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PLATO AND THE MATHEMATICAL OBJECTS

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1. Introduction

A way to see philosophy and the adventure of its development is to compare today's views concerning being (*το ον*) and its appearances (*φαίνεσθαι*) with the views which firstly were presented in the philosophical framework created by the Ancient Greeks. In this framework, the most basic question is the one concerning exactly the nature of being, as it is mirrored (assuming that such a thing is obtained) through the sensory and mental glasses that human beings happen to have. But what humans have at their disposal is not being in itself, but its mirroring. In the framework of such a mirroring they are obliged to invent, with reason, ways of representing questions, which do not belong entirely to the realm of appearances. Such questions can be rather in the area (if such an area really exists) between being and its appearances, and in order to be properly answered they should be questions asked by a superhuman having at his mental sensory disposal both being and its mirroring. Some of the questions we have in mind, as we talk about them in such generality, can be expressed as follows: What is that, which exists, and how it gets to be known? What is the structure of appearing and what sort of correspondence exists (if it exists) between being and our possible knowledge of it? Are the contents of our mind spatiotemporally relativized and if yes, how are they structured? Are our images of the world dependent exclusively on our sensory data? Are there any other entities spatiotemporally independent and unchangeable, immediately connected with our world's structure, and if such entities exist, which are they? Finally, if such entities exist, how do we manage (if we in fact do) to make them to be known to us?

Questions like the above mentioned are not new to the philosophers. They started being formed from the very beginning of the philosophical adventure. As a matter of fact, they are the sort of questions which from the very beginning determined both the limits and the content of philosophy. That is not to say that all possible philosophical questions were discovered and expressed from the very beginning of the human intellectual history. A lot of questions were added and a lot of others were dismissed throughout the years, as belonging to the realm of science. Nevertheless, the basic questions were asked almost from the very beginning of the philosophy's history. Such basic questions are all those, which cannot be framed scientifically, because they are questions which concern the limits of knowledge. Science can talk about its contents and not its limits. The limit problems of knowledge are not and cannot be or become scientific problems. That is why there is and there always be space (genuine space) for philosophy which is that of *knowledge's limit problems*. Such a problem is the one already mentioned, concerning being (*το ον*) and its appearances (*φαίνεσθαι*). Humans do not mentally contain being itself. So they can judge and talk about it through the images they have of it. In other words, they talk about being and its appearances through mental and linguistic representations of it. They talk about representations of being not from the basis of being itself, but from the only basis they have at their disposal, that is, through their representation of it. They talk, in other words, about the being as represented through representational self-referential predicates and categories.

For Plato the world, as it appears to us, should be based upon the world as it really is. Nevertheless, the world of appearances possesses characteristics which contradict the timeless and unchangeable nature of platonic ideal entities. It is a world based upon continuous spatiotemporal change governed by causality. Because of that, it cannot be the dominant factor for the stable and immovable realm of knowledge. Knowledge should be organized and finally formed obeying rules of permanence and unchangability. Its genuine object also should be the constants of representing the world as it really is, no matter whether or not such a representation is of a bijective[1] nature. So, for Plato the foundations of the world, both ontologically and epistemologically, are those constants which form his realm of *Ideas*. For instance, the Idea of "Good" exists as independent, immaterial and atemporal entity, because of which expressions like "the good Helen", "the good giant", "the good Frances" become meaningful. That is, Frances comes to be good, because, as good Frances, participates[2] to the Idea of Good. The ontological reason (which is not of a spatiotemporal origin) of the existence of the particular good Frances, as good, is his *μέθεξις* to the Idea of Good. Such *μέθεξις* is, so to speak, the be-causality for the existence of good Frances, as good.

The basic problem of the platonic system is not only that of knowledge per se, but also the problem of its sources. It is almost obvious that, according to Plato, such sources are not of empirical nature. Sense perception, therefore, is not such a source, but simply plays the role of the midwife during the process of transferring knowledge already existing at the level of the unconscious to the level of knowledge that we become aware of. Such a process of uncovering knowledge could take place according to Plato

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[1] The adjective "bijective" is used here with its technical, mathematical meaning. If we have two sets A and B, a function f from A to B is said to be bijective, if for every member a of A there exists exactly one member b of B such that f(a)=b and for every member b of B there exists exactly one member a of A such that, again, f(a)=b.

[2] We use here the most common translation of the ancient greek term «μέθεξις» into English, which is "participation".

always through and with the help of either sense perception or, of what he calls, the use of *dialectical* method[3]. An example of the usage of the dialectical method, together with the vivid support that what is uncovered was always there, in an unconscious manner, one can find in the platonic dialogue *Meno*[4]. There Meno calls upon a young slave with whom Socrates starts a conversation. The question that Socrates proposes to be examined is the one concerning knowledge as *recollection*, through the usage of a mathematical example, which has as follows: What is the length of the side of a square the area of which is double the area of a square, the length of each side of which is equal to 2 feet? In what follows in the dialogue, Socrates, in a dialectical manner, manages finally to get from the mouth of the absolutely ignorant young slave the right answer. At exactly this juncture, Socrates points out to Meno that the ignorant slave was dialectically led to the right answer, without the help of new empirical knowledge, and he continues:

> *Socrates*: What do you think Meno? Has he answered with any opinions that were not his own?
>
> *Meno*: No, they were all his.
>
> *Socrates*: Yet he did not know, as we agreed a few minutes ago.
>
> *Meno*: True.
>
> *Socrates*: But these opinions were somewhere in him, were they not?
>
> *Meno*: Yes.
>
> *Socrates*: So a man who does not know has in himself true opinions on a subject without having knowledge?
>
> *Meno*: It would appear so.
>
> *Socrates*: At present these opinions being newly aroused, have a dream-like quality. But if the same questions are put to him on many occasions and in different ways, you can see that in the end he will have a knowledge on the subject as accurate as anybody's.
>
> *Meno*: Probably.
>
> *Socrates*: This knowledge will not come from teaching but from questioning. He will recover it for himself.
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> *Meno*: Yes.
>
> *Socrates*: All the spontaneous recovery of knowledge that is in him is recollection, isn't it?
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> *Meno*: Yes.

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[3] The platonic dialogue *Theaetetus* contains all the necessary elements for the understanding of the role of the dialectical method in the uncovering of unconscious knowledge. See, *Theaetetus* 148e-151d.

[4] See *Meno* 81c-86b.

> *Socrates*: Either he has at some time acquired the knowledge which he now has, or he has always possessed it. If he always possessed it, he must always have known; if, on the other hand, he acquired it at some previous time, it cannot have been in this life, unless somebody has taught him geometry. He will behave in the same way with all geometric knowledge and every other subject. Has anyone taught him all these? You ought to know, especially as he has been brought up in your household.
>
> *Meno*: Yes, I know that no one ever taught him.
>
> *Socrates*: And has he these opinions, or hasn't he?
>
> *Meno*: It seems we can't deny it.
>
> *Socrates*: Then if he did not acquire them in this life, isn't it immediately clear that he possessed and had learned them during some other period?
>
> *Meno*: It seems so.
>
> *Socrates*: When he was not in human shape?
>
> *Meno*: Yes.
>
> *Socrates*: If then there are going to exist in him, both while he is and while he is not a man, true opinions which can be aroused by questioning and turned into knowledge, may we say that this soul has been forever in a state of knowledge? Clearly he always either is or is not a man.
>
> *Meno*: Clearly.
>
> Socrates: And if the truth about reality is always in our soul, the soul must be immortal, and one must take courage and try to discover – that is, to recollect – what one doesn't happen to know, or more correctly, remember, at the moment.
>
> Meno: Somehow or other I believe you are right[5].

The platonic theory of knowledge as recollection tells us that what is recollected, firstly, was always there, and, secondly, that what is recollected is, no matter how clear, basically true. In such a case what it would mean to recollect a wrong idea given that the knowledge of an idea means the right possession of it and therefore its truthfulness? Plato's answer to the problem can be found in the dialogue Theaetetus. There he recognizes its difficulty and he tries to solve it by transferring it to the level not of the semantics, but of the syntax of the used language. He is suggesting that the real source of the mistake comes from the fact that it is possible to be given, to a certain idea, the wrong name B instead of the right one A. This is, according to Plato, the basic source of the confusion and the mistakes. It is a confusion of a linguistic and not of a substantial origin[6].

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[5] Ibid. 85b-86b.

[6] See, *Theaetetus*, 196c-199c.

2. Plato's mathematical objects

It looks quite possible that Plato's knowledge of the mathematics of his time was good. It is quite certain, on the other hand, that mathematics and mathematical objects could and should be kept, for him, just outside the realm of what is perceptually accessible. What is important about them is that (a) their existence is independent of the existence of their possible knower, (b) they remain unchangeable and are not dependable upon spatiotemporal circumstances, (c) they are independent of the way we perceive and think about them, (d) they are deeply interwoven in the fabric our world is made of and (e) they are absolute in the sense that there are not relativistic representations of them. But, given all the above, let us see what is the possible connection between platonic ideas and Plato's mathematical objects.

Exactness, atemporality and perceptual independence of them are some of the basic characteristics of platonic ideas. It seems that Plato's mathematical objects have such characteristics. The question that arises at this point can be phrased as follows: What is the relation between platonic ideas and Plato's mathematical objects? What is, for instance, the existential status of the objects of geometry?

What appears as certain is that the discovery of the objects and the theorems of geometry have very little to do with observational procedures and measurements, and therefore truths about them are not (except by approximation) truths about objects of our sense perception. Plato believed that we cannot get a perfectly plane surface, or a perfectly straight line or even a dimensionless point through sense perception. In a way, nothing perceptible is as perfect as it is required in order to be an ideal entity. There are no perceptible ideal spheres or circles, for instance. Everything perceptible is necessarily three-dimensional. Therefore, perceptible lines cannot be one-dimensional and perceptible planes can only be approximately two-dimensional. This holds for everything which depends upon geometrical considerations for its description, like, for instance, planetary orbits[7]. It is, therefore, the case that geometrical and mathematical objects are for Plato like the Ideas. They do share with them a lot of common characteristics with the most basic ones those of spatiotemporal unchangability and of their non-perceptible nature. Nevertheless, they do differ in, at least, one important aspect. The Idea of the "Good" is unique, the notion of a "circle" as a non-perceptible entity is not. Ideal circles, as it were, are more than one, depending, not on sense perception considerations, but on strictly logical requirements, having to do with the center and the radius of each one of them. It is that difference which makes it quite difficult for Plato to consider mathematical objects as Ideas of a special sort. It seems that a better solution was always the adoption of postulating the mathematical and especially the geometrical objects as entities *intermediate* between perceptible things as chairs, tables etc. and forms. In Book VII of the *Republic*, for instance, Socrates seems to think that mathematics belong to the mental area of pure thought:

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[7] See, *Rep*. VII 529c-530a.

> It is befitting, then, Glaucon, that the branch of learning should be described by our law and that we should induce these who are to share in the highest functions of state to enter upon the study of calculation and take hold of it not as amateurs, but to follow it up until they attain to the contemplation of the nature of number, by pure thought, not for the purpose of buying and selling…[8]

And again:

> … this science is in direct contradiction with the language employed in it by its adepts … Their language is most ludicrous, though they cannot help it, for they speak as if they were doing something and as if all their words were directed toward action. For all their talk is of squaring and applying and adding and the like, whereas in fact the real object of the entire study is pure knowledge … It is the knowledge of that which always is and not of a something which at some time comes into being and passes away. That is readily admitted, he said, for geometry is the knowledge of the eternally existent. Then, my good friend, it would tend to draw the soul to truth, and would be productive of a philosophical attitude of mind, directing upward the faculties that now wrongly are turned earthward[9].

In modern terms, we would describe the platonic position concerning geometry and mathematics in general, as drawing a line between applied and pure geometry or mathematics. Applied geometry or mathematics have to do with the practice "of squaring and applying and adding and the like". Pure geometry or mathematics concerns "the real object of the entire study" which is "pure knowledge", "the knowledge" that is "of the eternally existent".

Concerning now the idea that the geometrical or, rather, the mathematical objects are entities *intermediate* between perceptible objects as chairs, tables etc. and forms, we should add that this is not accepted by the entire community of Plato scholars. It is Aristotle who first called them *intermediates*[10]. Nevertheless, there is no abundant textual support for such an interpretational viewpoint. For instance, Burnyeat[11] leaves the question of the existence of mathematical objects unanswered. He does it by insisting that Plato himself "leaves … [the problem] … tantalizingly open".

As we have already said, the mathematical objects are not existentially dependent upon the existence of somebody who thinks about them. Additionally, it is almost certain in the long run that all of us will discover (and not construct) basic mathematical objects as, for instance, the natural numbers. The multiplicity of mental representations

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8 Ibid. 525b.

9 Ibid. 527a-527b.

10 See, *Metaphysics*, 987b.

11 See, Burnyeat, M.F. "Plato on why mathematics is good for the soul", *Proceedings of the British Academy* 103, 2000, p. 22.

of such objects should not alarm us. Such a multiplicity is harmless, because those mental representations are basically isomorphic. That is due to basic similarities which characterize the way all of us think, as representatives of the same prototype, that is, of the human being. Such an isomorphism can be thought of as the basis upon which could be founded the unchangeable uniqueness of a universe of mathematical objects and, moreover, of a universe of platonic Ideas.

The necessity which characterizes mathematical truths and their non-relativistic absoluteness, is due to the fact that they are exact descriptions of unchangeable structural properties of a universe of also unchangeable non-spatiotemporally located objects. The mathematician, as it has already been mentioned, dos not construct or fabricate, but simply discovers them. It is in this sense that we can say, imitating Plato, that they already existed in the human mind, even though in an unconscious manner. Plato's dialectical method offers us a way through which we become aware of them. The discovery of unconsciously hidden truths about abstract objects already there, is a basic characteristic of the passage from the level of the *possible* to the level of the *actual*:

> Many times the discovery of such truths follows the paths of absolutely concrete algorithms. Nevertheless, such algorithms are not necessary for the characterization of a sentence as true or false. They are, of course, necessary for the discovery of truth without being an indispensable ingredient for the structure of truthfulness. A mathematical sentence is true or not, because it corresponds or not to certain structural characteristics of the universe of mathematical ideas, which are described correctly or not by the sentence we are talking about. The particular mathematical construction involved, or the algorithm which is going to be used, plays a purely secondary role and it represents Ariadne's thread for the exit from the labyrinth. The exit as a way out of it, nevertheless, exists independently of whether we can find it or not through the usage of the particular thread. Many times, we need intelligence and patience for succeeding in finding it. The existence, though, of a possible way out through a certain path exists independently of all that[12].

3. Two examples of contemporary platonic positions concerning the objects of mathematics

Modern mathematical platonists do not seem to be that much concerned with the notion of *μέθεξις* in order to support the idea of spatiotemporal independence of mathematical objects. Additionally, they do not care that much to separate pure Ideas like the one of the *Good* from mathematical Ideas concerning mathematical objects. It is, even, more or less accepted that *μέθεξις* (participation) is a characteristic of the

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[12] See Anapolitanos D. A. *Introduction to the Philosophy of Mathematics*, Nefeli Publ. Co., Athens, 1985, p. 34 (in Greek).

initial platonic system which, somehow, is not necessary to be used in order to talk about the ontological independence of mathematical objects from the particularities of their discovery. But let remind ourselves how Aristotle himself describes platonic *μέθεξις*. Having referred to some of the Presocratic philosophical systems, he continues:

> After the systems we have named came the philosophy of Plato, which in most respects followed these thinkers, but had peculiarities that distinguished it from the philosophy of the Italians. For, having in his youth first become familiar with Cratylus and with the Heraclitean doctrines … these views he held even into his later years. Socrates, however, was busying himself about ethical matters and neglecting the world of nature as a whole, but seeking the universal in ethical matters, and fixed thought for the first time in definitions: Plato accepted his teaching, but held that the problem applied not to sensible things but to entities of another kind – for this reason, that the common definition could not be a definition of any sensible thing, as they were always changing. Things of this other sort, he called Ideas, and sensible things, he said, were all named after these, and in virtue of a relation to these; for the many existed by participation in the Ideas that they have the same name as they. Only the name "participation" was new; for the Pythagoreans say that things exist by "imitation" of numbers, and Plato says they exist by participation, changing the name. But what participation or the imitation of the Forms could be they left an open question[13].

Independently of the use or not of the notion of platonic *μέθεξις* by contemporary philosophers of the realist school, the influence of platonic philosophical system on their own was quite substantial. Such an influence concerned especially the two main theses characterizing the realist branch of the philosophy of mathematics. First, the thesis that mathematical objects are existentially independent and do not bear the mark of their spatiotemporally oriented discovery or appearance and, second, that mathematical truths are of an unhistorical nature. Mathematicians, especially with philosophical tendencies or philosophers with mathematical sensitivities in opposition to, for instance, physicists or philosophers with empiricist tendencies, are much more inclined towards platonic considerations having to do with the possibly unchangeable nature of mathematical objects. Such a thing is probably due to the fact that physicists tend to have an empiricist point of view, because of their possible belief that the natural world (which is at the center of their scientific life) is continuously changing, while mathematicians are rather inclined towards the immovability and unchangability of mathematical objects and the universality of mathematical truths.

In the following we will present two examples of important mathematicians with platonic views concerning their way of seeing the world of mathematical objects and the mathematical truths. The first is Frank Drake, an expert in mathematical logic and

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[13] See *Metaphysics*, again, 987a30-b10.

especially in the theory of sets, who in his book *Set Theory: An Introduction to Large Cardinals* states that:

> Some comment is in order concerning the philosophical viewpoint taken in this book, of the foundation of mathematics and in particular set theory. Since set theory is sometimes regarded as important only because it provides a foundation for the rest of mathematics (and certainly much of the early impetus of its study derived from this aspect of set theory) philosophical positions on the foundations of mathematics have more marked impact on set theory than anywhere else in mathematics; and the reader should be well aware of the prejudices of the author he is reading. I have written this book from an uncompromisingly *realist* or *platonist* position; that is, I have taken the viewpoint that in some sense sets do exist, as objects to be studied, and that set theory is just as much about fixed objects as is number theory. This viewpoint is sketched in the introductory Chapter 1, and is the main reason for that chapter. It seems very difficult to me to give any reason for the study of large cardinals without taking a viewpoint of this sort[14].

Quite interesting is, also, the opinion expressed by Kurt Gödel, who draws our attention to the inappropriateness of sense perception for explaining the existence of mathematical objects. He develops an idea of mathematical intuition which is responsible for our unaided familiarity with important axiomatic truths of the theory of sets. It is that intuition and not sense perception which helps us to uncover mathematical truths which, as it were, "force themselves upon us as being true". But let us hear Gödel himself describing the situation:

> … despite the remoteness from sense perception, we do have something like a perception also of the objects of set theory as is seen from the fact that the axioms force themselves upon us as being true. I don't see any reason why we should have less confidence in this kind of perception, i.e., in mathematical intuition, than in sense perception, which induces us to build up physical theories and to expect that future sense perceptions will agree with them and, moreover, to believe that a question not decidable now has meaning and may be decided in the future[15].

The special attractiveness of realist considerations for the mathematicians or the mathematically oriented philosophers, as it has already been mentioned, should not surprise us. It is almost obvious that the research subject of mathematics has not the sense perceptional character, which one can find dealing with the natural sciences. The

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[14] Drake, F. R. *Set Theory: An Introduction to Large Cardinals*. North-Holland, Amsterdam, Oxford, New York, 1974, p. VIII.

[15] See Gödel, K. "What is Cantor's continuum hypothesis", in Benacerraf, P., and Putnam, H. *Philosophy of Mathematics: Selected Readings*. Second Edition, Cambridge University Press, Cambridge, 1983, pp. 483-484.

physicist, for instance, no matter how abstract could be the way he looks at the world, has the feeling that this world is just out there to be seen, touched or smelled. On the other hand, the path followed, especially by those who work on pure mathematics, is genuinely mental. Sense perception plays (if it does at all) a quite secondary role in the discovery of mathematical objects and mathematical truths. For the philosophically inclined towards platonism mathematician or mathematically oriented philosopher the mathematical objects and the mathematical truths are neither created nor constructed. They are somewhere out there waiting, as it were, patiently to be *discovered* (that is, to become known) at a certain moment when the time is right. This way of describing the situation has an air of paradoxicality without constituting a paradox. Timeless truths and objects come to be (in the sense of coming to be known by the human) when the time is right and the circumstances appropriate.

Bibliography

1. Αναπολιτάνος, Δ. Α. *Εισαγωγή στη Φιλοσοφία των Μαθηματικών*, Εκδόσεις Νεφέλη, Αθήνα, 1985.

2. Barnes, J. (Editor) *The Complete Works of Aristotle*, Princeton University Press, Princeton, New Jersey, 1984.

3. Benacerraf, P. – Putnam, J. (editors) *Philosophy of Mathematics: Selected Readings*, Second Edition, Cambridge University Press, Cambridge, 1983.

4. Benson, H. H. "The Problem is not Mathematics but Mathematicians. Plato and the Mathematicians", *Philosophia Mathematica*, Vol. 20 (2), Special Issue on Plato's Mathematical Methodology, 2012, pp. 170-199.

5. Burnyeat, M. F. "Plato on why Mathematics is Good for the Soul", *Proceedings of the British Academy*, 103, 2000, pp. 1-81.

6. Cornford, F. M. "Mathematics and Dialectic in the Republic", *Mind* 41, pp. 37-52.

7. Drake, F. R. *Set Theory: An Introduction to Large Cardinals*, North-Holland, Amsterdam, Oxford, New York, 1974.

8. Hamilton, E. – Cairns, H. (editors) *The Collected Dialogues of Plato Including the letters*. Bollinge Series LXXI, Princeton University Press, Princeton, New Jersey, Ninth Printing, 1978.

9. Jaeger, W. (Editor) *Aristotle: Metaphysics*, Oxford University Press, Oxford, 1957.

10. Shapiro, S. *Thinking About Mathematics*, Oxford University Press, Oxford, 2000.

11. White, N. *Plato on Knowledge and Reality*, Hackett Publ. Company, Indianapolis, 1976.