LOTTERIES, KNOWLEDGE, AND PRACTICAL REASONING

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ABSTRACT: This paper addresses an argument offered by John Hawthorne against the propriety of an agent’s using propositions she does not know as premises in practical reasoning. I will argue that there are a number of potential structural confounds in Hawthorne’s use of his main example, a case of practical reasoning about a lottery. By drawing these confounds out more explicitly, we can get a better sense of how to make appropriate use of such examples in theorizing about norms, knowledge, and practical reasoning. I will conclude by suggesting a prescription for properly using lottery propositions to do the sort of work that Hawthorne wants from them.

KEYWORDS: lottery paradox, knowledge norm, practical reasoning

I. The Argument

Suppose that Jane holds a ticket in a fair lottery with 10,000 tickets. She knows that the probability of her winning is a paltry 0.01%. She is offered a penny in exchange for her ticket, and reasons as follows:

The ticket is a loser.
So if I keep the ticket I will get nothing.
But if I sell the ticket I will get a penny.
So I’d better sell the ticket.

Hawthorne makes at least three comments about such reasoning.

“It seems clear enough that such reasoning is unacceptable.”

“It is clear that if one asks ordinary folk why such reasoning is unacceptable, they will respond by pointing out that the first premise was not known to be true.”

2 Hawthorne, Knowledge and Lotteries, 29.
“When we claim that no one can know that he or she will lose the lottery, part of what is going on is that we realize that no one is in a position, in advance of the lottery draw, to acceptably sell a lottery ticket for minimal return.”

In other words, Hawthorne argues not only that such reasoning is clearly unacceptable, but moreover that the source of the pathology is that the first premise is not known. He concludes that one ought not use premises in practical reasoning that one does not know.

Two observations about the interpretation and analysis of Hawthorne’s argument seem particularly salient. First, we must be careful not to construct the example so that the reasoning is found defective on the basis of an attendant decision’s being defective. Second, we must be careful to construct the example so that the pathology of the reasoning is best explained by whether the agent knows the premise(s) and not merely by appealing to decision-theoretic considerations.

II. Analysis

We must distinguish between Jane’s conclusion to sell the ticket for a penny and her reasons for doing so. This is because there are potential confounds in the sources of the intuitions to which Hawthorne appeals. Part of one’s intuition that Jane reasons irrationally might consist in one’s regarding her decision to sell the ticket for a penny as irrational; in rejecting her conclusion, one may look for a fault in her reasoning. But this is not how Hawthorne wants the example to function. He does not want his argument to depend on the rejection of her reasoning based on whether her conclusion is sub-optimal. His position is that, irrespective of whether it’s rational for Jane to sell her ticket for a penny, her reasons for doing so had better not be that the ticket is a loser (since she can’t know this).

This is important because Jane’s decision is not clearly unacceptable. One possible confound is that the propriety of her decision is partially a function of the expected value (EV) of her ticket. The standard decision-theoretic norm for reasoning in lotteries and wagers begins with specifying the EV for one’s ticket:

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4 Hawthorne, *Knowledge and Lotteries*, 178.
5 Christopher Hill and Joshua Schechter give a thorough and detailed analysis of Hawthorne’s argument but miss this other serious error. See their “Hawthorne’s Lottery Puzzle and the Nature of Belief,” *Philosophical Issues*, 17 (2007): 102-22.
the product of the probability of winning and the prize to be won.\footnote{Strictly speaking, for a single prize wager, \( EV = (Probability\ of\ Winning \times\ Prize\ to\ be\ Won) - Cost\ of\ Wager. \) Since Jane already owns the ticket, we ignore the cost of the ticket in calculating the EV.} If the EV in this case is below what Jane is offered for her ticket, then she should sell the ticket.\footnote{And assuming that one's utility function is sufficiently linear and that one is risk neutral. Being risk neutral means that an agent equally prefers a \textit{wager} with expected value \( w \) to the expected value of the wager (without having to take the risk of losing). These are not particularly good assumptions given research into how people \textit{actually} behave; but, at least for the purposes of decision theory, these are standard assumptions for ideally rational agents. Furthermore, these assumptions are in play for the norms of decision theory, even though they may fail to describe actual agents.} If the (solitary) first prize for her lottery were less than $100, then the EV of her ticket would be less than a penny. In such a case she should sell her ticket for a penny, if possible. So, contra Hawthorne's (3), Jane would be rationally justified in selling her ticket for ‘minimal gain’ in advance of information on the result of the lottery draw. It follows that Hawthorne’s example is not such a clear case of unacceptable reasoning; it depends on details that are not specified in his treatment. In many circumstances, such a decision would be rationally appropriate.\footnote{Most lotteries have a negative expected value: that’s how the corporations running lotteries make money.}

Furthermore, the potential pathology of Jane’s reasoning is not clearly to be identified in whether she \textit{knows} the first premise. If the prize structure for the lottery were specified, we would be in a better position to analyze the propriety of Jane’s reasoning. For example, if the (solitary) first prize were greater than $100, then Jane would be engaging in poor reasoning in coming to sell her ticket for a penny. Elsewhere Hawthorne suggests that the lottery has the following structure: 10,000 tickets and a (solitary) first price of $5,000.\footnote{Hawthorne, \textit{Knowledge and Lotteries}, 85.} Here Jane’s ticket has an EV of $0.50. If Jane were to sell her ticket for $0.01 she would be making a mistake since this is below the EV of her ticket. This raises the prospect that the intuitive problem with her selling the ticket is entirely a matter of her ticket's EV, given what she is offered in the sale. That is, what explains the pathology of her reasoning will sometimes be that she is selling her ticket for less than it is worth. No mention of whether she \textit{knows} the premises is required for this analysis. Hawthorne holds that in this case there is a difference between being offered the chance to sell the ticket for a dime rather than a cent.\footnote{Hawthorne, \textit{Knowledge and Lotteries}, 85 n91.} But since both are well
below the EV of the ticket, both decisions are irrational. By Hawthorne’s argument, if the reasoning in both cases depends on the premise that one’s ticket will lose, then both are irrational decisions (on decision-theoretic grounds) and both use bad reasoning (because the premise is not known). The key point is that the case should be constructed so as to avoid confounds based on intuitions surrounding the decision to sell a lottery ticket prior to information on the draw. The focus must be on an agent’s reasons.

One might think that Hawthorne (with Jason Stanley) has responded to this sort of objection. Hawthorne and Stanley argue that decision theory doesn’t properly account for our folk uses of ‘know’ in appraisals of practical reasoning. They argue for the Action-Knowledge Principle (AKP) and Reason-Knowledge Principle (RKP).

AKP: Treat the proposition that \( p \) as a reason for acting only if you know that \( p \).

RKP: Where one’s choice is \( p \)-dependent, it is appropriate to treat the proposition that \( p \) as a reason for acting iff you know that \( p \).

Hawthorne and Stanley argue that AKP “straightforwardly accounts for the use of epistemic terms in appraisals of practical reasoning” and that the most natural explanation is by appealing to AKP as the norm of practical reasoning rather than the expected utility norms of decision theory.

Of course, Hawthorne and Stanley’s position is meant to be consistent with the norms of decision theory, but they take their account to be more complete. They argue that a complete theory of practical reasoning must integrate an EV-based account with a reasons-based account, in order to accommodate cases in which people perform decision-theoretically correct actions for the wrong reasons. For example, suppose that in deciding whether to hire one of two candidates, a hiring committee chooses the stronger candidate – but only because her name starts with a B rather than a C. The right decision was made (it

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12 Hawthorne and Stanley, “Knowledge and Action,” 577.
13 Hawthorne and Stanley, “Knowledge and Action,” 578. An option is \( p \)-dependent iff the most preferable option on supposition of \( p \) is not the same as the most preferable option on the supposition of \( \neg p \).
14 Hawthorne and Stanley, “Knowledge and Action,” 578. I prefer using EV to expected utility or EU; but these terms are interchangeable for current purposes.
maximizes the firm’s EV) but the reason (other things being equal) was terrible. Hawthorne and Stanley argue that decision theory can only evaluate a decision based on EV considerations and is not able to account for our intuitions about making the right decision for bad reasons. So they argue that in the lottery case Jane may be coming to the right decision (to sell her ticket) if the EV of her ticket is less than what she is offered in the sale, but she’s doing it for poor reasons (because she does not know that her ticket will lose).\(^\text{15}\)

Yet even if we grant Hawthorne and Stanley’s criticism of decision theory as being inadequate for a complete theory of practical reasoning, this does not count against my argument that the analysis of the lottery case (at least as Hawthorne represents it) is confounded by decision-theoretic considerations. That Jane’s decision is a poor one is adequately explained by the observation that she ought not to sell the ticket because she’s selling it for less than it’s worth. If Jane’s ticket had an EV of $0.001, however, she would be making the right decision to sell her ticket. In neither case would we have to appeal to whether she knew the propositions that she uses in her practical reasoning.

**Conclusion**

I have argued that Hawthorne’s example of pathological practical reasoning is defective in at least two respects. First, there are possible confounds producing the intuition that such reasoning is defective, besides the intended interpretation that the problem consists in reliance on a premise that is not known. One of these confounds is that the perception that the decision is irrational may determine the rejection of the reasoning on which it was based. However, it is not clear that the conclusion of the argument is irrational. In some circumstances it may count as good reasoning. And second, when relevant details are added to make the example pathological, the problem with the reasoning is not necessarily that the agent lacks knowledge of a premise. Hawthorne’s purposes require a lottery case in which the best explanation is that the agent does not know the premise that the ticket will lose. That is, the case should not admit of a better explanation by merely appealing to decision-theoretic considerations. The problem is that Hawthorne (and Stanley) continues to feature the original problematic version of the lottery case as a

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\(^{15}\) On such an account the decision to sell a ticket would be correct if the EV of the ticket is less than what is being offered in a sale and the agent’s reasons for selling the ticket are that the EV is less than what is being offered in the sale. The reasons could not include propositions which the agent does not know (such as that the ticket is a loser).
paradigm example of pathological practical reasoning (involving an agent using premises that are not known).

Does this mean that Hawthorne’s argument is scuttled? That conclusion would be too strong. In fact, Hawthorne and Stanley mention a much better case but don’t feature it as their paradigm case. Suppose that Jane holds a ticket in a lottery with 10,000 tickets. The solitary first prize is $20,000. She knows that the EV of her ticket is $2. She is offered the opportunity to sell her ticket for $5. On decision-theoretic grounds she should sell her ticket. However, prior to information on the draw she had better not reason as follows:

The ticket is a loser.
So if I keep the ticket I will get nothing.
But if I sell the ticket I will get $5.
So I’d better sell the ticket.

Although she comes to the correct decision-theoretic conclusion, she does so for bad reasons. Why are her reasons bad? Hawthorne (and Stanley) argue that she does not know the first premise. This is a much better example of using the selling of a lottery ticket before information of the draw as a case of pathological practical reasoning than Hawthorne’s original case. The latter case avoids the confounds present in the former.

Ideally, the scope of these remarks will extend beyond simply analyzing a single case of reasoning about a lottery ticket. More generally, I hope to motivate the idea that we must make explicit what aspect of practical reasoning is to be the object of evaluation when we argue from cases: the decision, or the reasons for making a decision. If only the decision is the proper object of evaluation, then it is plausible that only decision-theoretic considerations are required for the analysis, and no appeal to whether premises are known is necessary. But if an agent’s reasoning in practical decision making is a proper object of our interests, then illustrative cases ought to be constructed so as to indicate that decision-theoretic considerations alone are insufficient to capture our intuitions. Abiding by this principle in the case at hand – by constructing it so that the proposed sale would be for more than the ticket’s EV – would remove the confound that poor reasoning in lottery cases is better explained by decision-theoretic considerations alone. Furthermore, the case should be constructed so as to not be confounded by

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16 Hawthorne and Stanley, “Knowledge and Action,” 575.
17 We’d specify that we are not concerned with endowment effects, nonlinear utility functions, loss aversion, transaction costs, and so on.
intuitions surrounding rejecting an agent’s reasons based on the impropriety of the attendant decision. The original lottery case suffers this confound but the latter case does not. Consequently, Hawthorne (and Stanley) should cease featuring the original case as the central example of poor practical reasoning.\textsuperscript{18}

\textsuperscript{18} I would like to express my thanks to Tim Kenyon, John Turri, Dave DeVidi, and Mathieu Doucet for their help on various versions of this paper.