CARTWRIGHT, GIORGIONE, AND
THE PRINCIPLE OF SUBSTITUTIVITY

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Philosophers have both produced as well as replied to a number of alleged "counter-examples" to the rule of substitution. Recently, Cartwright has urged that the standard reply to at least one of them is inadequate. The counter-example he singles out is:

1. Giorgioni is so-called because of his size.
2. Giorgioni = Barbarelli;
3. Barbarelli is so-called because of his size.

Cartwright argues that since 1) and 2) are true while 3) false, substitution has failed. It is argued in reply that, contrary to Cartwright's claim, substitution does not even occur in the above argument. Rather, the meaning of the predicate "is so-called because of his size" changes from 1) to 3), rendering the invalidity the result of equivocation, not the failure of substitution.
In an interesting paper,\textsuperscript{1} Richard Cartwright defends a principle which he terms the " Principle of Identity" (hereafter, PI). PI is formulated by Cartwright as:

\begin{equation}
\text{PI: if } x = y, \text{ then every property of } x \text{ is a property of } y. \textsuperscript{2}
\end{equation}

Cartwright considers the truth of PI to be so basic as to render it impossible to argue for directly. Yet philosophers have had doubts about PI. Cartwright believes that these doubts are ill-founded, that they are actually misplaced doubts about another principle—one he considers false, and hence distinct from PI—but that this other principle has not always been distinguished from PI. This other principle Cartwright terms the "Principle of Substitutivity" (hereafter, PS). Cartwright formulates PS as follows:

\begin{equation}
\text{PS: for all expressions } A \text{ and } B, \ 'A=B' \text{ expresses a true proposition if, and only if, for all sentences } S \text{ and } S_1, \text{ if } S_1 \text{ is like } S \text{ save for containing an occurrence of } B \text{ where } S \text{ contains an occurrence of } A, \text{ then } S \text{ expresses a true proposition only if } S_1 \text{ does also.} \textsuperscript{3}
\end{equation}

Notice that PI is about things and their properties, while PS is about expressions, sentences, propositions, and truth. One may indeed wonder if there are truly two different principles here or if one is merely a metalinguistic counterpart of the other. Cartwright, however, believes that the principles are not equivalent. He attempts to show this by demonstrating that \textit{there is} a counter-example to PS which fails to be a counter-example to PI. If correct, then Cartwright has shown

\begin{enumerate}
\item Cartwright, "Identity and Substitutivity", p.121.
\item Cartwright, "Identity and Substitutivity", p.120.
\end{enumerate}
that PS and PI are distinct. Though in sympathy with Cartwright's defense of PI, I don't believe that he has produced a counter-example to PS.

Cartwright's alleged counter-example is the following:

A. 1) Giorgione is so-called because of his size.
   2) Giorgione=Barbarelli
   3) Barbarelli is so-called because of his size.

Lines 1) and 2) are both sentences expressing true propositions, while line 3) is a sentence expressing a false proposition. Yet line 3) seems exactly like 1) save for the occurrence of "Barbarelli" in 3) where "Giorgione" occurs in 1). Cartwright thus believes he has provided us with a counter-example to PS. He argues that it does not follow that there is some property that Giorgione has and Barbarelli lacks; and thus we have a counter-example to PS which fails to be a counter-example to PI.

The standard way to handle this example is to claim that 1) should be written out in full. Substitution then yields another conclusion, namely, "Barbarelli is called 'Giorgione' because of his size." Happily, this conclusion is true. However, Cartwright points out that this new argument, though not providing us with a counter-example to PS, is simply a different argument from A. In that he is correct, though in his rejection of this way out, he completely shifts the focus of attention off of the meaning of the predicates involved, a shift which leads to an error.

Cartwright says of 1) and 3) that they are alike, save that 3) contains "Barbarelli" where 1) contains "Giorgione". But this is exactly where he errs; for the difference in names is not the only difference in the sentences under consideration. Put differently, in order to show that A is a counter-example to PS, Cartwright must assume that the expression "...is co-called because of his size" as attached to "Barbarelli" is one and the same as "...is so-called because of his size" as attached to "Giorgione". Only if this assumption were true would A be a counter-example to PS. But is it true? Crucial to the notion of substitution is that one begins with some term and predicate expression to which the term is attached or in which it is embedded. Substitution takes place only when one replaces the term and the predicate expression remains the same.

What Cartwright misses is that the proper logical form of A is not:

\[
\begin{align*}
f(a) \\
a = b \\
f(b)
\end{align*}
\]

as would be the case if substitution had occurred. Rather, the proper form of A is:

\[
\begin{align*}
f(a) \\
a = b \\
g(b)
\end{align*}
\]

Like A, this is an invalid argument — but not one in which substitution has occurred at all—so one could hardly count it as any sort of counter-example to PS. Put differently, what goes "wrong" in A is not that substitution fails, for it doesn't even occur; rather, we have in A a straightforward case of equivocation.

It remains to show that the predicate expression "...is so-called because of his size" as attached to "Giorgione" is different from the predicate expression "...is so-called because of his size" as attached to "Barbarelli". To do this we must give reasonable criteria which enable us to count the predicate expressions as two. Such criteria must be applicable to all expressions. I suggest at least two criteria for the sameness of expressions. 'Two' occurrences of an expression are to be counted as the same only if they fulfill both criteria. For the first, if two expressions are to be counted as one then they must be at least tokens of the same geometrical type. The predicate expressions with which we are concerned meet this first criterion. If difference of geometrical type is sufficient to count two expressions as different, is sameness of geometrical type sufficient to count two expressions as the same? Cartwright does not explore this, but it

5. It might be replied that this is exactly Cartwright's point; that PS, as formulated by Cartwright, allows for equivocation and hence invalidity. I don't believe this is Cartwright's intention, however. Surely it would not be a "counter-example" to say, the reflexivity of identity that the word "this" can be used to refer to different objects, thus rendering false some uses of the sentence "this=this".

6. Of course, these are not exhaustive. But they are enough to handle the case at hand.
is crucial to our case. An example might help. Consider the following:

B. 1) She is not feeling well.
   2) She = the Queen of England

If one is given further information that the ill person mentioned in 1) above is different from the Queen, one would not even consider concluding that the Queen is ill— for the simple reason that the "she" in 1) and the "she" in 2) are different words—even though they exhibit the same shape. Clearly, similarity of shape of two expressions is not sufficient to enable us to call them the same. Hence, the second criterion, namely, that the expressions must mean the same thing. This is not to appeal to some specific theory of meaning or invoke some problematic notion of "having the same meaning". Rather, anyone who argues within the context of counter-examples must implicitly employ such a notion. If he does not, then he can find "counter-examples" to any logical principle merely by equivocating on the meaning of the expressions involved. (An objector may claim that often we do count two expressions as the same if they are of the same shape. The objector says with respect to the uses of "she" in 1) and 2) above, however, that they are two uses of the same word. Note, however, that we both recognize the necessity of distinguishing some difference in the "she" in 1) and the "she" in 2), so the difference between us is only a verbal one.)

Return now to the original question: is the expression "...is so-called because of his size" as attached to "Giorgione" the same as the expression "...is so-called because of his size" as attached to "Barbarelli"? With respect to our second criterion, the answer is clearly no; the meanings of the expressions are different. As extra-linguistic items individuate the two uses of "she" in B, so too linguistic items individuate the two expressions under consideration. That is, the difference in women makes the two uses of "she" significantly different and the difference in names preceding "...is so-called because of his size" makes them different expressions.

One may insist, however, that there is a clear sense in which all occurrences of "...is so-called because of his size" mean the same thing. Although this is true, the sense of "meaning the same thing" here involved is that in which the two different uses of "she" in B mean the same thing.
Put differently, if the reference of "so-called" were fixed, that is, if we took it to mean "called 'Giorgione'", then 3) in A would no longer be false and we would have no counter-example. However, since the reference of "so-called" changes, then the whole predicate expression has changed. So the correct conclusion to draw is that not all uses of "...is so-called because of his size" express the same thing, as Cartwright assumes, but are as many expressions as there are different names to which they attach. If one does treat it as one expression, one is led into the difficulties that Cartwright suggests; but one would be led into such difficulties with any word or phrase whose meaning shifts from premise to conclusion.

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