtually logical necessity of the counterfactual condition in the major and the concrete results of testing its implications in the conclusion.

In reduction insight is no less important than in eduction. Its function is now, though, to discover the basis for an essential rather than an operational definition of an object. Only an essential definition can mediate between the results of confirmation and the kind of systematic theory needed for the "normal science" of puzzle-solving to begin or resume. The model appropriate to an essential definition is a "paradigm": a solution of a problem so conspicuously successful it prompts a similar approach to every other problem in a field. In the absence of a grand unified theory for the field, a paradigm can still lend coherence
to the field by the presage it offers of eventual comprehension. Kuhn has given the example of Copernicus's heliocentric hypothesis; Hanson, the example of Kepler's discovery of the orbit of Mars. For the ultimate goal of reduction—the assimilation of one theory to one another in a series of higher viewpoints culminating in a unified hierarchy of theory coterminous with the world—science itself has functioned as a paradigm in the modern world. Although there is no grand unified theory of any one science, much less of science as a whole, the conspicuous success of science in solving problems has changed the conception of the proper methodology for problem-solving from theology or philosophy to science itself. Insight into the structure of scientific method is presumed to be the necessary medium for framing an essential definition of knowledge.

Thus deduction can result in a genuine increase in knowledge. Formally, it can add a complete and consistent articulation of the consequences implicit—but not clear from induction—in the statement of an explanatory hypothesis. Materially, it can add a determination of the soundness of an explanation, an understanding of the nature of the objects to which the explanation applies, and the possibility of using the explanation to make accurate predictions about the future. Deduction, therefore, is not simply academic.

Yet for deduction to be formally apodictic, a theory would have to be framed in a formal language and based upon indubitable and indisputable observation. What is more, the observations themselves would have to have been made according to categorical and impartial criteria, while the language of the theory would have to be reducible, not just ideally but actually, to a formal system. This is the chimera pursued in the reductive analysis of reasoning.

But in the absence of a formal justification of reasoning, the validity of deduction can still be vindicated by virtue of its dialectic with induction in the self-corrective process of reasoning. By testing the precision and the realism of a supposition about the causality of an object, deduction can narrow the focus and expand the range for induction to search for appropriate signs of the object. Correspondingly, induction can provide deduction with more precise and more realistic hypotheses to test by looking for more immediate and more indicative signs of an object. This cyclical recursion between the two processes enables inquiry to become progressively more ingenious on the one hand and more critical on the other. The rationality of learning can be gauged, therefore, as much by the number and the magnitude of the problems it raises as by the aptness and certitude of the solutions it provides.

**SUMMARY**

A constructive approach to the analysis of reasoning has enabled us to view it as a self-corrective process of learning. We have seen that learning originates in the dialectic between inquiry and anomaly necessary for the perception of a problem. Wonder incorporates the search for a solution into a framework of questions corresponding to the kinds of answers to be sought. And to mediate between the openness of these questions and the cluture in the answers, insight into the patterns intelligible in empirical data must conspire with the inference necessary to articulate the meaning of these patterns into rational assertions. But for inference to reach a reflective equilibrium between theory and fact, it
must be a compound of analysis and synthesis—an analysis of the hypothesis presupposed in any insight; a synthesis of the concrete implications of any of these hypotheses. And since we must also learn to distinguish a realistic from a plausible hypothesis, the general form of inference, including analysis and synthesis, must have two functions, induction and deduction. Induction serves to get a good idea of an object from its apparent signs; deduction, to test the putative efficacy of the object to produce these signs. The cumulative effect of this dialectic—between inquiry and anomaly, between insight and inference, between analysis and synthesis, between induction and deduction—is to make of learning a self-corrective process. The logic of the process leads to a virtually unconditioned but perpetually corrigible solution of any problem we may choose to investigate.

ENDNOTES

1 See Norwood Russell Hanson, Patterns of Discovery: An Inquiry Into The Conceptual Foundations of Science (Cambridge University Press, 1958), 68–9.

2 Aristotle, Metaphysics, I (A) 982b 12.


4 See Nelson Goodman, Of Mind and Other Matters (Cambridge: Harvard University Press, 1985), for a discussion of how we can "construct" a theory within a stipulated version of the world.


Plato, Meno, 82a-86b. This passage so impressed Aristotle that he treated it twice, once in the Posterior Analytics (II 94b 20-35) and again in the Metaphysics (IX, 1051a 22-34), but he argued that it did not demonstrate, as Plato thought, the influence of anamnesis but rather the capacity of intelligence to reach understanding through appropriate images. Aquinas believed this analysis was one of Aristotle’s most important contributions to cognitional theory and he adopted for his own theory: see In II Posteriorum Analyticorum, lect. 9, and In IX Metaphysicorum, lect. 10. Modern Thomists have defended the importance of this analysis: see Karl Rahner, Geist in Welt: Zur Metaphysik der endlichen Erkenntnis bei Thomas von Aquin, 2d ed. (Munich: Kösel-Verlag, 1957); Lonergan, Verbum, 25-33; William Murnion, "St. Thomas Aquinas’s Theory of the Act of Understanding", The Thomist, 37 (1973), 115-6.

Plato, Meno, 80 d5-e5.

Norwood Hanson borrowed the example from C.S. Peirce, but he added to it the figures from Kepler’s notebooks and demonstrated the precise nature of Kepler’s insight: Hanson, 77-85.

Hanson, 87.

Hanson, 82.

Hanson, 83-5.

Lonergan, Insight, 103-245.

Bernard Lonergan, Understanding and Being: An Introduction and Companion to Insight, ed. Elizabeth Morelli and Mark Morelli (New York and Toronto: Edwin Mellen Press, 1980), 26-32. The irony is, though, that the modern turn to formalism to preserve mathematics from the contamination of casual intuitions derived not from a failure of insight but from the confusion of insight with hunch. And while formalism has succeeded in preventing constructivism, with its reliance upon intuition, from having any influence upon the academic development of mathematics, it has, however, been stymied by the paradoxes of set theory and Godel’s incompleteness theorem from reducing mathematics to an axiomatic system: see Allen Calder, "Constructive Mathematics", Scientific American, 241 (1979), 146-71.


Lonergan, "Forms of Inference", 3.

Lonergan, "Forms of Inference", 3.

The term, "reflective equilibrium", comes from John Rawls, A Theory of Justice (Cambridge: Belknap Press at the Harvard University Press, 1971), 20. The idea had already been analyzed by Nelson Goodman, Fact,

23 This set of outcomes is modelled on the combinatorial lattice Piaget devised to give a concrete interpretation to the logical alternatives represented in his quaternary group: see Inhelder and Piaget, 114-21.


28 For the significance of this distinction in the development of science, see Ernan McMullin, "The Goals of Natural Science", *Proceedings and Addresses of the American Philosophical Association*, 58 (1984), 37-64.


31 See note 6 above.


