Complexity, Existence and Infinite Analysis*[1]

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Abstract

According to Leibniz's infinite-analysis account of contingency, any derivative truth is contingent if and only if it does not admit of a finite proof. Following a tradition that goes back at least as far as Bertrand Russell, several interpreters have been tempted to explain this biconditional in terms of two other principles: first, that a derivative truth is contingent if and only if it contains infinitely complex concepts and, second, that a derivative truth contains infinitely complex concepts if and only if it does not admit of a finite proof. A consequence of this interpretation is that Leibniz's infinite-analysis account of contingency falls prey to Robert Adams's *Problem of Lucky Proof*. I will argue that this interpretation is mistaken and that, once it is properly understood how the idea of an infinite proof fits into Leibniz's circle of modal notions, the problem of lucky proof simply disappears.

0. Overview

According to Leibniz's infinite-analysis account of contingency, any derivative truth is contingent if and only if it does not admit of a finite proof. A popular interpretation of Leibniz's views on these matters explains this biconditional in terms of two other principles: first, that a derivative truth is contingent if and only if it contains infinitely complex concepts and, second, that a derivative truth contains infinitely complex concepts if and only if it does not admit of a finite proof. I will argue that this interpretation is mistaken. Anyone who accepts the second principle is confronted with what Robert Adams called 'the problem of lucky proof'. As for the first principle, there is ample textual evidence that some truths involving infinitely complex concepts were not regarded by Leibniz as contingent and that some truths regarded by Leibniz as contingent involve no infinitely complex concepts. In the light of these difficulties, I will propose to look at Leibniz's infinite-analysis account from a different angle: contingent truths require an infinite analysis not because of their inherent complexity, but because they are truths concerning existence. Their proof is infinitely long because it involves an infinitely long comparison, whose aim is to show that what exists is more perfect than anything else which is incompatible with it.

1. Infinite Analysis: The Basics

I will start by laying down and discussing two tenets concerning what Leibniz called 'derivative truths' or 'truths not known *per se*':

(i) Every derivative truth admits of an a priori proof.
(ii) A derivative truth is contingent if and only if it does not admit of a finite a priori proof.

As Leibniz explains in *Primary Truths*, a truth is derivative when it does not "assert the same thing of itself or deny the opposite of its opposite" (AG 30).[2] My focus in this paper will be primarily on derivative truths: for simplicity, I will call them simply 'truths' and reserve the term 'identities' for non-derivative or primary truths. Truth is a feature of propositions,[3] so I will use 'truths' and 'true propositions' interchangeably. In this context, the term 'propositions' refers to entities of which we can say- among other things- that they are simple or complex, affirmative or negative, universal or particular.[4] In other words, it refers to entities that resemble sentences in having a syntactic structure, but have concepts (rather than words) as their constituents. As we shall see, the proof of a proposition p proceeds by carrying out certain transformations on p. Since these transformations alter the structure of the proposition, they have to be thought of as preserving something like the *meaning* or the *content* expressed by the proposition, not the proposition itself. (I mention these subtle distinctions mainly to avoid confusion: while Leibniz was not completely unaware of them, he was not always overly careful in putting them to practice).[5]

According to Leibniz's Predicate-In-Subject theory of truth, every true proposition of the form 'A is B' rests on "a certain real connection between the predicate and the subject" (A6.4.806). Following Aristotle, Leibniz conceived of this connection as a form of *containment* of the predicate 'B' in the subject 'A'.[6] The texts are adamant that every true proposition to which the Predicate-in-Subject doctrine applies admits of an a priori proof (presumably, the same holds of any truth that can be formed by conjoining and disjoining propositions of the form 'A is B').[7] In principle, this would seem to be perfectly compatible with the falsity of (i), the general thesis that *every* truth admits of an a priori proof. It is clear, however, that Leibniz endorsed the general thesis as well.[8] Indeed, there is robust evidence that he regarded (i) as just an alternative formulation of the Principle of Sufficient Reason, whose scope of application is not restricted to any specific class of truths.[9] This is not surprising, given two facts. The first is that, according to Leibniz, *any* simple derivative proposition - whether essential or existential, affirmative or negative, universal or particular - can be transformed into a proposition of the canonical form 'A is B'. Here are some of the relevant transformations:

Every A is B $\leftrightarrow_{def}$ A-not-B is not-entity
Some A is not B $\leftrightarrow_{def}$ A-not-B is entity
No A is B $\leftrightarrow_{def}$ AB is not-entity
Some A is B $\leftrightarrow_{def}$ AB is entity[10]

The second fact is that, as the symbol '$\leftrightarrow_{def}$' signals, these transformations are precisely the kind of definitional equivalences that we are allowed to use when we carry out the proof of a true proposition. All this suggests that, ultimately, the Aristotelian doctrine that 'the predicate is in the subject' is pretty much all we need to explain Leibniz's commitment to (i).

Semantically, the procedure by which we can prove the truth of a proposition a priori consists in an analysis (*analysis*) or resolution (*resolutio*) or explication (*explicatio*) of the concepts involved in the proposition.[11] Formally, a proof can be seen as a particular sequence (*progressio*) of steps, each step being sanctioned by the rule that we can substitute definitions for the term they define (C 258).[12] Substitution after substitution, "all [true] propositions, even contingent ones, are resolved into identities" (A6.4.1449). But this 'resolution into identities' need not be finite, and the distinction between finite and infinite a priori proofs is the key to Leibniz's infinite-analysis account of contingency.[13] (ii) is the central tenet of this account: it affirms that a truth is contingent if and only if it does not admit of a finite a priori proof (hereafter, more simply: finite proof) . It is not difficult to find explicit support for both directions of this biconditional. In *The Source of Contingent Truth*, after explaining that the truth of a proposition can be shown "by giving reasons through the analysis of both terms into common notions", Leibniz affirms that "if [the analysis] is finite, it is said to be a demonstration and the truth is necessary" (AG 98-99). This entails that if a truth is contingent, it does not admit of a finite proof.[14] That the converse is also true can be seen as follows. Given (i), if a truth does not admit of a finite proof, it admits of an infinite one. But Leibniz says that "if the analysis proceeds to infinity and never attains completion, then the truth is contingent" (ibid.). Thus, if a truth does not admit of a finite proof, it is contingent.

When the analysis or resolution is finite, Leibniz speaks of a 'demonstration' (*demonstratio*) or 'reduction' (*reductio*) of the truth to identities.[15] So one could say that contingent truths are all and only the *indemonstrable* ones. Importantly, this does not mean that contingent truths contradict (i): like any other truths, contingent truths can be proved a priori, but they admit of no *finite* proof, that is, of no 'demonstration' or 'reduction' to identities.[16]

(ii) will be my focus for the rest of the paper. Since it constitutes the cornerstone of Leibniz's infinite-analysis account, I will hereafter refer to it as the Core Thesis.

2. Infinite Analysis and Infinite Complexity

Clearly, Leibniz did not regard the Core Thesis as *stipulative* of what it is for a truth to require an infinite analysis (or for it to be contingent). Nor did he take the truth of the Core Thesis to express a *brute fact* about contingency. The question, therefore, naturally arises: why did he think that every contingent truth requires an infinite analysis and, conversely, that every truth requiring an infinite analysis is contingent? The challenge is to explain how the Core Thesis and the idea of infinite proof fit into Leibniz's circle of modal notions.

One might attempt to deal with this challenge in two steps. First, it might be supposed that, according to Leibniz, it is a distinctive feature of contingent truths vis-à-vis necessary ones that they involve infinite complexity:

[Complexity] A truth is contingent iff it contains infinitely complex concepts.

Bertrand Russell must have had in mind something like this thesis when he ascribed to Leibniz "the view that infinite complexity is the defining property of the contingent" (1903, 183) and that "the world of contingents is characterized […] by the fact that everything in it involves infinity by its infinite complexity" (1992, 72). These suggestions are not at all unmotivated and have been embraced by many contemporary scholars.[17] After all, Leibniz's preferred examples of contingent truths are singular propositions like 'Adam sins' or 'Peter denies Christ'. And on at least one occasion he speaks as if the expressions "contingent truths", "truths of fact" and "truths about individual things" could be used interchangeably (L 264). But the concept of any individual is infinitely complex.[18] So if any contingent truth is a truth about individual things, any contingent truth will involve at least one infinitely complex concept. On the other hand, it seems plausible to think that no concepts other than the concepts of individual substances are infinitely complex.[19] Plausibly, then, every truth involving infinitely complex concepts is a truth about individual things- that is, a contingent truth. This simple reasoning might have led Leibniz to endorse Complexity.

Complexity brings us relatively close to explaining Leibniz's commitment to the Core Thesis. All we need is some thesis like:

[Decomposition] A truth does not admit of a finite proof iff it contains infinitely complex concepts.

Together with Complexity, Decomposition would provide us with a key to understanding Leibniz's infinite analysis account of contingency: contingency and indemonstrability go hand in hand because contingent truths are all and only the truths about individual things and these latter are all and only the truths whose resolution is infinitely long, depending on the decomposition of infinitely complex concepts.

Unfortunately, there are well known problems with Decomposition. Robert Adams observes that "even if infinitely many properties are contained in the complete concept of Peter, at least one of them will be proved in the first step of any analysis" (Adams 1994, 34). Imagine, for instance, that we need to prove the true proposition 'Peter denies'. Since the concept 'Peter' is infinitely complex, it resolves into infinitely many simple or primitive positive concepts. Suppose, for good measure, that 'denies' is one such simple or primitive positive concept. Then the probability of hitting upon the concept 'denies' in the first step of the analysis will be at least as little as the probability of drawing the marble marked with the number '2' from a bag containing a marble for each and every natural number. The problem is that, however little the probability, one might always be lucky and hit upon the concept 'denies' after a finite number of steps. This has become known as the *Problem of Lucky Proof*. Rodriguez-Pereyra and Lodge (2011) have recently argued that the problem is not just one of luck:

> Even if we are unlucky and it takes a long time to uncover a particular predicate in the definition of the subject, it will always be uncovered in some finite number of steps. The point can be seen more clearly if we associate each one of the infinitely many concepts constituting Peter's concept with a natural number and we imagine that our analysis uncovers those constituent concepts

> according to the order of natural numbers. Then no matter what number the concept 'denier of Christ' is associated with, it will take only a finite – but probably very large – number of steps to reach this concept from the beginning of our analysis. (223)

I am not entirely convinced that things for the advocate of Decomposition are as bad as Rodriguez-Pereyra and Lodge suggest. Think again of a bag containing infinitely many marbles, each numbered with a different natural number. Sure enough, we can imagine a sequence of draws in which marble number '2' is hit upon after finitely many attempts (here is one such sequence: 1, 3, 7, 11, **2**, 34,…). But of course there are many 'unlucky' sequences as well: think of any sequence going from marble number '150' onward. Rodriguez-Pereyra and Lodge invite us to "imagine that our analysis uncovers [the] constituent concepts according to the order of natural numbers". Now, sure enough, if our analysis unfolds according to the order of natural numbers, concept number '56' will be hit upon after 56 steps. But the point is precisely that whether or not our analysis unfolds according to the order of natural numbers is a matter of *luck*: there are vastly many 'unlucky' analyses that evolve randomly and take infinitely long detours (lottery addicts will be especially familiar with this phenomenon). So even if there is a problem of lucky proof, I do not think this problem generalizes into a problem of *guaranteed* proof.[20]

Still, the problem of lucky proof is bad enough to derail our attempt to explain the Core Thesis by appeal to Complexity and Decomposition. One might hope to bypass the problem by placing further constraints on what should count as a correct 'analysis' or 'resolution' of a proposition. This is precisely the solution Rodriguez-Pereyra and Lodge propose. Their suggestion is that the analysis of a proposition p is not complete (or indeed cannot even start) until one has carried out a 'consistency check' by fully decomposing each of the concepts involved in p and showing that it contains no hidden contradiction. Since individual concepts are infinitely complex and cannot be decomposed and shown to be consistent in a finite number of steps, the suggestion entails that all propositions about individuals require infinite steps to be proved.

The idea that in order to prove 'A is B' one needs to establish that the relevant concepts are consistent by fully decomposing them is not without textual support. But this support is not exceedingly strong[21] and- perhaps more revealingly- it comes at least in part from passages in which Leibniz does not talk explicitly about infinite analysis. It should also be observed that over the years Leibniz became increasingly sceptical of the general availability of a priori proofs of possibility based on consistency checks,[22] while he remained stably committed to the a priori provability of every true proposition. This seems to suggest that he was not overly confident in the idea that the analysis of every proposition requires a consistency check of its component concepts. I will return to this delicate point towards the end of this paper (§ 5).

Even setting these exegetical worries aside, however, it is hard not to find some consequences of Rodriguez-Pereyra and Lodge's proposal highly problematic. It is a familiar point-and one that the authors of the proposal explicitly acknowledge- that if proving a proposition about Peter requires a full decomposition of the concept 'Peter', then not only the proposition 'Peter denies Christ' but also propositions like 'Peter is possible' or 'Peter is Peter' will require an infinite analysis.[23] So both these propositions will have to be classified as contingent, on pain of contradicting the Core Thesis. This is surely an unfortunate result. Perhaps there are reasons to think that Leibniz did not take propositions like 'Peter is Peter' to be necessary.[24] Maybe he worried about the existential import of these propositions and thought that they would fail to be true in a world where Peter does not exist. But what about the contingency of 'Peter is possible'? As is well known, Leibniz's treatment of iterated modalities raises delicate issues,[25] but the claim that the proposition 'Peter is possible' is necessary finds at least indirect support in the texts. For instance, in a 1707 letter to Burnett, Leibniz affirms that, although the actual world is a product of God's decrees, "the idea of this world as possible does not cease to be eternal and necessary" (G III 315). I take this to mean that, although the existence of this world is contingent, its possibility is necessary. More likely than not, something similar is true of created individuals: it is contingent that Caesar exists, but not that he could have existed. This should not come as a surprise. For Leibniz, the possibility of something coincides with its clear and distinct conceivability by the infinite intellect of God. Since the conceivability or inconceivability of something is not at the disposal of God's free will, there does not seem to be room for contingency when it comes to the very possibility or impossibility of a certain created individual. So the proposition 'Peter is possible' is not contingent, but necessary.

Careful reflection on these problems can't but invite a reconsideration of Complexity, and not just of Decomposition. For Complexity affirms that every truth involving infinitely complex concepts is contingent and this is already sufficient to yield the perplexing conclusion that 'Peter is Peter' and 'Peter is possible' are contingent, no matter what Decomposition and the Core Thesis jointly imply.

3. Against Complexity

Complexity has highly implausible consequences, but this is not the only reason to think that Leibniz did not endorse it. There is textual evidence that some propositions involving infinitely complex concepts were not regarded by Leibniz as contingent and that some propositions regarded by Leibniz as contingent do not involve infinitely complex concepts.

Let us begin with the former kind of counterexamples: necessary propositions involving infinitely complex concepts. At the forefront of this class are propositions like 'God exists' and 'God loves himself'. Along with many of his contemporaries, Leibniz regarded the first as a necessary truth (in fact, he famously offered an explicit and, obviously, finite proof of God's necessary existence).[26] As for the proposition 'God loves himself', one text contrasts it with 'God chooses what is best' as an example of a necessary truth that can be demonstrated from the definition of God.[27] Notice that both propositions involve an infinitely complex concept, namely the concept 'God'. Although, in some early writings,[28] Leibniz ventured the idea that the only truly primitive concepts are those of God and Nothingness, this can hardly be regarded as his official position. In fact, there are at least two compelling reasons for thinking that the concept 'God' is infinitely complex. The first is that God is an individual substance- the individual substance, *par excellence*- and Leibniz affirms that "each and every individual substance contains the whole series of things in its complete notion and harmonizes with everything else, and to that extent contains something of the infinite" (AG 100).[29] The second is that, if there are any infinitely complex concepts, then the concept of God must be one of them. For a concept is infinitely complex if and only if it resolves into infinitely many simple or primitive positive concepts. But we know that the simple or primitive positive concepts that enter into any complex concepts are all and only the absolute attributes of God.[30] So, to the extent that any concept is infinitely complex, the absolute attributes of God must be infinitely many and, since they are all contained in the concept of God, the concept of God, far from being primitive or simple, must be infinitely complex, too. But- as any friend of Complexity will readily acknowledge- it is clear that there are infinitely complex concepts: for example, the complete individual concept of Peter. Hence, the concept of God is infinitely complex.

It might be suggested that God is the exception that proves the rule. Never mind, because there are at least two more counterexamples to the thesis that all necessary propositions involve only finitely complex concepts. We find them in the *New*

Essays: the propositions 'I shall be what I shall be' and 'I have written what I have written' (NE 180). Theophilus, Leibniz's *alter ego* in the dialogue, characterizes these propositions as "primary truths of reason" and calls them "identities", putting them on a par with 'A is A' or 'An equilateral rectangle is an equilateral rectangle'. So he clearly regards both of them as necessary. We know that Leibniz took the proposition 'I exist' to be only contingently true,[31] so it must be presumed that the relevant reading of 'I shall be what I shall be' and 'I have written what I have written' is one on which neither of them has any existential import. One possibility is to interpret them as roughly equivalent to 'If I shall be something, I shall be what I shall be' and 'If I have written something, I have written what I have written'. In any case, the main point to note here is that these necessarily true propositions violate Complexity. For it's entirely unclear on what grounds one could argue that the concept 'I' featuring in them is only finitely complex, given that the referent of 'I' is an infinitely complex individual.[32] (Let it be noted in passing that, given the close similarity between the proposition 'Peter is Peter' and the proposition 'I shall be what I shall be', the fact that Leibniz explicitly characterized the latter as necessary tells strongly against the hypothesis, mentioned above, that he might have regarded the former as contingent).

The propositions considered so far seem to me plausible counterexamples to the thesis that any proposition containing infinitely complex concepts is contingent. There is even ampler textual evidence against the converse of that thesis: the claim that any contingent proposition contains infinitely complex concepts.

One can start from the true proposition 'No pentagon exists'. According to Leibniz, this truth is contingent, "for the pentagon is not absolutely impossible [...] even if it follows from the harmony of things that a pentagon can find no place among real things" (AG 21).[33] This is certainly a valid counterexample to Complexity, because the concept 'pentagon' is not infinitely complex. There is, however, some leeway to object to the relevance of this counterexample. For 'No pentagon exists' is a negative proposition and it might be urged that it's not clear how Leibniz's infinite-analysis account of contingency deals with negative propositions.

This objection is entirely unconvincing, for, as we've seen in § 1, we can easily translate a negative proposition like 'No pentagon exists' into a simple affirmative proposition of the form 'A is B'. Moreover, this can be done without introducing any infinitely complex concepts in a proposition that contains none.[34] But, once again, we can rely on more direct counterexamples. Consider the propositions 'There are bodies in nature that actually appear to have right angles' and 'Every man is liable to sin'. Neither of these propositions involves any infinitely complex concept. Yet Leibniz described both as contingent.[35] More to the point, he did so in passages where the salience of the infinite analysis account is beyond question.

It will be objected that Leibniz's recognition that these propositions are contingent is in tension with his inclination to identify contingent truths or 'truths of fact' with 'truths about individual things': on the face of it, neither 'There are bodies in nature that actually appear to have right angles' nor 'Every man is liable to sin' are truths about individual things. One might be tempted to explain away this tension by supposing that Leibniz would analyze propositions of this kind as complex conjunctions or disjunctions of propositions about individual things. In support of this hypothesis, one might cite a text from 1690 in which Leibniz wrote that "'Every man is an animal' is the same as [*idem est quod*] 'Man A is an animal', 'Man B is an animal', 'Man C is an animal', and so on" (G VII 212; P 116). Coherently with this interpretation, one could devise a version of Complexity that applies not only to propositions involving infinitely complex concepts, but also to propositions that are definitionally equivalent to propositions involving infinitely complex concepts.

I find this way of dealing with the problem unsatisfactory. There are good reasons to think that Leibniz would deny that, in analyzing the proposition 'Every man is an animal', one is free to replace it with a long conjunction of propositions concerning man A, man B, man C,…etc. Maybe he was attracted by the idea that 'Every man is an animal' does not express any *fact* over and above the facts jointly expressed by 'Man A is an animal', 'Man B is an animal',…etc. Indeed, it is not too much of a stretch to read this idea into Leibniz's claim that the proposition 'Every man is an animal' *is the same as* (in Latin: "*idem est*" or "*nihil aliud dicit*") the conjunction of 'Man A is an animal', 'Man B is an animal',...etc. In any case, I strongly doubt that Leibniz would take 'Every man is an animal' to be *definitionally equivalent* with a long conjunction of singular propositions. He did not even remotely suggest that a correct analysis of a universal proposition of the form 'Every A is B' should or could proceed by analyzing the individual concepts of all the As. In fact, he explicitly suggested the opposite: the containment connection is supposed to be observable directly at the level of the concepts 'A' and 'B' (e.g. 'man' and 'animal'), without any need to carry out an analysis of the individual concepts of each of the individuals that fall under 'A'.[36] This lends clear support to the view that neither the proposition 'Every A is B' nor any proposition definitionally equivalent to it contains those individual concepts. So, to go back to our initial example, the contingency of the propositions 'Every man is liable to sin' and 'There are bodies in nature that actually appear to have right angles' constitutes further evidence against Complexity.

4. Infinite Analysis and Existence

A Complexity-based explanation of why Leibniz endorsed the Core Thesis is unlikely to succeed. On the one hand, such an explanation would have to appeal to some principle like Decomposition and, as we've seen, Decomposition is problematic: there is no fully satisfactory answer to why a proposition containing infinitely complex concepts should not be provable in finitely many steps. On the other hand, Leibniz's writings contain several counterexamples to Complexity, not to mention the fact that Complexity has the implausible (and unLeibnizian) consequence that propositions like 'Peter is Peter' and 'Peter is possible' are contingent.

If Complexity-based explanations fail, how are we to make sense of the correlation posited by Leibniz between contingent and indemonstrable truths? My proposal is to replace Complexity and Decomposition with the following two principles:

[Existence] A truth is contingent iff it concerns existence.

[Comparison] A truth does not admit of a finite proof iff it concerns existence.

It is obvious how Existence and Comparison would jointly explain the truth of the Core Thesis.[37] So let me move directly to explaining why I think Leibniz would endorse both these principles.

There are good textual and philosophical reasons to think that Leibniz would endorse Existence. I begin with the textual evidence:

> Propositions of fact involve existence. The notion of existence is such that what exists is the state of the universe that pleases God. (A6.4.1449)
>
> All contingent propositions have reasons to be one way rather than another […] But […] these reasons are based only on the principle of contingency or the *principle of existence of things*, that is, based on what is or appear to be best among several equally possible things. (AG 46, my emphasis)
>
> All existential propositions, though true, are not necessary, for they cannot be proved unless an infinity of propositions is used, i.e. unless an analysis is carried to infinity. (C 376; P 66)

> There are propositions that concern the essence, and others that concern the existence of things. The essential ones are clearly those that can be demonstrated through the resolution of terms [...]. Totally different from these are the existential or contingent ones [*existentiales sive contingentes*], whose truth is understood *a priori* only by an infinite intellect and cannot be demonstrated by any analysis. (C 18)

The first two excerpts militate in favour of the left-to-right direction of Existence. The third and the fourth (this latter in conjunction with the Core Thesis) yield the right-to-left direction.

Leibniz was well aware that not all contingent propositions wear their existential nature on their sleeve. 'Peter denies', 'Adam sins' and 'Every pious man suffers' do not immediately strike us as propositions about existence. Here, however, the definitional equivalences introduced in § 1 become, once again, relevant. Take, for instance, 'Every pious man suffers'. By the first equivalence, this gets transformed into 'Pious-man-not-suffering is not-entity'. But 'is not-entity' here cannot mean 'is not possible' or 'is not conceivable', otherwise the proposition would not be true: it simply means that a pious-man-not-suffering is not existent.[38] Thus, the contingent proposition 'Every pious man suffers' is equivalent to 'Pious man not suffering is not an existent', which clearly concerns existence (A6.4.1631). By the same token, 'Peter denies' and 'Adam sins' are equivalent, respectively, to 'Peter-denying exists' and 'Adam-sinning exists' (C 371).[39]

The way in which Leibniz sponsored the idea of a direct correlation between existence and contingency is somewhat perplexing, given that he took the proposition 'God exists' to be necessarily true. One's perplexity deepens as one realizes that the problem is not confined to propositions concerning God's existence. For surely the conditional 'If God exists, something exists' is necessarily true. From this and the necessity of 'God exists', one obtains the necessary truth of 'Something exists' (the inference is valid in any system of modal logic containing the K-axiom). And doesn't the proposition 'Something exists' concern existence? Another puzzling case is the proposition 'A pentagon with six sides is not an existent': on the face of it, it closely resembles 'Pious man not suffering is not an existent', but surely it is necessarily, not contingently true. These obvious difficulties argues for a qualified reading of Existence. When he talks about existential propositions being contingent, Leibniz cannot have in mind just any proposition containing the concept 'existence'. So what propositions are 'existential', in the relevant sense?

I submit that a proposition is 'existential' whenever it entails that some kind of possibilia (i.e. possible individuals or possible worlds) exist and others don't. For instance, 'Peter denies' is existential because it entails that Peter-denying exists and that any world (or 'state of the universe') in which Peter-denying is missing doesn't. 'Every pious man suffers' rules out the existence of any world containing non-suffering pious men and (given that there must exist something) it entails the existence of a world free of non-suffering pious men. Consider, by contrast, 'God exists'. The only scenario whose existence is ruled out by the truth of 'God exists' would be one in which God doesn't exist. But *there is no such possible scenario*. Hence, according to the criterion above, 'God exists' is not existential. By the same token, 'A pentagon with six sides is not an existent' is not existential, because it doesn't rule out the existence of any possible individual or possible world.

This interpretation, while perhaps not mandated by the texts, is naturally suggested by Leibniz's remark that the principle of existence of things is "based on what is or appears best *among several equally possible things*" (AG 46, my emphasis): the scenario in which God exists is *not* one among several equally possible scenarios, for there are no possible scenarios alternative to it. It is also important to keep in mind that Leibniz's main or only good reason to posit a correlation between existence and contingency is his idea that, at the moment of creation, God freely chose to bring into existence some kind of possibilia and not others.[40] So the only version of Existence he had good reasons to endorse is one on which a proposition 'concerns existence' in the sense that it entails the existence of some kind of possibilia and not others. Propositions that concern existence in this sense are contingent because their truth reflects God's initial choice as to which series of individuals should be actualized. Conversely, if a true proposition does not entail an existence-based discrimination among possibilia (as in the case of 'God exists', 'A pentangon with six sides is not an existent' or 'Something exists'), that proposition must be necessary.[41]

It will be objected that Existence is false even under the qualified reading I've just sketched, for, strictly speaking, not *all* contingent propositions reflect God's choice to create one series of individuals rather than another. Arguably, the proposition 'God chooses what is best' is contingent, but it does not directly concern the actualization or creation of a given series of individual. My response to this worry is twofold. First, I agree with Adams that "there seems to have been more vacillation and uncertainty in Leibniz's mind about whether it is necessary or contingent that God chooses what is best than about any other main issue in the problem of contingency" (1994, 36). While it is relatively clear that Leibniz denied the necessity *de re* of God's giving to the world the form it *actually* has, a plausible case could be made that he admitted the necessity *de dicto* of God's choosing *whatever* is best. But the contingency of the proposition 'God chooses what is best' under the *de re* reading is not incompatible with Existence, for that proposition (under that reading) means nothing else than that God chose *this* world or, equivalently, that *this* series of individuals exists. Second, and perhaps more interestingly, on the few occasions where Leibniz denied the necessity of 'God chooses what is best' read as *de dicto*, he by and large refrained from explaining the contingency of that proposition (under that reading) in terms of infinite analysis.[42] So even if I'm wrong about the modal status of 'God chooses what is best' under the *de dicto* reading, this would constitute no decisive evidence against the correctness of my interpretation of Leibniz's infinite analysis account of contingency.

All in all, there's far more evidence in favour of the hypothesis that Leibniz endorsed (a qualified version of) Existence than there is against it. What about Comparison? Did Leibniz think that existential truths are all and only the indemonstrable ones? Once again, let us begin with the textual evidence.

On Freedom and Contingency (1684) contains one illuminating passage concerning the analysis or resolution of existential truths:

> All existences, except God's existence, are contingent. The reason why some particular contingent exists rather than others should not be sought in its definition alone [*non petitur ex sola ipsius definitione*], but in a comparison with other things. For since there are an infinity of possible things which, nevertheless, do not exist, the reason why these exist rather than those should not be sought in their definition (for then nonexistence would imply a contradiction, and these others would not be possible, contrary to our hypothesis) but from an extrinsic source, from the fact that the ones that do exist are more perfect that the others. (AG 19)

In this passage, Leibniz affirms that in order to find the reason for the truth of a particular existential claim, one cannot look only at the definition of the particular thing which is said to exist. One has also to engage in a comparison between the thing which is said to exist and all the possibilia that could have existed in its place. Notice that the process of looking for the reason why a certain truth is true is the very process of 'analysis' or 'resolution' of the truth in question. For the reason why a certain truth is true is, of course, nothing else than the fact that the predicate is contained in the subject.[43] So the passage above says that the analysis or resolution of an existential proposition requires a comparison between the things whose existence the proposition affirms and all the things that might have existed in their place. I will return to this apparently puzzling result shortly.

In the following passage, Leibniz is even more explicit that the proof of an existential proposition somehow proceeds by way of a comparison between the actual and the merely possible:

> All propositions involving existence and time also involve the entire series of things. [...] Whence the fact that these propositions do not allow for a demonstration, i.e. of a finite resolution from which their truth would be made apparent. [...] And even if one could know the entire series of things, one could not give a reason for it, *if not by comparing it with all the other possibilities*. Whence it appears clear why it is not possible to find any demonstration of a contingent proposition, however far we push the resolution of the notions involved. (C 19, my emphasis)

The reasoning here goes as follows. Given the interconnection of all existents, the reason why a particular existential claim is true cannot be grasped if not "by a perfect cognition of every portion of the universe" (ibid.). But suppose one is granted the most complete knowledge of the interconnected series of individuals that inhabit the actual world. This would still not allow one to see why *this* series exists rather than another. What is needed is a comparison between the actual series and all the possible alternative series. This, Leibniz concludes, is what explains why existential propositions cannot be demonstrated, i.e. resolved into identities in a finite number of steps.

Notice that the explanation Leibniz sketches in these passages of the indemonstrability of existential truths works equally well for infinitely and finitely complex propositions. The proposition 'There are bodies that actually appear to have right angles' is, I've argued, only finitely complex. But it is contingent and, therefore, according to Leibniz, existential (by the fourth definitional equivalence, we can transform it into 'Body-appearing-to-have-right-angles is an existent', which entails the non-existence of any world free of bodies that appear to have right angles). So in order to prove its truth a priori, we need to engage in a comparison between the worlds where some bodies appear to have right angles and the worlds where no bodies appear to have right angles. Presumably, what we need to prove is that at least one world of the former sort is more perfect than any world of the latter sort: this would explain why God did not choose to bring into existence any of the worlds of the latter sort. Unsurprisingly, the task of accomplishing this comparison is infinite: it requires us to go through infinitely many possible series of individuals, each of which may well include an infinite number of infinitely complex members. No wonder one has to "continue the analysis to infinity through reasons for reasons" (AG 28).

At this point, however, a doubt may be raised: weren't we told that the 'analysis' or 'resolution' of a proposition proceeds simply by substituting definitions for the analysanda? And so isn't Leibniz changing the topic when he explains the indemonstrability of existential truths in terms of the idea of an infinitely long comparison? Well, yes, but then again, no. Analysing an existential proposition of the form 'AB is an existent' is tantamount to eliciting the concept of 'existence' from the concept 'AB'. So, for instance, the contingent proposition 'Peter denies' is equivalent to 'Peter denying is an existent' and proving the truth of 'Peter denying is an existent' is tantamount to eliciting the concept 'existence' from the concept 'Peter denying'. Now, the first thing to notice is that the concept 'existence' gives the analysis of existential propositions a somewhat unexpected comparative twist. The reason is, quite simply, that Leibniz thought of existence as a comparative notion. He *defined* 'existens' as "that which is *compatible with more things than* anything else which is incompatible with it" (C 376; P 51, my emphasis). Even more explicitly he suggested that:

> Existence is the difference between the degree of reality of each thing and the degree of reality of its opposite, that is to say, what is more perfect than all the mutual incompatible alternatives exists and, conversely, what exists is more perfect than everything else. So [...] it is not true that existence is itself a perfection, for it consists only in some sort of mutual comparison among perfections [*quaedam perfectionum inter se comparatio*]. (A6.4.1354)

So the idea that the 'analysis' or 'resolution' of an existential proposition requires a comparison between possibilia is not an arbitrary addition to Leibniz's account of what the 'analysis' or 'resolution' of a proposition consists in. It is somehow connected with the way Leibniz glossed the notion of 'existens'.

Still, one would expect to hear more about this connection. For the worry remains that the reasons cited by Leibniz in the two passages above for why the proof of an existential proposition is infinitely long are somewhat extraneous to the notion of analysis we started out with (§ 1). When, at some stage in the analysis of a proposition, we replace the *definiens* 'is compatible with more things than anything else which is incompatible with it' for the *definiendum* 'exists', the subsequent stages of the analysis can be *interpreted as* or *likened to* a very long comparison between the subject of 'exists' and anything else which is incompatible it. But, strictly speaking, an analysis is *not* a comparison and, for reasons that should be familiar from § 3, even if one can substitute 'is compatible with more things than anything else which is incompatible with it' for 'is an existent', it is *not* as if one can then go on to substitute infinitely many individual concepts for the expression 'anything else which is incompatible with it' (just as one cannot substitute 'Man A is an animal', 'Man B is an animal',…etc. for 'Every man is an animal').

To understand Leibniz's ultimate reasons for endorsing Comparison, we have to do some reading between the lines. A closer look at the two passages above reveals that what makes the analysis of 'Peter denying is an existent' infinite is not so much the difficulty of carrying out a comparison between Peter and his (infinitely many) competitors in the race for existence as the fact that the predicate 'is compatible with more things than anything else which is incompatible with him' is *not* one that applies to Peter *essentially* or *by definition*.[44] Leibniz puts us on the right track when, in the first of the two passages, he admits that "the reason why some particular contingent exists rather than others should not be sought in its definition alone [*non petitur ex sola ipsius definitione*]". Even more revealingly, he writes that "[he] use[s] the term 'contingent', as do others, for that whose essence does not include existence" (A6.4.1775). Given that substituting definitions for the terms they define is all we are allowed to do in the analysis of a proposition, these remarks can be seen as providing a deeper or more rigorous explanation of why, in the analysis of contingent propositions, there is no hope of arriving at a primitive truth: the end of the resolution *does not exist* and what we are left with, at every stage, is a more and more fine-grained comparison between Peter and 'anything else which is incompatible with him', one that gives us more and more reasons, but no demonstration, of the fact that Peter had to be created. It is not difficult to see that, given this setup, the problem of a 'lucky proof' simply disappears.

What has been said so far supports the claim that every existential proposition requires an infinitely long proof. What about the converse of that claim? Are all truths requiring an infinitely long proof existential truths? To my knowledge, Leibniz never explicitly said so. Nevertheless, the evidence collected so far points heavily in this direction. We know for sure that Leibniz endorsed the Core Thesis. And I've argued and provided textual evidence that he also endorsed Existence. The Core Thesis and Existence jointly entail Comparison, which says that a truth cannot be analyzed in a finite number of steps if *and only if* it concerns existence. As we've just seen, the right-to-left direction of this biconditional finds independent textual support in Leibniz's writings and fits very well with Leibniz's general conception of what it is for something to have 'existence'. Overall, the hypothesis that Leibniz's infinite analysis account of contingency rests on the combination of Existence and Comparison seems to me compelling. I will conclude by considering some further possible objections to this hypothesis.

5. Objections & Replies

Objection 1: Consider an uncreated substance Caesar*. The proposition 'Caesar* is less perfect than infinitely many possible substances' is either necessary or contingent. Suppose it is necessary. Then it has to admit of a finite proof. But arguably showing that Caesar* is less perfect than infinitely many possible substances requires us to go through an infinitely long comparison. So infinitely long comparisons are not synonymous with infinitely long analyses. Suppose, on the other hand, that 'Caesar* is less perfect than infinitely many possible substances' is contingent. Then we have a counterexample to Existence: a proposition that requires an infinitely long comparison, but does not seem to concern existence.

Reply to Obj. 1: The proposition 'Caesar* is less perfect than infinitely many possible substances' must be contingent. This follows from three very plausible assumptions, namely:

(i) God could have created a world containing only Caesar*;

(ii) Necessarily, if there are infinitely many possible substances more perfect that x, then there are infinitely many possible worlds more perfect than a world containing only x.

(iii) Necessarily, if there are infinitely many possible worlds more perfect than a world containing only x, then God does not create a world containing only x.

I submit Leibniz would have accepted (i). For clearly a world containing only Caesar* is possible and how could it be possible if it were impossible for God to create it? (ii) seems very plausible: any world containing only one substance and such that that substance is more perfect than x would be better than a world containing only x. And I am inclined to think that (iii) is true, because it follows from God's nature that God creates whatever is best.[45] But it can easily be seen that, given (ii) and (iii), if it is necessary that Caesar* is less perfect than infinitely many substances, then it is necessary that God did not create a world containing only Caesar* (the inference relies, once again, on the K-axiom). This contradicts (i). So, by reductio, 'Caesar* is less perfect than infinitely many substances' is contingent.[46]

Is this a counterexample to Existence? Not really. For 'Caesar* is less perfect than infinitely many possible substances' *does* concern existence. In particular, its truth entails that a world containing only Caesar* does not exist. And notice that the same can be said of any true proposition to the effect that Caesar* is less perfect than another possible substance Caesar** (if Caesar* is less perfect than Caesar**, then a world containing only Caesar* does not exist). So 'Caesar* is less perfect than Caesar**' is, at most, contingently true. This is just what we should expect, for even the resolution of 'Caesar* is less perfect than Caesar**' is bound to take the form of an infinitely long comparison between Caesar*'s and Caesar**'s infinitely many perceptions, attributes,…etc.[47]

Objection 2: Consider the proposition 'Caesar* is shorter than infinitely many possible substances'. Arguably, its truth requires an infinitely long comparison. But surely it is not contingent! And suppose, per absurdum, that it is contingent. Then we have another counterexample to Existence, for surely 'Caesar* is shorter than infinitely many substances' does not concern existence.

Reply to Obj. 2: The truth of 'Caesar* is shorter than infinitely many possible substances' does *not* require an infinitely long comparison and is *not* contingent. It is a necessary truth concerning Caesar*. For no matter how tall Caesar* is, he must be finitely tall. But then, necessarily, there is a possible substance that is ever so slightly taller than him (just as, necessarily, for every body moving at a finite speed, there is a body moving ever so slightly faster than it).[48] And if there is one substance ever so slightly taller than Caesar*, there are infinitely many substances taller than him (for every substance taller than Caesar* we can find a shorter substance that is still taller than him,…).

Objection 3: What about 'Caesar* has more perceptions as of green than Caesar**'? How could we establish the truth of this proposition, if not via an infinitely long comparison of all of Caesar*'s and Caesar**'s perceptions?

Reply to Obj. 3: 'Caesar* has more perceptions as of green than Caesar**' is necessarily true, if true at all. If Caesar* has more perceptions as of green than Caesar**, this follows from the *definition* of Caesar* and Caesar** (which, of course, does not mean that we couldn't have decided to use the names 'Caesar*' and 'Caesar**' to talk about two possible substances that do not stand in the relevant relation). So the proposition 'Caesar* has more perception as of green than Caesar**' is reducible to a primitive truth in a finite number of steps. Here the difference with existential propositions couldn't be more evident: it is only because the predicate 'is an existent' is not included into the definition of any substance other than God that the analysis of any particular truth concerning existence ends up taking the form of an infinitely long comparison.

Objection 4: Comparison affirms that *only* existential propositions require infinitely long proofs. But occasionally Leibniz suggests that "the possibility [of something] cannot be demonstrated, unless we resolve it into [its] primitive requisites [*nisi resolutio in requisita primitiva facta*]" (A6.4.277). It seems to follow that it would take us an infinite number of steps to prove the proposition 'Peter is possible', for the concept 'Peter' has infinitely many primitive requisites.

Reply to Obj. 4: I concede that it is somewhat puzzling why Leibniz did not realize that showing the consistency of an infinitely complex concept would take us an infinite amount of steps, if nothing less than a full decomposition of the concept were required to accomplish the task. I also concede that, according to what several texts suggest, one can only prove the consistency of a concept by fully decomposing it. Yet I simply cannot believe that Leibniz would have classified 'Peter is possible' as contingent. Leibniz's commitment to the Core Thesis being beyond question, I have no choice other than to conclude that the texts in which proofs of possibility are said to *require* a full decomposition of the concept should be distrusted or, at the very least, taken with a grain of salt. This does not seem to me an unbearable cost. For one thing, I've already noted that over the years Leibniz became increasingly sceptical of the general availability of a priori proofs of possibility based on consistency checks. For another, there are texts implying that the full decomposition of a concept is *not* the only a priori method for showing its consistency. Another a priori method consists of giving what Leibniz calls a 'causal definition', a definition by which we "understand how the thing [defined] can be produced" (G IV 425).[49] For instance we define a circle as the figure described by the motion of a straight line about a fixed end (A6.4.541). Third, and perhaps more important, Leibniz *did* give a finite a priori proof of the consistency of an infinitely complex concept, for he showed that the concept 'God' is consistent. It's not quite clear what to make of this. Maybe we should conclude that in some cases a less-than-full decomposition is sufficient to establish the consistency of a concept.[50] Or maybe we should not exclude the possibility that the complete notion of a certain individual might be amenable to a finitely long decomposition even if that notion is 'infinitely complex' in the sense that it 'involves the whole series of thing'.[51] A full assessment of these alternatives falls well beyond the scope of this paper.

Received 8 August 2012

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Notes

**This paper won the 2012 Leibniz Society of North America essay competition.*

[1]I am greatly indebted to Gonzalo Rodriguez-Pereyra, who gave me detailed and insightful comments on earlier drafts of this paper. Many thanks to Elia Zardini and Francesco Berto for useful discussion. The research leading to these results has received funding from the European Community's Seventh Framework Programme FP7/2007-2013 under grant agreement no. FP7-238128.

[2] In addition to the abbreviations for Leibniz's works standardly used in this journal, I will use 'P' for G. W. Leibniz, *Logical Papers: A Selection*, edited by G.H.R. Parkinson (Oxford: 1966).

[3] A6.6.396-398.

[4] Leibniz borrowed these distinctions from the Scholastic tradition. For an early exposition, see A6.4.206.

[5] But see A6.2.480.

[6] Some of the most explicit passages are L 236; C 16-17; G VII 300, 309. The view is ascribed to Aristotle at C 518-519.

[7] "There is no [true] proposition in which there is no connection between the subject and the predicate, that is, no [true] proposition which cannot be proved a priori [*non probari possit a priori*]" (AG 19); "The predicate or consequent is always in the subject or antecedent and the nature of truth in general or the connection between the terms of a [true] statement consists in this very thing. […] A priori demonstration rests on this." (AG 31). That contingent truths are no exception to

this rule appears clear from this passage from the *Discourse on Metaphysics*: "All contingent propositions […] have a priori proofs of their truth which render them certain and which show that the connection between subject and predicate of these propositions has its basis in the natures of both" (AG 46).

[8] "In general, every true proposition that is not identical or true by itself can be proved a priori, by means of axioms (or propositions that are true by themselves) and definitions (or ideas)" (A6.4.805).

[9] "The principle that a reason must be given is this: that every true proposition not known per se has an a priori proof, or that a reason can be given for every truth, or, as is commonly said, everything has a cause" (G VII 309).

[10] G VII 212; A6.4.780.

[11] The three terms are used interchangeably: see, for instance, A6.4.935.

[12] It is true, however, that when giving an actual example of a proof, Leibniz freely uses the Aristotelian syllogisms (AG 96). The point is made by Velasco (Ms).

[13] AG 98-99; A6.4.862. A *finite proof* of a given proposition p consists in a finite sequence of steps having p as its first element and a primitive truth as its last element. The notion of an *infinite proof* is, of course, more problematic. We know that an infinite proof of a given proposition q is an infinite sequence of steps having q as its first element. But obviously not just *any* infinite sequence of steps having q as its first element counts as a proof of q and, since "the end of the resolution does not exist" when the resolution is infinite (A6.4.1653), there is no intuitive sense in which certain infinite resolutions as opposed to others can be said to *lead* to identities. Leibniz must have been well aware of the problem: his idea was that an infinite sequence of steps having q as its first element counts as a proof of q only if it obeys a certain kind of 'law' (*lex progressionis*). The details here are somewhat tentative, but the general idea is that it has to appear "from the law of the sequence [*ex regula progressionis*] that the reduction has reached a point at which the difference between what should coincide is less than any given difference" (A6.4.761) or that "the further we push the resolution, the closest we get to identities, though we never reach them" (A6.4.776). Of course, it is one thing to say that an infinite resolution of q has to obey a certain 'law' in order to count as a proof of q and another to say that a finite mind can find out whether or not q admits of an infinite resolution of the relevant type. The fact that the latter claim is explicitly denied in *Necessary and Contingent Truths* (A6.4.1516) does not immediately entail that Leibniz was not consistent with himself on the former point.

[14] "In contingent truths, even though the predicate is in the subject, this can never

be demonstrated [*demonstrari*]" (A6.4.1653); "In contingent propositions one continues the analysis to infinity through reasons for reasons, so that one never has a complete demonstration" (AG 28). See also A6.4.776.

[15] AG 98-99. This use, however, is not always consistent.

[16] Confusion on this point led Blumenfield to define contingent propositions as propositions that cannot be proved a priori (1985, 488). This is a mistake: Leibniz denies that contingent propositions can be 'demonstrated', but he does not deny that contingent propositions can be proved a priori.

[17] Rodriguez-Pereyra and Lodge explicitly endorse the right-to-left direction of the thesis I called Complexity (2011, 235). Blumenfield ascribes to Leibniz the view that 'Peter denies' is contingent because "the concept *Peter* is infinitely complex and [...] no finite analysis will suffice to elicit the concept *denial* from it" (1985, 495). Sleigh (1982, 227-228) takes the thesis that there are concepts whose analyses are infinite to be the cornerstone of Leibniz's infinite-analysis account of contingency. Lodge and Puryear (2006/2007) take it that an essential requisite of the infinite-analysis account is that "the concepts of many things we actually conceive are composed of other concepts ad infinitum" (189).

[18] "Each and every individual substance contains the whole series of things in its complete notion and harmonizes with everything else, and to that extent contains something of the infinite" (AG 100); "Every singular substance involves the whole universe in its perfect notion" (C 521). See also C 375. Russell argues for the infinite complexity of the concept of any individual substance as follows: "Every substance is infinitely complex, for it has relations to every other, and there are no purely extrinsic denominations, so that every relation involves a predicate of each of the related terms" (1992, xvi). Moreover, according to Russell, "[that every substance has an infinite number of predicates] is evident from the mere fact that every substance must have a predicate corresponding to every moment of time" (1992, 71).

[19] One relevant issue here is whether merely possible individuals have infinitely complex concepts. If so, even truths about them will come out contingent, as Russell himself pointed out. But Ishiguro argues that "only the individuals in this world can be logically treated as individuals and have corresponding individual concepts, and that "Pegasus" or "Zeus" express only a general concept" (1972, 134). For a discussion, see Mates (1986, 66).

[20] A possible rejoinder is that, if the analysis of a concept obeys a certain *order* and is not allowed to evolve randomly in the way I suggest, the problem of guaranteed

proof is restored. But things are more complicated than this. For suppose that the order in question is *dense*: for any two steps x and y in the analysis of Peter's concept, there is a step z (distinct from x and y) which comes *after* x and *before* y according to such order. If its steps are subject to a dense ordering, a proper analysis of Peter's concept will have to go through infinitely many steps before hitting upon any particular constituent concept (there will be literally *no* first step in the analysis, just as there is not such a thing as the first positive rational number after 0). Notice that this would be possible even if the steps in question are infinitely, but countably many (the obvious comparison is with the rationals, which are both dense and countable). Of course, Leibniz would owe us an explanation of why the analysis of certain concepts obeys a dense order. But the point remains that, given how little the texts say on the topic, the burden of the proof of showing that the order is *not* dense rests with the proponent of the problem of guaranteed proof.

[21] Rodriguez-Pereyra and Lodge cite four main passages in favour of their interpretation: two passages from the *General Inquiries* (A6.4.760 and A6.4.746), one from the *Critical Thoughts on the General part of the Principles of Descartes* (G II, 359) and one from the *Meditations on Knowledge, Truth and Ideas* (A6.4.588). In the latter two, Leibniz says that whenever we use definitions for drawing conclusions, the definitions we use call for a consistency check. But it's not clear to me that using definitions in a process of analysis is to use definitions 'to draw a conclusion'. Nor is it obvious that the particular *definitions* one substitutes for 'Peter' during the analysis (e.g. the definition 'the denier of Christ') cannot be shown to be consistent in a finite number of steps, notwithstanding the impossibility of showing, in a finite number of steps, that the *individual concept* 'Peter' is consistent. As for the two passages from the *General Inquiries*, Rodriguez-Pereyra and Lodge readily acknowledge that the first (A6.4.760) is not totally suggestive of their interpretation. I shall add that the passage might not be immediately relevant, for it concerns propositions that can be demonstrated (*demonstrari*) and, as we've seen, contingent propositions for Leibniz are not among these.

[22] See Parkinson (1966, xxxv).

[23] The point is made, among others, by Hawthorne and Cover (2000).

[24] See Rodriguez-Pereyra and Lodge (2011, 229-232).

[25] See Lenzen (2004) for a discussion.

[26] AG 235-240.

[27] Gr 288. See also G VI 256. On the modal status of 'God chooses what is best', see below.

[28] I am thinking, in particular, of the *De Organo Sive Arte Magna Cogitandi* (A6.4.156-160).
[29] See footnote 18 above.
[30] G IV 425 (AG 26) and G VII 310.
[31] AG 193.
[32] One is immediately tempted to say that 'I' is a finitely complex (or perhaps even simple) concept referring to an infinitely complex individual. But this temptation reflects a picture of the meaning of 'I' that does not square with Leibniz's Predicate-in-Subject theory of truth. Moreover, it should be noted that if 'I' is not an infinitely complex concept, Leibniz's claim that "it is a contingent truth that I exist" (AG 193) will constitute a counterexample to the thesis that only propositions involving infinitely complex concepts are contingent. So whichever option one chooses, Complexity appears to be in trouble
[33] As Leibniz points out, further complications arise from the fact that, although a pentagon could exist, it could only exist "abstracted from time" (ibid.). For the sake of simplicity, I set these complications aside.
[34] Notice, in particular, that the concept 'existence' is not infinitely complex: in a 1677 letter to Arnold Eckhard Leibniz gives 'existence' as a *notio incomplexa sive irresolubilis*. This remark cannot be trusted completely, for Leibniz did provide us with what looks like an analysis of the concept of 'existence' (see § 4). However, it seems very unlikely that he would have characterized as '*irresolubilis*' a concept that admits of no finite analysis.
[35] AG 193, A6.4.779.
[36] "When I say 'All pious are happy' I mean by this only that the connection between piety and happiness is such that whoever understands the nature of piety perfectly will see that the nature of happiness is involved in it" (L 236).
[37] In this context, I use the term 'explain' rather loosely: I do not mean to suggest that, besides endorsing these principles, Leibniz took them to be more explanatorily fundamental than the Core Thesis. More plausibly, he regarded Existence, Comparison and the Core Thesis as three mutually illuminating analyticities.
[38] See Mates (1986, 55) for a discussion of the ambiguity of the copula in Leibniz.
[39] Let it be noted in passing that all this fits quite nicely with Leibniz's famous doctrine that the real question for God when he created the world was not whether Adams should be allowed to sin but whether or not Adam-who-sins should be admitted to existence: "God can be understood as determining not whether Adam should sin, but whether the series of individuals including the Adam whose perfect

individual notion involves sin should nonetheless be preferred to other series" (A6.4.1619).

[40] Notice that the existence of a certain kind of possibilia is tightly linked to the non-existence of others, for "whatever is possible has, by itself, a tendency to exist, and there are no other reasons why something possible doesn't exist than those which arise from the joint reasons of the existence [of other kind of possibilia]" (A6.4.1635).

[41] "And the eternal truths are those that not only will hold as long as the world exists, but that would have held even if God had created the world otherwise"(A6.4.1517). Notice that I am not committing myself to saying that things could have been different *because* God could have chosen otherwise. A good case could be made that the explanation goes the other way around: God could have chosen otherwise *because* things might have been different. All I am committing myself to is the existence of a correlation between the logical space of possible worlds and the moral space of God's possible choices.

[42] It is not easy to find passages in which 'God chooses what is best' is both characterized as contingent and unequivocally construed *de dicto*. To my knowledge, the main places to look at are the *Theodicy* (G VI 256; G VI 284) and *On Freedom and Contingency* (in particular, A6.4.1447): here, as well as at Gr 333, the emphasis lies on the voluntariness and freedom of God's actions, more than on the indemonstrability of the relevant proposition. One notable exception is Gr 288.

[43] Incidentally, this is why Leibniz so often equated his Principle of Sufficient Reason with the thesis that every truth can be proved a priori. See, for example, G VII 309 and A6.4.806.

[44] Unlike Mates (1986, 74f and 112f), I do *not* think that certain true predications involving 'is an existent' or 'is compatible with more things that anything else which is incompatible with it' represent an exception to the Aristotelian doctrine that the predicate is in the subject: even when they do not apply to a term *essentially* or *by definition*, these predicates are contained in every subject of which they can be truly predicated. At A6.6.358 Leibniz is explicit about this: "When it is said that something exists [...], this existence itself is the predicate; that is, the notion of existence is linked with the idea in question, and there is a connection between these two notions". I simply fail to see any tension between this thesis and the claim that, in actualizing an individual concept, God does not change it (G VII, 390).

[45] See Adams (1994, 36-42) and Rescher (2002).

[46] Leibniz himself comes very close to providing an argument very similar to the

one I use to establish that 'Caesar* is less perfect than infinitely many substances' is contingent. It is in *On Freedom and Possibility* (AG 20). Part of what Leibniz argues in that passage is that from the necessity of the hypothesis 'A is more perfect than B' we get the necessity of 'B doesn't exist' (given the starting assumption that either A or B exists and that A and B are incompossible). But Leibniz wants to rescue the assumption that B *could* have existed. So (he seems to suggest) we should deny the necessity of 'A is more perfect than B'.

[47] C 19, C 375.

[48] AG 238. Note that the argument I'm giving does *not* rely on the assumption that, tallness is not capable of a highest degree. It only relies on the principle that, necessarily, for any property P other than perfection, if x has P to a finite degree, then there is some created or uncreated substance which has P to a higher degree. Presumably, this principle applies also to particular moral perfections, which are capable of a highest (albeit, presumably, infinite) degree (AG 35). So it also rescues the necessity of 'Caesar* is less knowledgeable/courageous/good/… than infinitely many substances'.

[49] That the two methods of proof are distinct is strongly suggested in § 24 of the *Discourse on Metaphysics* (AG 57).

[50] Here one might appeal to the notion of *lex progressionis* (cf. footnote 13 above). At A6.4.757, Leibniz suggests that one can demonstrate "from the way the resolution proceeds [*ex ipsa progressione resolutionis*] or from some general relation between each step of the resolution and its successor that a contradiction will never arise, no matter how far we push the resolution". He then goes on to point out that "*this is certainly remarkable, for in this way we can often free ourselves from a long iteration*" (my emphasis).

[51] The idea might be that a concept C which has finitely many atomic constituents is nonetheless such that one can know infinitely many truths of the form 'C is F' just by virtue of understanding C. C would 'contain' all these truths in the sense that all these truths would be knowable a priori. And nevertheless a full decomposition of C would be of finite length.