FREGE’S “BEDEUTUNG” AND MILL’S “DENOTATION”

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Frege refines his theory of content in what is his major and most influential contribution to the philosophy of language, namely his essay *Über Sinn und Bedeutung* (1892). There is quite a bit of discussion about how to translate this title into English, particularly Frege’s use of “Bedeutung” (see Frege 1997, 36-46). In ordinary German, “Sinn [sense],” “Bedeutung [meaning]” and “Inhalt [content]” are used interchangeably, as are their ordinary English translations. So in ordinary German the title “Über Sinn und Bedeutung” is redundant and thus it seems a bit odd to the ordinary German reader, as would the title “Of Sense and Meaning” in English. Since Frege aims to draw a distinction using German words that ordinarily are used synonymously, his use of those terms in the body of the essay is bound to be odd, as would be the case when using the synonymous English terms “sense” and “meaning” to draw a distinction. These are all reasons for translating this essay as “On Sense and Meaning.”

But this is not the translation that has prevailed. The translations that have prevailed are the ones that aim to capture the technical meaning Frege gives to these terms. This is the approach of Bertrand Russell, the first philosopher to mention Frege’s work in English. Russell first used “indication” for “Bedeutung” (1903) but in 1905 in his own landmark essay “On Denoting,” he writes that Frege “distinguishes...two elements, which we may call the meaning and the denotation,” where “meaning” translates Frege’s “Sinn” and “denotation” is the term for Frege’s “Bedeutung” (1905). Russell goes on to write that when phrases have meaning and denotation, they “express a meaning and denote a denotation: (ibid.). Max Black and Peter Geach in their translation of the title as “On Sense and Reference” (Frege 1952, 56-7) continue this approach, but use “reference” instead of “denotation” to translate “Bedeutung” for stylistic reasons. They believe that “denotation” is a “philosophical technicality[y]” that “would give a misleading impression of Frege’s style” (Frege 1952, ix). Although they change the title to “Sense and Meaning” in the 3rd edition of their translation, the title “On Sense and Reference” has become the standard title for this essay and these terms are widely accepted as the terms that capture the distinction that Frege is making in this essay.

I believe that the more accurate translation is “On Sense and Denota-
Frege, like Mill before him, notices that linguistic meaning is complex and has several components, and one of the components Frege isolates is Mill’s denotation. There are good reasons to believe that Frege’s reading of Mill had some role to play in Frege’s thinking about meaning. Frege’s main work between *Begriffsschrift* and “On Sense and Reference” was *The Foundations of Arithmetic* (1884), where Frege addresses the second step of the overarching project that he began in the *Begriffsschrift*, namely of trying to see how much of arithmetic he can generate from logic alone. While in the *Begriffsschrift* he wanted to show that mathematical proofs are logical ones, in the *Foundations* he aims to show that the concept of a number can also be reduced to the concepts of logic. To motivate his theory of number, Frege examines the views of his predecessors and Mill’s empiricist account of number is a primary target. (In the Introduction and Parts I-III of the *Foundations*, which comprises the historical and critical discussion prior to Frege’s presentation of his own theory in Part IV, Frege uses Mill’s name more than anyone else’s name, including that of Leibniz. Mill’s name is used 31 times while Leibniz’s name is used 25 times. There is a large gap between the occurrence of Mill’s and Leibniz’s names, and the next most frequently occurring names, which belong to Stanley Jevons (9 times) and Kant (8 times)).

Now, since for Mill all truths about matters of fact are synthetic truths known empirically or *a posteriori* and arithmetic is a source of truths about matters of fact, Mill believed that arithmetic consisted of synthetic, *a posteriori* truths. Accordingly, the concept of number had to be defined empirically. Mill believes that we repeatedly recognize groups of objects, say three pebbles, three trees, etc., and “we term all such parcels Threes” (1974, VII: 257). Moreover, we recognize that these groups can be separated into parts—for instance three pebbles can be separated into a group of two and a single pebble—and this is the basis the arithmetic operations. Frege is particularly critical of this account. Already in the Preface to the *Foundations* Frege describes this as “gingerbread or pebble arithmetic” and that “the only thing missing is to ascribe to the flavor of the case a special meaning for the concept of number” (1884, vii).

Later he responds to Mill’s text sentence by sentence (1884, 9-11). The section Frege critiques includes this sentence by Mill:

The expression “two pebbles and one pebble,” and the expression, “three pebbles”, stand for the same physical fact. They are names of the same objects, but of those objects in two different states: though they denote the same things, their connotation is different. (Mill 1974, VII: 256).

Although Frege does not quote this particular sentence, since he is reading
this section so carefully, Frege must have read this passage (Beaney 1996, 308n51). Mill returns to counting pebbles in his discussion of the laws of nature, where he writes that each name of a number “denotes physical phenomena, and connotes a physical property of those phenomena” (1974, VII: 610). A bit later Mill summarizes his discussion as follows:

What, then, is that which is connoted by a name of a number? Of course, some property belonging to the agglomeration of things which we call by the name; and that property is, the characteristic manner in which the agglomeration is made up of, and may be separated into, parts. (1974, VII: 611)

Taking some liberties by transforming Mill’s question into an answer, Frege quotes this text as follows:

The name of a number connotes [bezeichnet] some property belonging to the agglomeration of things which we call by the name; and that property is, the characteristic manner in which the agglomeration is made up of, and may be separated into, parts. (1884, 29-30)

Since he quotes this passage, Frege must have been aware of Mill’s distinction between denotation and connotation.

Nevertheless, Frege’s choice of words for Mill’s “connotation” in his paraphrase ignores this distinction. The German translation of Mill’s System of Logic by J. Schiel, the translation that Frege cites, clearly distinguishes between connotation and denotation. Schiel consistently used “bezeichnet” for denotation and “mitbezeichnet” for connotation, preserving in German the fact that in English “connotation” and “denotation” have “notation” as a common root (Mill 1863, I: 35). So Schiel very appropriately uses “mitbezeichnet” when translating “connoted” in Mill’s question “What, then, is that which is connoted by a name of a number?” while Frege ignores this in his paraphrase. Instead, Frege, as noted in the quoted text above, uses “bezeichnet.”

This could be taken to mean that in the Foundations of Arithmetic Frege was still completely confused about the components of meaning and that he did not distinguish denotation from other components of meaning, as he does eight years later in “On Sense and Reference.” But this attributes more confusion to Frege than is warranted. A more plausible interpretation is that Frege simply ignored Mill’s concept of connotation, a concept that was too closely tied to subjectivity to be useful for Frege, and instead Frege focussed on denotation and assimilated Mill’s connotation to denotation. On this reading, the view Frege is considering in this passage is that number terms denote properties of objects.

In German, it would be very natural to suppose that connotation
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[Mitbezeichnung] is just a kind of denotation [Bezeichnung]: one can denote only an object and nothing else, or one can also denote along with it other things, such as its properties. Mill himself suggests this interpretation by how he uses italics, for example, when he writes, as we saw above, that “though [the expressions ‘two pebbles and one pebble’ and ‘three pebbles’] denote the same things, their connotation is different.”

The opening sentence of the Foundations of Arithmetic clearly shows that here Frege is concerned with the denotation of number terms:

To the question, What is the number one, or what does the sign 1 denote [bedeuten], one usually gets the answer: a thing. (1884, i).

Frege is paraphrasing the question “What is the number one?” in terms of the second question about the relationship between a sign and an object. That is, Frege is asking what is it that a number term stands for or names, and this is precisely the relation that the verbs “denote” in Mill’s English and “bezeichnen” in the German translation of Mill signify. It is for this reason that the verb “bedeuten” in this context is synonymous with the English verb “to denote.”

That denotation is what Frege had in mind in this opening sentence is made clear in a discussion of the Foundations in an unpublished draft written in the years 1891-1892 and called “On the Concept of Number.” Smarting or even smoldering from the fact that the Foundations was being ignored, he chastises an author who writes about the basic concepts of arithmetic “as if the third section of my Foundations had never been written” (1983, 82).

In the course of this acerbic critique, Frege repeats his opening question from the Foundations: “One could ask: what does the word ‘one’ actually denote [bedeutet]? Does it denote [bezeichnet] a number?” (ibid.). Similarly, in a summary of his life’s work that Frege wrote in July 1919, six years to the month before his death, he asks “Is arithmetic a game or a science?” and ties this question to the problem of what number terms denote. “Is the visible the thing arithmetic is concerned with,” he asks, “or is the visible only a sign for it...? Is the denotation [das Bezeichnete] a number and if not, what is it?” (1983, 277).

The verb “denote [bezeichnen]” also appears in key paragraph §62 of the Foundations, which has been described as “arguably the most pregnant philosophical paragraph ever written” (Dummett 1991, 111). This paragraph lays out the general strategy Frege will use for defining number terms and it is the opening paragraph of a section Frege calls “To obtain the concept of number, the sense [Sinn] of a numerical equation must be determined” (1884, 73). As the use of “sense [Sinn]” in this title indicates, this is also the text where Frege distills content into two components. The
way Frege arrives at sense as the semantic companion to denotation is as follows.

Prior to paragraph §62, Frege had concluded that numbers are neither physical things, physical properties of things, nor subjective entities such as mental representations (1884, 58). But if a number “is neither something sensible nor a property of an external thing,” we cannot mentally represent numbers as such (1884, 70). Moreover, if we cannot mentally represent numbers, which are the denotations of number terms, and denotation is all there is to meaning the meaning of a word, it follows that we do not have mental representations of the meanings of number terms. This raises a problem for Frege with which he begins paragraph §62 and the section on “the sense of a numerical equation”:

How, then, shall a number to be given to us, when we cannot have any representations or intuitions of it? (1884, 73).

This is the central question that leads Frege to a notion of linguistic sense in addition to denotation.

The pivot that gets Frege from denotation to sense is what has come to be called Frege’s “context principle” (Dummett 1973). Frege maintains that the lack of a mental representation for the meaning of a number word can suggest that number terms do not have meanings, but this, for Frege, is a mistake. The fact that no representation can be formed of the meaning of a word, Frege writes, “is...no reason for denying it any Bedeutung” (1884, 71). We might be inclined to deny such words meaning if we look at words in isolation, but this is a mistake. “Only in a complete sentence [Satz] do words really have Bedeutung” Frege writes, appealing to a principle he highlighted in the Introduction to the Foundations as one of the three “fundamental principles” of his inquiry into the nature of number (1884, x and 71).

What is the meaning of “Bedeutung” in these passages? I think that the discussion so far already suggests that Frege has denotation in mind, but this reading is clinched, I believe, by what Frege writes to elucidate the context principle. After pointing out that this principle “throws light on quite a number of difficult concepts...and its scope is certainly not restricted to mathematics” (1884, 71), Frege addresses the appearance of a contradiction between the context principle and a view he defended earlier in the Foundations that a numbers is an “independent object [selbstständiger Gegenstand]” (1884, 68). The independence of numbers might suggest that a number term denotes by itself in isolation from a complete sentence, but Frege rejects this suggestion:

The independence that I am claiming for number is not to be taken to mean
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that a number word denotes [bezeichnen] something when removed from
the context of a sentence...(1884, 72).

Although numbers are independent objects (and not properties of objects,
for example), the context principle still holds for number terms. Frege’s use
of “denotes [bezeichnen]” to make this clarification regarding the context
principle shows that Frege uses “Bedeutung” in the context principle
synonymously with “Bezeichnung,” the German term used for “denotation”
in Schiel’s translation of Mill.

Once it is clear that the context principle is about denotation or
reference, it is clear how the context principle serves as a strategy for
answering his question about how numbers are given to us. Although we
have no mental representations or intuitions of numbers, they are given to
us by means of number terms in the context of whole sentences. Hence,
understanding how numbers are given to us “depends on defining the sense
of a sentence in which a number word occurs” (1884, 73). Frege argues that
the sentences we must look for in this inquiry are ones that “express
recognition” of the number. Frege continues: “if the sign a is supposed to
denote [bezeichnen] an object for us, then we must have a criterion
[Kennzeichen] that decides in all cases whether b is the same as a,” and
sentences of identity will yield such a criterion (ibid.). “Our aim,” Frege
writes, “is to construct the content of a judgment that can be understood as
an equation on each side of which is a number” (1884, 74). In other words,
we will understand how numbers are given to us if we understand the sense
of identity sentences involving number terms, e.g. “7+5=12.” Frege takes
for granted that we have a general concept of identity that is expressed in
identity statements, and “by means of this already known concept of
identity, obtain that which is to be regarded as identical” (ibid.). So a number
term has a denotation, but the way this denotation is given is always in the
context of a whole sentence, particularly those expressing numeric identi-
ties.

This analysis yields for Frege a notion of meaning in addition to
denotation. It is not the case that “an object can be given in one single way”
and the “versatile and meaningful use of identities rests... on the fact that
something can be reidentified even though it is given in a different way”
(1884, 79). So although in an identity statement there is only one denotation,
nevertheless the identity statement expresses distinct ways in which the
object is given, and the way an object is given is another component of
meaning besides denotation.

I believe that in writing the Foundations of Arithmetic Frege began to
distill the raw concept of meaning or content into two components. More-
over, the space and effort Frege devotes in the *Foundations* to responding to Mill's views about the meaning of number terms as well as his use of "Bezeichnung," the standard German translation of Mill’s term "denotation," strongly suggest that Mill’s *System of Logic* played a key role in this distillation process. I argue that a close reading of the text suggests that in responding to Mill in the *Foundations*, Frege first narrows the concept of meaning to denotation or *Bezeichnung*, but this leaves Frege with a remainder that he then isolates in his study of identity statements: how objects are given to us. Thus Frege is left at the brink of the distinction he draws in his essay "On Sense and Reference."

**Bibliography**


