**DISCUSSION**

**Infinite Number and the World Soul; in Defence of Carlin and Leibniz**

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In last year's *Review* Gregory Brown took issue with Laurence Carlin's interpretation of Leibniz's argument as to why there could be no world soul. Carlin's contention, in Brown's words, is that Leibniz denies a soul to the world but not to bodies (themselves infinite aggregates) on the grounds that "while both the world and [an] aggregate of limited spatial extent are infinite in multitude, the former, but not the latter, is infinite in respect of magnitude and hence cannot be considered a whole" (Brown 1998, 115). Brown casts doubt on this interpretation—or rather, he begins by questioning its adequacy as an interpretation of a central passage, only to concede its essential correctness as an interpretation of Leibniz's position, and to turn his attack on the latter itself. In this note I shall argue that Brown underestimates the subtlety of Leibniz's views on the infinite, and that Carlin is basically correct in his suggestion that the infinite magnitude of the world is what precludes it from having even the phenomenal unity that a body does.

The central difficulty, clearly identified by both Carlin and Brown, is this. Leibniz claims, in the *Theodicy* and elsewhere, that "what serves to confute those who make of the world a God, or who think of God as the soul of the world" is that "infinity, that is to say, the accumulation of an infinite number of substances is, properly speaking, not a whole, any more than infinite number itself". However, Leibniz maintained that corporeal substances are true wholes, consisting in an organic body and a principle of unity (a soul or entelechy), despite the fact that their bodies—like all bodies—are infinite aggregates. (Indeed, directly prior to the remarks in the *Theodicy* quoted above, Leibniz had just reasserted his characteristic doctrine that there are infinitely many creatures in the smallest particle of matter.) So, if a corporeal substance can have an body that is an infinite aggregate of substances, why can't the universe be a corporeal substance?

It is as a solution to this dilemma that Carlin offers his interpretation: unlike finite bodies, the universe is infinite in magnitude, and this is why it cannot be a whole. Against this Brown argues that Leibniz's denial of wholeness to the world appears to ride not on this distinction, but directly on his denial of infinite number. Leibniz, he suggests, denies that the actual infinite in magnitude can be a whole "for the same reason that he seems to think that an actual infinite in multitude..."
cannot be a whole, namely, that the existence of such wholes would imply that there is an infinite number, which, according to Leibniz, implies a contradiction" (116). But if this is so, then the above stated difficulty remains, and Leibniz's position is deeply inconsistent. He allows an infinite multiplicity in the case of bodies, whilst denying that this is the same as allowing infinite number. (In this connection, Brown approvingly quotes Russell's remark that Leibniz embraces the actual infinite "on the express ground that it does not lead to infinite number.") Yet in the case of the world, Leibniz equates the accumulation of infinitely many substances with infinite number, and uses this to deny that the world is a whole.

Brown is not reticent in suggesting a solution. Leibniz should have fully embraced infinite number, as was later done by Georg Cantor (who, ironically, cited Leibniz as his inspiration). Brown claims that Leibniz's "failure to embrace infinite number was due to an uncharacteristic failure of mathematical imagination on his part" (121), a failure due to his assumption that infinite numbers, like finite numbers, should obey the axiom that the whole is greater than the part. Applied to infinite sets, this leads to contradiction. Had Leibniz simply realized, as Dedekind was later to do, that an infinite set can be defined as one having a proper subset all of whose members may be put into 1-1 correspondence with all the members of the whole set, he would not have insisted on infinite sets being ruled out by their contradicting the part-whole axiom (122). Not only would this have been consistent with Leibniz's characterizations of bodies and systems of bodies as wholes (118), but, Brown contends, "Leibniz's other commitments should have forced him to accept that the world is a whole in a sense that does imply infinite number" (118), citing his numerous references to "the whole universe", to the interconnection of all things, to monads "representing the whole universe" with greater or lesser distinctness, and so forth. Brown ends by suggesting that to have accepted infinite wholes would not have committed Leibniz to admitting a world soul, for he could have blocked this inference by denying that the world is a body, for which he had independent arguments.

These are trenchant arguments, and it is apparent that Carlin and Brown, in their attempts to tackle the vexed issue of Leibniz's rejection of the world soul, take us right to the heart of Leibniz's whole philosophical position. But there are at least three independent issues raised by Brown's criticisms: (1) did Leibniz even have a consistent position in allowing infinite multiplicities but denying infinite number? (2) whether or not he did, shouldn't his other metaphysical commitments have persuaded him to accept that the world is infinite in a sense that does entail infinite number? and (3) even if he had a perfectly consistent position on the infi-

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finite, did he have a sound argument against the possibility of the world soul? I shall treat these issues in the order stated.

At the root of the first issue is Galileo’s Paradox, and Leibniz’s resolution of it. Leibniz first proposed his resolution in 1672 while examining Galileo’s *Discorsi*, and its centrality to his philosophy of the infinite is signified by his habitual references to it when discussing the infinite. The argument in question is not original with Galileo, but in the *Discorsi* he proposes it with particular clarity in the course of demonstrating that neither greater, smaller, nor equal to, are attributes befitting the infinite. Leibniz summarizes the demonstration as follows, albeit reaching a different conclusion:

Among numbers there are infinite roots, infinite squares, infinite cubes. Moreover, there are as many roots as numbers. And there are as many squares as roots. Therefore there are as many squares as numbers, that is to say, there are as many square numbers as there are numbers in the universe. But this is impossible. Hence it follows either that in the infinite the whole is not greater than the part, which is the opinion of Galileo and Gregory of St. Vincent, and which I cannot accept; or that infinity itself is nothing, i.e. that it is not one and not a whole (A VI.iii 168; cf. also A VI.iii 550-551).

That is, given 1-1 correspondence and the part-whole axiom, treating “all the squares” or “all the numbers” (or any other infinite collections) as wholes leads to contradiction; therefore “there is no maximum in things, or, what is the same thing, the infinite number of all unities is not one whole” (A VI.iii.98), i.e. no infinite aggregate is truly one or a whole, and there is no such thing as infinite number. The contradiction can also be avoided if, with Galileo and St. Vincent, one rejects the part-whole axiom; then infinite collections can be treated as wholes, but ones not comparable with respect to quantity. The third (Dedekindian) option proposed by Brown, namely to discard the part-whole axiom and define equality in terms of 1-1 correspondence, does not occur to either Galileo or Leibniz. The reason for this, I suggest, is that for them the part-whole axiom is constitutive of quantity.

This is perhaps easier to see if we turn to a consideration of the continuous magnitudes of geometry, to which the part-whole axiom unambiguously applies. Here one can construct an analogous paradox, as Leibniz does immediately prior to expounding Galileo’s Paradox in each of two major pieces on the problem of the continuum in the 1670s (“On Minimum and Maximum” (1672), A VI.iii 97-8, “Pacidius to Philalethes” (1676), A VI.iii 549-550). (I shall call this “Leibniz’s Diagonal Paradox”, though again it is not original with him.) Suppose lines to be

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infinite aggregates of indivisible points. Now the points on the diagonal of a rectangle can be put into 1-1 correspondence with the points on one of the sides by drawing lines parallel to the bottom line of the rectangle, so there are as many points in one as the other; yet the magnitude of the side is clearly less than the magnitude of the diagonal, i.e. is equal to a part of it. Thus the part is equal to the whole, contrary to the part-whole axiom assumed as a premise. Since we began with well-defined wholes, the contradiction must result from supposing them to consist in infinitely many indivisible points. It therefore follows that the lines cannot be composed of points or indivisibles.

Here if one seeks to avoid the contradiction by taking the 1-1 correspondence between the points in the side and the diagonal to establish their equality in number, then, in default of a theory of measure, it appears the indivisibles of unequal lines must be of unequal magnitudes. Galileo, acutely aware of the paradoxes of the composition of the continuum attendant on that supposition, instead declares that the indivisible points composing a line are *parti non quante*, i.e. do not have quantity, even though an infinite number of them do. Thus whereas Leibniz proposes that an infinity of discrete units do not compose a true whole, and that there are no actual infinitesimal parts of a continuous whole, Galileo proposes that infinite totalities of discrete units are wholes, but incomparable with respect to quantity, and that continuous wholes have actual infinitesimal parts that are similarly quantitatively incomparable. Both positions are consistent, and avoid the paradoxes.

Nevertheless, Galileo was not able to maintain his position consistently. In order to demonstrate perhaps the most important and influential theorem of the *Discorsi*, the mean-speed theorem (EN 208-9), he found it necessary not only to assume infinitesimal elements with quantity (the “degrees of speed” at different instants), but that infinite aggregates of “all the lines” in two figures representing these degrees of speed could be equated on the grounds of their 1-1 correspondence, in contradiction to his claim about equality not applying to infinite aggregates. However, when Cavalieri founded his *Geometria Indivisibilibus* on similar notions, taking proportions between infinite aggregates of indivisible lines passed through one at a time by a moving line, Galileo had refused to countenance them, citing the paradoxes attendant on assuming indivisibles with quantity.

Such paradoxes were a matter for concern to Leibniz already in 1672 when he devised his Diagonal Paradox, for the previous year he had published his “Theory of Abstract Motion” containing a theory of the composition of the continuum from indivisibles that owed much to Cavalieri. There Leibniz had ascribed quan-
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tities to his infinitely small magnitudes, so that unequal points or conatuses can be in the same ratio as the lines or speeds they compose. But the above demonstration with the diagonal proves that these infinitely small elements of lines or speeds cannot be minima or indivisibles, after all. Leibniz subsequently resolves this difficulty (to cut a long story short) by his creation of the differential calculus, in which infinitesimals are infinitely small fictional parts into which a continuous whole can be resolved, but not infinitely small actuals or elements that compose into the whole.

In sum, Leibniz had good reasons for upholding the part-whole axiom as constitutive of quantity, and for rejecting the assumption that an infinite aggregate is a whole. We have seen how this entails the impossibility of the actually infinitely small as well as the infinitely large. But this doesn’t yet dispose of the issue of consistency. How, one might object with Brown, can Leibniz deny that his infinite multiplicities have an infinite number of terms? How can a body be infinitely divided and yet not have an infinite number of parts?

To answer this I need to say a few words about infinite series, since these serve Leibniz as a mathematical model for his idea of infinite division (as do continued fractions). In his early work with infinite series he had become convinced (following Gregory of St. Vincent) that Galileo and Gassendi were wrong to assert that an infinity of finite quantities would necessarily result in an infinite quantity. A geometrically decreasing series, such as $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$, has a sum, in this case, 1, as does any converging infinite series, such as Leibniz’s own alternating series for $\pi/4$. In the same way, then, any part of a body can be further divided to infinity, and yet the body can have finite extent. But doesn’t the fact that converging series have a sum show that they are wholes, and have an infinite number of terms? By April 1676 Leibniz had a ready answer:

Whenever it is said that a certain infinite series of numbers has a sum, I am of the opinion that all that is being said is that any finite series with the same rule has a sum, and that the error always diminishes as the series increases, so that it becomes as small as we would like. For numbers do not go absolutely to infinity, since then there would be a greatest number. But they go to infinity when applied to a certain space or to an unbounded line divided into parts.

In the same vein, Leibniz is easily able to talk of infinite multiplicities, for instance, of the divisions within a body, without committing himself to infinite number: in such a case there are more divisions than any assignable number. However, there is no last division, just as “there is no last number of an [infinite]..."
series, since it is unbounded”, so we must conclude “that an infinity of things is not one whole, i.e. that there is no aggregate of them”. 14

Thus Leibniz adopts the subtle position that there is an actual infinity of things, if infinity is understood syncategorematically, so that there are more things than any assignable number, but there is no infinite collection of things. This allows him, as he says in the passage quoted by Carlin (23, n. 23), to enunciate things about the infinite, provided he does so “in distributive mode”, and not collectively. “So it can be said that every even number has a corresponding odd number, and vice versa; but it cannot on that account accurately be said that the multiplicities of odd and even numbers are equal.” There is nothing inconsistent about this position. Indeed, as Russell remarks, the principle that infinite aggregates have no number “is perhaps one of the best ways of escaping from the antinomies of infinite number” (which we can assume Russell knew all too well by the time he published the second edition of his famous book). 15

Turning now to the issue of the compatibility of the denial of infinite number with Leibniz’s wider metaphysics, we can see that Brown systematically underestimates the subtlety of Leibniz’s position when he takes his references to “all the substances in the world”, “the whole universe (tout l’univers)”, and so on, to commit him to infinite number. To take one example, Leibniz’s much beloved doctrine of the “connection of all things” is easily interpreted on the above principles: if each thing is connected with those with which it is in contact, and there is no vacuum, then “all things are connected” is perfectly intelligible without the supposition of an infinite collection. Leibniz’s other references to a thing’s “having a relation to all of the other substances in the universe”, and so forth, can be taken in the same vein. Moreover, the same principles can be used to explain how Leibniz can speak of matter as an infinite aggregate of substances without it committing him to an infinite collection. For the argument for this infinitude is that every portion of matter, however small, contains a substance or substances within it. This argument involves only the syncategorematic infinite, and the idea of monads distributed everywhere in matter, but not matter as a collection of monads. Thus one can talk about an infinite by division, where the parts have some law governing their connection, and by means of which they can be integrated into a whole, provided this whole is not an infinite collection of substances. But then this unity is determined by the law or principle governing the connection of the parts, not by an accumulation of elements. However, one cannot talk about the infinite by addition, such as would be necessary for an infinite magnitude to exist.

Thus we see that for Leibniz the denial of infinite number, in the sense of a
completed collection or whole, is not a result of “failure of mathematical imagination”, but lies at the creative heart of his system. It is equivalent to denying the existence of a last term, even an infinitieth, in an infinite series, which is consequently conceived as approaching its sum as a kind of ideal limit. Similarly, there is no bound to an infinite division, such as occurs in any body, so that the infinite parts of a body do not constitute it as an infinite collection or true whole. Consequently, a Cartesian body, regarded as pure extension, is not something complete, and cannot be a substance. Such a body is a purely phenomenal whole, whose unity comes only through its being perceived as one whole by an act of perception external to it. A corporeal substance, on the other hand, has a non-material unifying principle internal to it, organizing its constituent parts into one in a similar way to which the law of a converging series allows its infinite multiplicity of terms to be summed into a finite quantity.

This concludes my brief historical excursus into the genesis of Leibniz’s philosophy of the infinite, which I hope is enough to establish the material equivalence for him of the following: No infinite number \(\equiv\) no infinitely large magnitude \(\equiv\) no infinitely small actual \(\equiv\) no last term in an infinite series \(\equiv\) no totality of parts in an infinite division. Correlatively, we are now in a position to see what the consequences would have been had Leibniz followed Gregory Brown’s recommendation and embraced infinite numbers and wholes: There would be an infinite number of infinitesimals in a finite quantity; therefore infinitesimals would be actual parts, and Leibniz’s philosophy of mathematics would have been completely different. Bodies would be real wholes, since there would be a determinate, though infinite, number of parts into which they were divided. Matter would therefore be real, and would not need immaterial principles to complete it. The monadology would be unneeded. By extension, human bodies would form real unities without the need of immaterial souls, and likewise there wouldn’t even be a need for a world soul—materialism and atheism would reign triumphant! Now I do not say that Leibniz might not have found other ways to block these inferences, as did his contemporaries; merely that, had he done so, he would have produced a system unrecognizable as the one we know as Leibnizian.

Of course, establishing the failure of Brown’s suggested remedy does not automatically resolve the difficulty both Carlin and Brown were trying to address, (the third issue identified above), so let me return to that now. Leibniz denies the world could have a soul on the grounds that it is an infinite accumulation of substances, and is therefore, as an infinite totality, not one and not a whole. But how then can a finite body, also an infinite aggregate of substances, have a corre-
sponding soul?

I believe Brown is correct to suggest that Leibniz’s denial of infinite number (infinity understood collectively) precludes a body’s being a true whole just as surely as it precludes the world’s being a true whole. But Brown appears to assume, falsely, that Leibniz wants finite bodies to be true wholes, which is why he insists Leibniz would have done better to have accepted infinite number. The question, though, is why the material universe cannot be the body of a corporeal substance (whose corresponding soul would be God). Here I think Carlin gave the right answer when, after quoting Leibniz to the effect that the world, being infinite in magnitude, “would not be a single body, nor could it be regarded as an animal”, he explained that, unlike finite bodies, “the actual infinite [in magnitude] does not even possess what Leibniz would call arithmetical unity” (Carlin, 11).

This tallies with my analysis above. A finite body can comprise an arithmetical unity, though not a true one, because the parts within parts, each of which contains either a substance or an aggregate of substances, are progressively smaller. Consequently they can “sum” to a finite quantity in the same way that a converging infinite series can, without there being an infinite number, or, equivalently, a last part or last term of the series. But if every corporeal substance is contained within a larger body, the analogous series for the whole world is divergent. Consequently such a world would not possess even the arithmetical unity requisite for its semi-reality as a body or well-founded phenomenon. Thus Leibniz’s philosophy of the infinite does allow him to conclude that the world cannot be a body: if it were infinite, there would be infinite magnitude, and therefore infinite number, which he has rejected.

This argument, however, is only as strong as its main premise, that the universe is actually infinite in magnitude. Leibniz could have held the world to be of finite size without violating any of the principles of his philosophy of the infinite—but not without violating his principle of plenitude: wherever God could have made more things without compromising the rest, he would have done so. To have rejected this principle, however, would have been no small concession, for not only is this the same principle Leibniz used to justify infinite containment—“given the plenitude of the world, it is necessary that there exist some globules smaller than others to infinity”—it is closely related to the principle of sufficient reason. As he wrote concerning space in 1676, “since there is no reason determining or limiting its size, it will be the greatest it can be, i.e. absolutely infinite” (A VI.iii.585).
In conclusion, I believe that Leibniz had a consistent position on the infinite, and (given his characteristic principles) a sound argument on that basis against the possibility of a world soul.

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Notes
1 I am indebted to Pauline Phemister for her perceptive criticisms (not all of which I can claim to have met) in an engaging correspondence on the issues treated here, and also to Laurence Carlin for his comments on an earlier draft of this (unsolicited) defence, which I received just as I was finishing the final version. All translations given here of Leibniz, Galileo, and Gassendi are from my book, *Leibniz: the Labyrinth of the Continuum*, forthcoming with Yale University Press.
3 I am trying hard not to misrepresent Brown. He says “I don’t see how Leibniz’s apparent argumentative strategy here [viz, in *Theodicy*, §195] can be squared with Carlin’s claim that the reason Leibniz thought that an aggregate of infinitely many substances of finite extent could possess a soul but that the world could not is [that] the latter, but not the former, is infinite in extent” (117). But he ends by saying: “I do not deny that the argument that Carlin elucidates in his paper is one that Leibniz himself adopted, for it clearly is. Rather I have argued that it is not an argument that fits well with Leibniz’s other commitments...” (124).
4 T, p. 249.
5 For a different perspective on this issue, the reader is encouraged to consult O. Bradley Basser’s stimulating article, “Leibniz on the Indefinite as Infinite”, *Review of Metaphysics* 51 (June 1998), 849-874.
6 Leibniz alludes to this demonstration (or simple variants on it), for instance, in the letters to Malebranche and to Johann Bernoulli quoted by Brown (Brown, 122), and misattributes his own resolution of it to Galileo in the passage from Grua quoted by Carlin: “Galileo demonstrated [infinite number] to be neither one nor a
whole (Grua 558)” (Carlin, 7).

7 “I believe that these attributes of greatness, smallness and equality do not befit infinities, about which it cannot be said that one is greater than, smaller than, or equal to another. To prove this I recall a similar discussion I once had...” (EN 77-78; references for Galileo’s Discorsi in the text and notes are keyed to the Editione Nazionale—EN—of his Opera).

8 Negative roots are excluded, since we are considering only natural numbers. Of course, one could easily adapt the argument for integers.

9 Galileo has Salviati conclude: “the attributes of equal, greater, and smaller have no place among infinities, but only among bounded quantities” (EN 79). Gregory of St Vincent’s opinion is to be found in his Opus Geometricum, 1647, lib. 8, pr. 1, theorema, p. 870 ff.

10 “Given that the line, and every continuum, are divisible into ever-divisibles, I do not see how to escape their composition being from infinite indivisibles. ... And the existence of infinitely many parts entails as a consequence their being unquantified” (EN 80).


12 “[I]ninitely many quantified things make an infinite extension... But it is clear that quantified parts actually contained in their whole, if they are infinitely many, make it of infinite magnitude”, Galileo Galilei, EN 80-81; “when a finite body is assumed, if the number of parts into which it is divided is nonetheless not finite, we may then understand there really to be in it infinite parts; whence the whole resulting from these parts is infinite, which is contrary to the supposition”, Pierre Gassendi, Animadversiones (Guillelmus Barbier: Lyons, 1649), 411. Johann Bernoulli makes a similar argument in his correspondence with Leibniz (GM iii.529); see Bassler’s discussion, “Leibniz on the Indefinite as Infinite”, 861.

13 “Infinite Numbers”, A V.i.iii 495-504: 503. In this paper Leibniz even anticipates Bernoulli’s later objection that although the last number of an unbounded series could not be finite, it might be an infinite number: “I reply, not even this can exist, if there is no last number. The only other thing I would consider replying to this reasoning is that the number of terms is not always the last number of the series. That is, it is clear that even if finite numbers are increased to infinity, they never—unless eternity is finite, i.e. never—reach infinity” (504).

14 Here Leibniz says that the fact that an infinity of things is not one whole means that there is no infinite aggregate of them; elsewhere he expresses the same view by saying that there is an infinite multiplicity (multitudino) of things, but not an
infinite number of them. This is the same thought, expressed in different terminology. I think Leibniz would have done better to stick with the former way of expressing himself, since “aggregation” connotes “collection”.

15 Bertrand Russell, A Critical Exposition of the Philosophy of Leibniz, 2nd edition (1937), London, Routledge, 1992, p. 117; this remark about the antinomies of infinite number is not among the points he corrects for their mathematical naivety in his preface to that edition.

16 For a fuller elaboration of these points, see my contribution to last year’s Leibniz Society Review, “Infinite Aggregates and Phenomenal Wholes: Leibniz’s Theory of Substance as a Solution to the Continuum Problem”, 8, 25-45.


18 “[T]he same argument should apply as well against there being any infinite accumulation of substances that could possess a soul—including such accumulations [as] appear to have only a finite magnitude. And if [Leibniz] should attempt to avoid this consequence by asserting that ... [infinite accumulations of substances] that don’t give rise to infinite magnitudes can be wholes, then it seems that his argument against infinite number would collapse.” (117).

19 Strictly speaking, such a sequence of bodies being contained by larger bodies would not have to be divergent; but not only would there be no reason for a limiting magnitude of body, but the idea of a sequence of large corporeal substances encased in bodies that are greater than them by decreasingly small amounts, goes against the whole spirit of Leibniz’s speculative metaphysical vision. On the other hand, as Laurence Carlin has pointed out to me, Leibniz did conceive of the world as approaching an asymptote temporally (see G.ii.507; AG 192). Also, most of the analogies with infinite series that he gives in his correspondence with De Volder are to the states of monads and the world unfolding through time like a preprogrammed mathematical series (see e.g. G.ii.259; L 533).

20 Note that my argument is couched in terms of a reading of Leibniz that takes for granted the reality of corporeal substance, i.e. an immaterial soul together with its organic body. On the usual idealist reading of Leibniz, where bodies are mere aggregates of purely immaterial monads, there appears to be no basis for the dis-
tinction between infinite aggregates yielding (i) finite bodies, or (ii) an infinite non-entity. But I take that as further evidence against the immaterialist position and for the corporeal substances view, sketched in my “Infinite Aggregates” paper, and expounded independently and more eloquently by Pauline Phemister, “Leibniz and the Elements of Compound Bodies”, *British Journal for the History of Philosophy*, 7, 1, (February 1999), 57-78.

21 This example of his characteristic doctrine is drawn from an unpublished piece written in late April 1676 (A VI.iii 525).

22 Leibniz’s use of the term ‘absolutely’ here is not very happy. Note, however, that he does not contradict himself in saying that the world is infinite in magnitude. As he says in the passage quoted by Carlin, this means “that it extends beyond any magnitude that can be assigned” (G.ii.304): the world is syncategomatically infinite, but not categorematically so.