This discussion addresses a minuscule bit of philosophical history focusing on one aspect of the work of one particular philosopher. Oddly enough it is written by the very philosopher involved. Writing it has made me only too clearly conscious of the fact that in an historical inquiry of this sort the subject himself does not enjoy any particular advantages over others. The only thing that does separate such an historian-and which may well set him apart from the great majority of others-is the firm determination to be fair and indeed generous to the individual involved.

Some sixty years have now passed since my high-school junior year when I began to be seriously interested in the study of philosophy. Since then, I have written more than one hundred books and published over 350 papers in this field, most of them pretty substantial. In this body of work I have articulated and expounded a considerable variety of theses and theories. But there is one single small item, something that I called the 'plurality quantifier'-an idea initially articulated in a brief 1962 note that was no more than two pages long-with which my name has become most intimately associated. This is the so-called 'Rescher-quantifier,' which has made a name for itself in present-day symbolic logic. And so, the irony here is that, in a long lifetime of creative philosophical work, the creation most closely associated with my name-and which more than any other thing seems likely to be associated with it over the years to come-is embodied in a diminutive note formulating a rather simple idea tossed off in some idle moments as a jeu d’esprit.

Classical quantification theory in symbolic logic is based on two quantifiers only, namely 'existential quantifier' where something holds of SOME and the 'universal quantifier' where something holds of ALL. And these are in one very important way sufficient—that is, they suffice for all the purposes of mathematics. But the contrasting 'plurality quantifier' MOST not only resists definition in terms of the standard quantifiers, but also has various interesting formal features (such as the duality implication that MOST $\Rightarrow$ not-MOST-not and also the failure (in contrast to SOME and ALL) to be self-commutable. Moreover, it is just this sort of
qualification, standing apart from all-or-something approach of the usual quantifiers, that is interestingly applicable and useful in the domain of human affairs.

This idea of plurality quantification might possibly have dropped into oblivion were it not for a follow-up discussion in which David Kaplan drew attention to the fact that it opened the doorway to a significant shift of prospective regarding the role and modus operandi of quantification. But this is speculation, and in actual fact during the subsequent period plurality qualification under the name of ‘the Rescher quantifier’ made its way into the agenda of logical research and established itself as a useful instrumentality for elucidating and systematizing a significant sector of quantificated logic. Odd though it seems (to me, at any rate), it seems fair to say that no other single concept or thesis of my work has found such extensive resonance in the research literature. In substantiation of this contention it can be observed that currently (January, 2004) the search engine Google offers under the rubric ‘Rescher quantifier’ several hundred references to the work of a wide variety of logicians at work in many countries.

In view of this development, it seems appropriate to give a brief account of how the idea of plurality qualification initially entered into my thinking. While (rather regrettably) I have no specifically concrete recollection of this, I feel pretty confident that at the back of my mind in thinking on this issue there figures Aristotle’s idea of generalizations that hold on-the-whole (or ‘for the most part,’ epi to polu.)

A word as to background. My work on plurative generalization was pre-figured on views about the human sciences, developed in the mid 1950’s, which pointed towards the idea of exception-tolerant generalizations as an important instrumentality in social and in particular in historical contexts came to the fore in my thought regarding the epistemology of the inexact sciences and history in particular.

In the wake of my return to philosophy in 1957, after some years spent in other pursuits, I worked rather intensely on issues relating to Aristotle’s logic. Additionally, during this period I was carrying out a variety of investigations regarding nonstandard uses of quantifiers in relation to such matters as many-valued qualification, many-sorted quantifiers, quantification in modal and many-valued systems, and the like. And so, proceeding not just by memory but by the documentary record, I feel confident that it was the confluence of these two streams of historical and logical studies that led me to conceive the idea of plurality quantification.

Moreover, as that 1964 itself note indicates, immediately upon projecting the idea of pluratative generalizations, the idea came to me of adding such propositions to the standard manifold of universal and existential propositions in Aristotelian syllogisms. And from this it was an easy step to extend the mechanism of so-called Venn diagrams to monitor the validity of syllogisms in this extended syllogistic. The
details of this approach were worked out in collaboration with my student Neil Gallagher, and the results formulated in a little paper published in the mid-1960's. Ultimately, the ideas at work in these deliberations about plurality-quantification in formal logic ultimately exerted a significant influence upon my work on larger philosophical issues—and indeed upon my views regarding the nature of philosophy itself. For—as Aristotle already contemplated—the shift from ‘all’ to ‘most’ in science suggests the prospect of a parallel shift from ‘always and invariably’ to ‘normally and standardly’ in matters afforded by chance and choice. On such a perspective, philosophical generalizations would be construed as holding not with regard and rigorous universality, but would instead manage to characterize the usual and normal course of things. In unusual or extraordinary circumstances, such generalizations would admit of exceptions which would be seen as such exactly because the circumstances are un-usual and extra-ordinary. The program of ‘philosophical standardism’ which I elaborated in a 1994 book of that same title was dedicated to working out the details of such a metaphilosophical approach geared to generalizations of this sort. Thus while the idea of plurality qualification was principally active in the world of logicians, its influence on me moved rather in the direction of social and humanistic concerns. It is not that there are no pluralistic facts in mathematics and logic. (After all, ‘Most primes are odd,’ with 2 forming the sole exception.) But the pluralistic truths of these disciplines can always be restated without going beyond the resources of standard quantification. Whereas in the social and human sciences this does not seem to be the case.

And so in the end the ideas at work here came round in a full circle-moving from the initial inspiration of Aristotle’s qualified ‘generally and on the whole’ generalizations towards the general idea in matters of human affairs and humanistic studies of an Aristotelian (quasi-) science geared not to how matter stand always and invariably, but rather merely ‘on the whole’ (epi to polu).

Appendix 1

PLURALITY-QUANTIFICATION

We introduce the new mode of plurality-quantification, represented by the quantifier M, with \((Ma)\forall a\) to be construed as: ‘For most individuals \(a\) (in the non-empty universe of discourse \(D)\forall a\).’ This is to be taken to say that the set of individuals for which \(f\) is true has a greater cardinality than the set for which it is false (so that it is clear that the applicability of the notion is not restricted to finite universes). This \(M\) quantifier qualifies as a quantifier in the sense of Mostowski \((Fund. Math.,\) vol. 44 (1957), pp. 12-36; cf. T. Hailperin in \textit{Zentralblatt fur Mathematik} etc., vol. 78 (1959), p. 244). The semantical theory of this mode of quantification is obvious.

Some of the logical principles governing this quantifier are:
(i) \((x)q x \supset (Mx)q x\),

(ii) \((Mx)q x \supset (\exists x)q x\),

(iii) \([(Mx)q x & (Mx)\psi x] \supset (\exists x)(q x & \psi x)\),

(iv) \((Mx)q x \supset \neg (Mx)\neg q x\),

(v) \([(Mx)(q x \supset \psi x) & (x)q x] \supset (Mx)\psi x\),

(vi) \([(x)(q x \supset \psi x) & (Mx)q x] \supset (Mx)\psi x\).

(It is clear that the converse of (iv) does not obtain - think of a domain D with an even number of members, exactly half of which have \(q\).) Note, however, that all these theses, as well as all ensuing remarks about the M-quantifier, are unaffected if \((M\alpha)q \alpha\) were construed-for finite universes-as ‘For over 80% [instead of 50%] of all individuals \(\alpha\), \(q \alpha\)' , etc.

Since a semantical interpretation of the M-quantifier is in hand, we may raise (but will not here discuss) the question of a complete set of axioms for the ‘lower predicate calculus’ involving this quantifier (i.e., principles which, taken as axioms, yield as theorems all logical truths of the (M-enriched) quantification theory based on 1-place predicates alone).

An interesting feature of the M-quantifier is that it is not self-commutative, i.e., unlike the analogous case with universal or existential quantification, we do not have:

\[(Mx)(My)q xy \equiv (My)(Mx)q xy\].

This is readily shown by considering a universe of discourse of three members, \(x_1, x_2, x_3\), with the relation \(R\) taken such that \(x_1\) bears \(R\) to nothing, \(x_2\) only to \(x_1\) and \(x_2\), and \(x_3\) only to \(x_1\) and \(x_2\).

If the identity-relation is given, so that we can define the familiar operator for ‘there are exactly \(i\) distinct individuals \(\alpha\) such that \(q \alpha\), and further if the domain of discourse \(D\) is finite and its cardinality is specified, then one can of course define \(M\) by the remaining machinery of the system. However, if the cardinality of the (finite) domain \(D\) is unspecified, or if this domain is infinite, then \((M\alpha)q \alpha\) cannot be defined in terms of the familiar resources of standard quantificational logic. The theory of quantification based on universal and existential quantification alone cannot render: ‘For most x’s (of the non-empty domain \(D\)), \(q x\).’

Observe that ‘\((Mx)(Ax \supset \psi x)\)’ does not represent ‘Most A’s are B’s’ (and indeed would not do so even if ‘\(\supset\)’ were replaced by an implication-relation stronger than material implication). It is readily shown that (if the A’s are a proper subset of the entire domain of discourse, and the cardinality of this domain is not specified as
some finite number) ‘Most A’s are B’s’ cannot be defined by means of the usual resources of quantificational logic; not even when these are supplemented by our plurality-quantification, or any other type of quantification, for that matter. Nevertheless, the logic of the propositions ‘Most S is P’ and ‘Most S is not P’ is an extremely simple matter. For example, syllogisms involving these quasi-categorical statements are subject to a validity test using Venn-diagrams (by a suitably elaborated employment of arrows to indicate the extension of one region of the diagram to be less than that of another). More generally, the machinery needed for the analysis of such syllogisms is much less than is required for De Morgan’s ‘numerically definite syllogisms’ (*Formal Logic*, Chapter VIII).

Consider the arguments:

\[
\begin{array}{ccc}
\text{(1)} & \text{(2)} & \text{(3)} \\
\text{All A’s are B’s} & \text{Most things are A’s} & \text{Most C’s are A’s} \\
\text{All parts of A’s are parts of B’s} & \text{Most things are B’s} & \text{Most C’s are B’s} \\
\text{Some A’s are B’s} & \text{Some A’s are B’s} & \text{Some A’s are B’s}
\end{array}
\]

Textbooks often charge that traditional logic is ‘inadequate’ because it cannot accommodate patently valid arguments like (1). But this holds equally true of modern quantificational logic itself, which cannot accommodate (2) until supplemented by something like our plurality-quantification. And even such expanded machinery cannot accommodate (3). Powerful tool though it is, quantificational logic is unequal to certain childishly simple valid arguments, which have now featured in the logical literature for over a century (i.e., since De Morgan). (To be sure, once the quantificational system is elaborated in some way to the point where arithmetic is possible so that we can count the extension of a property P and compare the results of such countings-then this impotence is overcome.)

Notes

1 For details regarding this mass of publications see my web site: <http://www.pitt.edu/~rescher/>.
Nicholas Rescher, ‘Plurality Quantification,’ The Journal of Symbolic Logic, vol. 27 (1964), pp. 373-74. Since this note original piece in which the Rescher quantifier made it first appearance is so brief, there is space to reproduce it in full as an Appendix here.


It took some time for this work to bear fruit in publications. Among the earlier of these were: ‘New Light for Academic Sources on Galen and the Fourth Figure of the Syllogism,’ Journal of the History of Philosophy, vol. 3 (1963), pp. 27-41 and ‘Aristotle’s Theory of Modal Syllogisims’ in M. Bunge (ed.,) The Critical Approach of Science and Philosophy (London and New York: Free Press of Glencoe, 1964), pp. 152-77.


Philosophical Standardism (Pittsburgh: University of Pittsburgh Press, 1994).