The Form of Inference

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MR. JOSEPH'S thorough *Introduction to Logic* consistently opposes the idea of reduction. In convincing analysis are set forth the three or four figures and nineteen moods of syllogism. But the admission that the fourth (or indirect first) figure moods need validation is canceled by the contention that these moods never occur in actual thinking. The second and third figures are found not only to conclude in their own right but also to involve distinctive processes of thought; their reduction, at times easy and at others ludicrously cumbrous, is always superfluous. A similar position is maintained with regard to other formal types of inference. If a hypothetical argument contains only three terms, it can be reduced to syllogistic form; but it may contain more than three, and then reduction is a useless tour de force. Occasionally mathematical reasoning is syllogistic as when an Euclidean proof appeals to an earlier theorem; but such appeals arise only when insight into the data is imperfect and, in general, the mathematician has perfect insight.[1] If, finally, one cares to complete the brief against reduction, one need only turn from Mr. Joseph to Cardinal Newman. By definition the latter's illative sense proceeds along ways unknown to syllogism from a cumulation of probabilities—too manifold to be marshaled, too fleeting to be formulated—to a conclusion that nonetheless is certain.

I have recounted these views not because I hope to refute them but because I wish to present a problem. Is the human mind a Noah's ark of irreducible inferential forms? Is there no general form of all inference, no highest common factor, that reveals the nature of the mind no matter how diverse the materials on which it operates? Is everything subject to

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[1]See Joseph, *op. cit.* (Oxford, 1931[2]), pp. 330, 339, 341, 545.

measure and order and law except the mind which through measurement and comparison seeks to order everything with laws? One has only to raise such questions to grasp how paradoxical it is to deny reduction. But if this point is granted, there immediately follows another. Neither Cardinal Newman nor Mr. Joseph has attacked reduction as such. Their sole contention is that a particular reduction, reduction to syllogism, does not fit the facts. Thus it remains to be seen whether there exists some other type of formally valid inference that possesses both the radical simplicity and indefinite flexibility necessary to embrace all other types within itself.

I

Any language has a number of syntactical forms that are peculiarly inferential. Most obvious is the causal sentence, because A, therefore B, where A and B each stand for one or more propositions. Next comes the concessive sentence, although A still not B, which is the natural instrument of anyone ready to admit the propositions, A, but wishing to deny that A implies B. To meet such denial, to give separate expression to the implication of B in A, there is the host of conditional sentences, if A then B, which may be past or present, proximate or remote future, particular or general, actually verified or the pure inter-connection grammarians call contrary-to-fact. It is not hard to see that these three syntactical forms are peculiarly inferential. Just as "so that" and "in order that" express the relations of efficient and final causality, so also "because," "although" and "if" are the special tools of reasoning man.

Closely related to these linguistic tools is the transition from informal to formal inference. It appears a fact that spontaneous thinking sees at once the conclusion, B, in apprehending the antecedents, A. Most frequently the expression of this inference will be simply the assertion of B. Only when questioned do men add that the "reason for B" is A; and only when a debate ensues does there emerge a distinction between the two elements in the "reason for B," namely, the antecedent fact or facts, A, and the implication of B in A (if A then B). Thus the transition from informal to formal inference is a process of analysis: it makes explicit, at once in consciousness and in language, the different elements of thought that were present from the first moment. For when B simply is asserted, it is asserted not as an experience but as a conclusion; else a question would not elicit the answer, B because of A. Again, when this answer is given, there would be no meaning to the "because" if all that was meant was a further assertion, A. On the contrary, the causal sentence (because A therefore B) compresses into one the three sentences of the formal analysis (if A, then B; A; ∴ B).

No doubt these considerations throw some light both on the prevalence of enthymeme and on the awkwardness of a logical theory that overlooks the normal syntax of inference to design a Procrustean bed with predication. But at any rate it is from the syntactical forms that the logician derives his simple hypothetical argument. This is of the type:

If A, then B; but A; ∴ B.

Its indefinite flexibility is apparent: A and B each stand for one or two or any number of propositions; the propositions may be categorical, disjunctive or hypothetical; and there is no reason why any of them should be forced into the mould of subject, copula and predicate. No less apparent is the radical simplicity of this type. Every inference is the implication of a conclusion in a premise or in premises: the conclusion is B; the premise or premises are A; the implication is, if A then B. Thus a study of language has given us a working hypothesis: the form of inference is the simple hypothetical argument.

II

What language suggests, symbolism confirms. For if one analyzes a symbolism one finds two distinct elements. First there is abbreviation: eight hundred and thirty-seven reduces to 837, a paragraph is compressed into the equation "sin i = m.sin r," and at least a page into any expression involving the nabla operator. But symbolism is much more than abbreviation. Of the millions who would have no difficulty in finding the square root of 1764, not a few would be at a loss if required to use Roman numerals in performing the same operation. Why? Not because 1764 is shorter than MDCCLXIV but because they work by rule of thumb and have never grasped the algebraic theorems, underlying the rules of thumb. Their understanding has been shortcircuited. Like adding machines which do not understand addition, like integrating machines which never were puzzled by the calculus, they have acquired through class-room drill not an intellectual insight into arithmetical operations but an ability to get answers.

Now these two elements in symbolism correspond to the two elements in the simple hypothetical argument. Because symbolism is abbreviation, it gives a terse expression to the minor premise, the data, A. But because it is more than abbreviation, because it involves pattern, association, convention and rule of thumb, the symbolism not only expresses the minor premise, A, but also its implication, if A then B. Indeed only because machines and schoolboys possess the implications in automatic routines are they capable of obtaining right answers without understanding what they are doing. Nor is there any other explanation of the fact that the inventive mathematician, who is at once master and schoolboy, occasionally finds his symbolism taking the initiative and leading to theorems or methods that otherwise would not have occurred to him. Between the crucial experiments of these extremes, both of which are somewhat abnormal, there is the everyday function of symbolism, the function of reducing to a compact routine the use of multitudinous theorems which the mathematician has understood, which now he wishes to employ, but which he wishes to employ without retracing the countless steps that once for all were taken in the past.

A further point is to be made by adverting to the limitations of symbolism. The mathematician deals with ideal entities, with things that are exactly what he defines them to be; this makes it possible to abbreviate without falsifying. Again, the mathematician studies correlations that not only are universally valid but also are employed over and over again; this makes it worth while to reduce these correlations to habitual patterns of thought and to automatic routines of notation and operation. But at the opposite pole to such inquiry stands Newman's illative sense. Thus, a general will estimate his own and the enemy's resources, opportunities, preparedness, methods, drive, staying power, to conclude principles of strategy, the merits of different dispositions of forces, the measure of success and the ulterior effects of given lines of action. In another field the diplomat studies persons, problems, movements to predict reactions to given policies. In still another field the broker examines both general trends and the actual position of, say, Broadcast Bounty, Inc., to foresee that Broadcast Bounty will rise. In such inferences the data are not ideal but real; they are known not by the decision of a definition but only by the intimate familiarity of long-standing experience; and so far are they from admitting abbreviation of statement that they tend to be too multitudinous, too complex, too nuanced to be stated in any adequate fashion. Similarly the implication of the conclusion in the data is not any general principle or rule. It arises from the intuition of the moment; its ground is the objective configuration of the moment as interpreted through the accumulated insights of experienced judgment; its value is just the value of that judgment; its only court of appeal is the event and when the event has come then, except on a theory of identical historical cycles, its day of usefulness is over forever. To attempt to apply symbolism to such inferences would be to misunderstand symbolism. The data can hardly be stated, much less abbreviated. The implication is not a general correlation to be employed repeatedly but the unique coincidence of a complex objective configuration and a complex subjective interpretation and judgment.

But however vast the interval that separates mathematical and concrete inference, both have a common form. Both proceed from data through implication to conclusion; and so both are of the type,

$$\text{If A then B; but A; } \therefore \text{ B.}$$

It may be that only the conclusion, B, can be stated in a concrete inference. But this does not prove that there are no data, A, or no implication, if A then B. Again such conclusions are usually probable and only in limiting cases certain; but this is irrelevant to formal logic, for the form of the inference is exactly the same whether one diffidently concludes, "probably B," or downrightly asserts, "certainly B." On the other hand, the mathematician regularly states his data, A, and with equal regularity omits the implication, if A then B. Still the implication is an essential moment in his thought or in the routines of his symbolism, nor does it make the slightest difference whether the implication be obvious as in the step

$$A = B; B = C; \therefore A = C$$

or not so obvious as in the stride

$$y = \sin t; x = \cos t; \therefore dy/dx = -\cot t.$$

For the function of formal logic is not to make explicit the elements of thought that are not obvious to everyone; its function is to make explicit all the essential elements whether they are obvious or not.

III

If the simple hypothetical argument appears a plausible form of inference from the syntax of language, the significance of symbolism and the structure of Newman's illative sense, it still has to undergo comparison with the other formally valid types recognized in manuals on logic. Deferring syllogism to the next section, we here examine the *modus* *tollens* of the simple hypothetical argument, the dilemma, the disjunctive argument, the compound hypothetical argument and the hypothetical sorites.

From the hypothetical premise, if A then B, one can always draw two and sometimes draw four conclusions. Always, if one affirms A in the minor, one can affirm B in the conclusion. Always, if one denies B in the minor, one can deny A in the conclusion. Sometimes one can deny A or affirm B in the minor and so deny B or affirm A in the conclusion. This last case arises when A is the unique ground of B: thus all organisms and only organisms are mortal; hence if the major premise is,

If X is an organism, X is mortal,

one can argue that a stone is not an organism and so a stone is not mortal, or again that men are mortal and so men must be organisms. However, in the general case, the antecedent, A, is not the unique ground of the consequent, B, but only one of many possible grounds; if Fido were a man, he would be mortal; Fido is mortal and yet not a man. Hence, in the general case, it is invalid to argue through a denial of the antecedent or an affirmation of the consequent. On the other hand, "if A then B" always implies "if not B then not A," because the absence of B proves the absence of all grounds of B; hence it is always valid to argue through a negation of the consequent to a negation of the antecedent. Thus the very justification of the *modus tollens* reveals it to be an implicit form of the *modus ponens.* One can argue

If A, then B; not B; ∴ not A

not because of a special form of inference but because the explicit major implies the major of the *modus ponens,* because "if A then B" necessarily implies "if not B then not A."

The disjunctive argument yields to analysis in similar fashion. For the disjunctive premise,

Either A or B or C or D or . . . ,

is ambiguous. It may have only the minimal meaning that at least one of the alternatives is true, that is,

A if neither B nor C nor D nor . . .
B if neither A nor C nor D nor . . .
Etc.

But it may also mean that the truth of any alternative is incompatible with the truth of any of the others, and that gives the additional bases of argument,

If A, then neither B nor C nor D nor . . .
If B, then neither A nor C nor D nor . . .
Etc.

It follows that the *modus tollendo ponens* is always valid, that the *modus ponendo ponens* is sometimes valid, and that in either mood the real argument is in virtue of an implicit premise and so in the *modus ponens* of the simple hypothetical argument.

Perhaps it will suffice to deal only with the most symmetrical forms of the dilemma, trilemma, tetralemma, etc. These employ a series of hypothetical propositions to proceed from one disjunction to another; thus from the major

If A then P; if B then Q; if C then R . . .

one may argue constructively by adding

Either A or B or C . . . ∴ Either P or Q or R . . .

or destructively by adding

Neither P nor Q nor R . . . ∴ Neither A nor B nor C . . .

In these instances it should seem that one has simply a combination of several simple hypothetical arguments and so no solid reason for affirming a distinct form of inference.

The compound hypothetical argument is a particular case of the hypothetical sorites; the type is

If A, B; if B, C; if C, D; if D, E; ∴ if A, E

where the premises may be any number greater than one. Illustrations of such argumentation abound in mathematics in which the data, A, are transformed to B, C, D, and finally E which is the solution; and, as anyone familiar with mathematics is aware, much more complex patterns than the single track of the sorites are common. But the question arises, Are we to suppose an implicit premise:

If "if A, B; if B, C; if C, D; if D, E;" then "if A, E" and so reduce the sorites to the simple hypothetical argument, or should one say that the sorites by itself expresses the whole process of thought? We think the former alternative preferable: the implication of the conclusion in the premises is distinct from the set of implications that constitute the premises, as may be made evident by constructing a fallacious sorites; the function of formal logic is to make explicit all the elements of thought essential to the conclusion, and therefore even the awkward implicit premise stated above.

IV

Syllogism is open to different interpretations. Thus we have Euler's circles in vivid illustration of the view that syllogism concludes in virtue of the coincidence or non-coincidence of the denotations of its terms. Only on such a view can one have the conversion of propositions, rules regarding distribution, the argument showing that there are nineteen and only nineteen valid moods, and the reduction of the imperfect figures by means of converting propositions or of substituting contradictory premises. Hence if arguments from denotational coincidence never occur elsewhere, at least they occur in books on logic. What then is the form of such inference?

It seems to be enthymematic. No one can consistently advance that the argument

$$A = B;\ B = C;\ \therefore A = C$$

is an enthymeme which fails to express a factor in the mental procedure, while the argument from denotational coincidence

All S is some M; All M is some P; ∴ All S is some P

is not an enthymeme but formally complete. It should seem evident that both arguments suppress the statement of the implication and, indeed, that the implication is less obvious in the denotational coincidence or non-coincidence than in the geometrical argument.

On a second possible interpretation of syllogism the denotations of the terms are considered quite irrelevant. The inference arises from the connotational relations between a middle, M, and a predicate, P. Thus, either M implies P, or M excludes P, or P implies M, or P excludes M. If these four cases are combined with the merely material fact that the subject, S, may be distributed or undistributed, there result the eight direct moods of the first two figures of syllogism. When M implies P, the mood is Barbara or Darii; when M excludes P, it is Celarent or Ferio; when P implies M, it is Camestres or Baroco; when P excludes M, it is Cesare or Festino.

However this connotational interpretation, no less than the denotational, leads to the hypothetical argument as the form of inference. In the first place a purely connotational relation between M and P cannot be expressed in the categorical propositions, All M is P, No M is P, All P is M, No P is M, for the subject of a proposition is meant materially or in denotation and not formally or in connotation. The same point may be put differently by asking the logician, If when you say that all organisms are mortal you do not mean to speak of "all organisms" but of the nature of "organism," then why on earth do you say "all organisms?" To that query I have never heard a sensible answer and on the present hypothesis of connotational interpretation there is no answer possible. Thus one is forced to replace

Barbara and Darii by

If S is M, S must be P; S is M; ∴ S is P.

If our enemies are men, they must be mortal; they are men; therefore they must be mortal.

If some capitalists ae fraudulent, they ought to be punished; some are fraudulent; they ought to be punished.

Celarent and Ferio by

If S is M, S cannot be P; S is M; ∴ S is not P.

If angels are pure spirits, they cannot have bodies; angels are pure spirits; they cannot have bodies.

If some employers demand evil, they are not to be served; some do; therefore some are not to be served.

Camestres and Baroco by the *modus tollens*

If S were P, it would be M; S is not M; ∴ S is not P.

If John had a vote, he would be twenty-one; but he is not twenty-one; therefore he has no vote.

If all guests were to enter, they all would have tickets; but not all have tickets; so not all are to enter.

or by the *modus ponens*

If S is not M, it is not P; S is not M; ∴ it is not P.

If John is not twenty-one, he has no vote; etc.

If not all guests have tickets, not all are to enter; etc.

Cesare and Festino by the *modus tollens*

If S were P, S would not be M; S is M; ∴ S is not P.

If hydrogen were a compound, it would not be an element; but it is an element; so it is not a compound.

If all aquatic animals were fish, none would be mammals; but some are mammals; so not all are fish.

or by the *modus ponens*

If S is M, S is not P; S is M; S is not P.

If hydrogen is an element, it is not a compound; etc.

If some aquatic animals are mammals, they are not fish; etc.

Now the foregoing reduction is not merely a tour de force in the interests of a theory on the form of inference. If a connotational interpretation of the first two figures of syllogism is possible at any time and sometimes actually occurs, then it has to be expressed in the hypothetical form for the very good reason that categorical expression would be saying what is not meant; there is no reason why so daintily precise a person as a logician should speak of "all men" and "all frauds" and "all voters" when he is thinking of the connotational aspect of humanity, fraudulence and the right to vote. Further the reduction to hypothetical form reveals the exact significance of the reduction from second to first figure syllogisms. A glance at the examples given above will show that Cesare and Festino in the *modus ponens* are identical with Celarent and Ferio; thus these instances of syllogistic reduction are really a transition from the *modus tollens* to the *modus ponens;* and such reduction is easy because if P excludes M, as in Cesare and Festino, then M must exclude P, as in Celarent and Ferio. In other words, connotational incompatibility is a mutual relation. On the other hand, if one wishes to substitute the direct movement of thought from S through M to P for the round-about movement from S to P through M back to P in the moods Camestres and Baroco, then the substitution of a modus ponens for a *modus tollens* is perfectly simple while a syllogistic reduction is an almost incredible feat of denotational acrobatics. The reason for this is plain from the more ultimate reduction to hypothetical form, for that reduction reveals that there is no first figure mood identical with the *modus ponens* of Camestres and Baroco; if P implies M, then it does not follow that M implies P while it is false that M excludes P; what does follow is that not-M implies not-P, which denotationally is the acrobatic contrapositive but hypothetically the quite obvious and natural premise, If S is not M, S is not P.

A third interpretation of syllogism is partly connotational and partly denotational. The classic formula of this view is the *dictum de omni et nullo,* namely, that what is true of a class of objects is true of all the members of that class. Here both the subject, S, and the middle, M, are taken in denotation while the predicate, P, is connotational. This seems to provide the most natural interpretation of third figure syllogisms, for, as Professor Joseph has observed, the third figure is an appeal to an instance in refutation of a hasty generalization. Thus, when the revolutionary calls for the confiscation of all property, the heckler asks, What about savings? The argument is in Felapton:

No savings are to be confiscated;
All savings are property;
∴ Some property is not to be confiscated.

But really one may doubt that the argument is as described; for if it is true that the subject of a proposition is to be taken denotationally and the predicate connotationally, then the above syllogistic expression implies that the subject of the argument, property, is at once both connotational and denotational. It should seem much more plausible that the expression is mistaken than that the thought is confused, and so again we are led to the hypothetical form:

> If all property is to be confiscated, then savings are; but savings are not to be confiscated; therefore, not all property is to be confiscated.

The hypothetical major gives the implication of the revolutionary thesis; the minor premise gives the bourgeois antithesis; and the conclusion gives the bourgeois answer. I submit that that is the real process of thought, and anyone caring to make the induction will find that arguments in Felapton, Ferison and Bocardo are expressed unambiguously and naturally by the *modus tollens*

> If S were P, M would be P; M is not P; ∴ S is not P.
>
> If all domestic animals had horns, cats would have horns; but cats have no horns; so not all domestic animals have horns (Ferison).
>
> If all ruminants had horns, all goats would have horns; but some goats have no horns; etc. (Bocardo).

while Darapti, Disamis, Datisi are in the *modus tollens*

> If S were P, M would not be P; M is P; ∴ S is not P
>
> If no woman could be a statesman, Maria Theresa was not; but she was; so a woman can be a statesman (Darapti).
>
> If no quadrupeds had horns, no goats would have them; some have; etc. (Disamis).
>
> If no revolutionary is intelligent, no communist is intelligent; but some communists are intelligent; therefore some revolutionaries are (Datisi).

It will be objected that the hypothetical form is longer than the syllogistic. But this objection merely confirms our position, for in actual thinking these arguments are always enthymemes and what is omitted is the hypothetical major premise; such omission is natural since "because A therefore B" is equivalent to the formally complete "if A then B; but A; therefore B;" on the other hand the exponents of categorical syllogism have still to explain why at least one of their premises is always omitted in actual thinking.

The forms we have given for the third figure are in the *modus tollens;* if they are reduced to the *modus ponens,* there result arguments in the first figure as interpreted by the *dictum de omni et nullo.* This is not equivalent to the connotational interpretation of the first figure which makes the middle term, M, not a class of objects but an attribute or meaning. However, in all cases except the moods Bocardo and Disamis (in which M neither implies nor excludes P) it is possible to re-think the argument from denotational coincidence to connotational implication. Thus one can conceive "savings" as an attribute of some property and as excluding the further attribute "deserving of confiscation." This re-thinking will give as *modus ponens*

If some property is savings, it is not to be confiscated . . .

instead of the *dictum de omni et nullo's*

If savings are not to be confiscated, some property is not . . .

The difference between the two is obvious. The latter is an argument from denotational coincidence; the former is what Aristotle calls scientific thinking in which the middle term is the *causa essendi* of the predicate: savings precludes confiscation; the argument turns on the meaning of terms and not on their denotation. Such re-thinking of the third figure moods is possible even when the middle term is an individual; thus the appeal to the instance in

If a pious man is a sissy, Jogues was a sissy . . .

becomes scientific in the form

If a pious man is a Jogues, he is not a sissy . . .

This, I submit, reveals a rather obvious difference between the *Posterior Analytics* and pseudo-classical *dictum de omni* *et nullo*. But the revelation comes through the form we have found in all inference, the hypothetical argument.

So much for syllogism. Three distinct interpretations of it have been considered and all have led away from syllogism to the hypothetical form. There are other interpretations of minor importance, such as the view that syllogistic inference is a matter of second intentions with S a logical part of M and M a logical part of P so that S must be a logical part of P as is evident from Porphyry's tree. No doubt one can perform an inference in this or in various other fashions if one makes up one's mind to do so. But the mere existence of so many different interpretations of syllogistic thought is proof that the mind really is proceeding in virtue of some more general and ultimate law that can be given a variety of less general interpretations.

V

To conclude, our aim has been an empirical investigation of the nature of inference. Just as the physicist working out a theory of light will not repeat the established experiments on reflection, refraction, colour, interference, spectral lines and the like, but rather will accept the results of such prior investigations in an effort to discover their ultimate unity, so too we have taken as our empirical basis not particular instances of inference but generally recognized types, and from them as starting-point we have worked to the ultimate unity of the simple hypothetical argument. Thus our conclusion has to do with the nature of the human mind. We have not sought the reduction of one inferential type to another because we thought one more valid or more obviously valid than the other. On the contrary we assumed all to be valid, and our concern with reduction has been a concern with the one law or form of all inference.

We have not considered inductive conclusions. To correlate the movement from data through hypothesis to verified theory with the movement from implier through implication to implied, and both of these with the more ultimate process from sensa through intellection to judgment, is indeed a legitimate inquiry; but it is more general than the present and presupposes it. For the same reason we have not aimed at explaining inference but rather at finding the highest common factor of inferences no matter how they are explained. Indeed, it is precisely in our attitude towards the explanation of inference that we differ from the approach of the more traditional manuals on logic; the latter presupposes an explanation of conceptualization and of inference; we on the contrary have aimed at taking a first step in working out an empirical theory of human understanding and knowledge.