Infinite Aggregates and Phenomenal Wholes: Leibniz's Theory of Substance as a Solution to the Continuum Problem[1]

Richard Arthur, Middlebury College

I

There have been some excellent analyses of Leibniz's theory of substance of late.[2] And although there are still disputed questions, the mutual agreement has been so great, and the studies themselves so scholarly and thorough, that one has to wonder whether the remaining disagreements might simply be a consequence of trying to force more consistency on Leibniz's thinking than it can sustain.[3] This may be so; but even if it is, I do not think we can permit ourselves the luxury of assuming it, especially when a slight change of perspective might help to resolve some of the remaining difficulties. It is in that spirit that I offer this contribution to the debate, an attempt to get a fresh angle on some of the difficulties with Leibniz's theory of substance by considering how it emerges in connection with his attempts to resolve the difficulties of the composition of the continuum in the formative period of his philosophy. I shall be concentrating particularly on some very interesting papers Leibniz wrote in Paris in February-April of 1676—his Paris Spring, as it were.

The main problem of interpretation I wish to address is as follows. Leibniz claims that bodies are phenomena, and also that bodies are aggregates of substances. As Robert Adams points out, many commentators have seen these two theses as being irreconcilable, and have accused Leibniz either of vacillation or inconsistency, or have sought to explain the incompatibility by attributing to him different theories of substance at different periods in his intellectual career. In opposition to this, Adams holds that Leibniz believed the two theses to be consistent, and continued to believe this till the end of his career. I concur. Adams also maintains that "Leibniz thought that bodies are phenomena precisely *because* they are aggregates of substances" (*Leibniz*, 218). Again, I concur; only I would add that on Leibniz's account bodies as phenomena are necessarily *infinite* aggregates; and that this is a consequence of the fact that Leibniz's theory of substance is considered by him to be a solution to the *problem of the continuum*.[4]

There is a second related problem of interpretation that I shall come back to at the end of this paper. This concerns the status of *corporeal substances* in Leibniz's ontology. As is well known, Leibniz claimed (as in the "Monadology" of 1714) that the primary constituents of the created world are immaterial, partless, simple substances, which he called *monads*, each of whose states (or perceptions, in a suitably wide sense of 'perception') represents the whole of the rest of the universe. Each monad consists in a series of such states, together with a primitive active force, or endeavour to pass into the next state in the series, also called by Leibniz an 'appetition'. All else—the bodies, motions, and other phenomena we perceive—are said by Leibniz to "result" from simple substances and their modifications.

But the clarity of this ontic landscape is immediately obscured by Leibniz's further claims—often made in the same passages in which he depicts monads as the basic reality—that there exist *corporeal substances*. A corporeal substance is a unity per se "consisting in a simple substance or monad (that is, a soul or something analogous to a soul) and an organic body united to it" (G.vii.501). This is similar to (and confessedly indebted to) the Aristotelian notion of a substance as a union of form and matter.

This raises obvious difficulties of consistency. One response is to see this accommodation to Aristotelian realism as characteristic only of the middle period of Leibniz's philosophy. Thus Garber sees Leibniz's robust commitment to corporeal substances in the 1680s and '90s giving way in his later years (from about 1704) to the idealism of the monadology ("The Middle Years", 91), and likewise Wilson sees a conflict between corporeal substances and monads being resolved in favour of the monadology after about 1703 (*Leibniz's Metaphysics*, 192-4). In contrast, Adams denies that the Aristotelian elements in Leibniz's thought are inconsistent with the monadological foundation, or that there is any "major change in Leibniz's thought from his middle to his later years on this point" (*Leibniz*, 308). On his reading, though, an organic body is an aggregate of immaterial substances, so that a corporeal substance, being a composite of an organic body and an immaterial soul, is a construction solely from simple substances and their modifications. The apparent concession to realism in Leibniz's upholding of corporeal substances is dismissed as heteronomous to his philosophy, "an accommodation to traditionalist concerns of others, especially Roman Catholics" (307). Similarly, Rutherford concludes his study by relegating corporeal substances to Leibniz's *discours éxoterique*, a message tailored "to suit the needs and expectations of his audience" (*Rational Order*, 282). "Leibniz's deep metaphysics" he claims, "is the metaphysics of monads, in which all other beings, including living creatures, are no more than 'phenomena' and 'results'" (282).

In opposition to both these readings, I shall argue that Leibniz's commitment to the reality of corporeal substances is steadfast. He neither abandons them later in life, nor is he less than earnest in maintaining that they are true unities.[5] Although "ultimately" all God needs to posit are the simple substances from which compound substances result, once he has created simple substances he has *ipso facto* created corporeal substances too. And since each corporeal substance is formed into a true unity by its dominant monad, it is a substance, and not a mere phenomenon.

Let me begin my argument with two preliminary questions. First, why are bodies infinite aggregates, according to Leibniz? And second, how, and in what sense, does this make them phenomenal?

The answer to the first question is actually quite simple: bodies are infinite aggregates because every body is infinitely divided.[6] The infinite dividedness of matter is a thesis that Leibniz adopts very early in his development, in 1666, if not earlier still.[7] Its origins are perhaps Aristotelian; but the only argument Leibniz ever offers depends on premises of Cartesian provenance: (i) each body or part of matter, being acted on by all others in the plenum, is agitated by its own motion; and (ii) matter is at each moment actually divided by the differing motions of its parts. From these premises both Descartes and Leibniz conclude that matter is infinitely divided. But whereas Descartes had allowed that some bodies might be undivided, and that it is only necessary for the fluid matter between them to be *indefinitely* divided in order to preserve the continuity of matter flowing through unequal spaces, Leibniz concluded that, since every body will have parts moving with different motions, every body is actually divided into an infinity of smaller bodies. The argument is given most explicitly in a fragment of the early 1680s, "Created things are actually infinite":

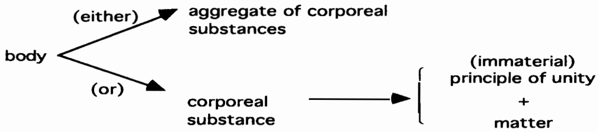
> Created things are actually infinite. For any body whatever is actually divided into several parts, since any body whatever is acted upon by other bodies. And any part whatever of a body is a body by the very definition of body. So bodies are actually infinite, i.e. more bodies can be found than there are unities in any given number (Ve254: 1129).

Now by itself this yields no conclusion as to the phenomenality of bodies. So this brings us to my second preliminary question: how, and in what sense, is the phenomenality of bodies supposed to follow from this? Here Leibniz avails himself of a second premiss: "substance is indivisible"; or, in an alternative formulation, "no being that is really one is composed of a plurality of parts" (Ve284, ca. 1685).[8] From this together with the infinite dividedness of bodies it follows that no body is a substance. For, by the above argument, every body is composed of a plurality of parts. So body, qua body, is not "really one", is not a true whole. Now by Leibniz's lights, any entity that appears to the senses as one whole, but is not really one, is a phenomenon (a phenomenon is an entity appearing to the senses that is not a substance; or, is something that exists only in relation to us). Ergo, bodies are phenomenal.

Here it must be stressed that this argument does not preclude a body's *containing* substances, true unities, organizing principles. What it entails is that if there is nothing more to body than the usual defining properties—extension, motion, shape, magnitude—in short, if body is taken as mere *bulk*, then it does not contain any indivisible units.

Naturally, Leibniz does not believe that there are no unities in bodies. He himself is a unity whose principle of unity is the soul, and so by analogy are other thinking things. And why would God give only human bodies a true unity if he could give all kinds of bodies unity too, thereby increasing the amount of perfection in the world? "To attribute a soul only to man and a few other bodies is as inept as believing that everything is for the sake of man alone" (Ve 445: 2039, ca. 1680). Here the appeal to the principle of plenitude is complemented by an implicit argument that Leibniz makes explicit elsewhere. Suppose only humans (and perhaps angels) are true unities. Then all other apparent unities, whether rocks, planets or animals, will be only aggregates of their parts; but worse, since they contain no true principles of unity whatsoever, there are no substances of which they could be aggregates. So they will be completely insubstantial, mere phenomena. Thus on the one hand Leibniz thinks it probable that we are not the only corporeal substances, or bodies endowed with a principle of unity. However, he knows that he has no means to prove this to the skeptic. Nor can he prove from the appearances whether any particular body, such as a sheep, contains a soul or some other organizing principle, or is only an accidental unity. Hence the customary hypothetical formulations, "either bodies are mere phenomena, or there must exist in them some true unites" (cf. Ve125: 487), "if beasts are not mere machines, it is necessary for them to have substantial forms, and these are called *souls*" (Ve107: 418). Nevertheless, Leibniz believes there is a strong presumption—one that he expects his correspondents to share—that bodies are not mere phenomena, and that there must ultimately be substances of which they are aggregates.

Now if a body is not to be a mere phenomenon, it must either be constituted as a true unity by some internal organizing principle, or be an aggregate of such unities. But by the argument from infinite division, no body that is a true unity can be purely material. It therefore follows that whatever confers unity on a given body, its principle of unity, must be non-material. This then gives:



This corresponds to what Sleigh calls "Leibniz's corporeal substance theory"—except that I see no reason to accept Sleigh's stipulation that in Leibniz's middle period "extension, motion and shape...are not [further] analyzable into properties of soul-like entities" (*Leibniz and Arnauld*, 99). In fact, as I hope to show here, Leibniz gives arguments for the non-primitiveness of these "primary qualities" based on the infinite division of bodies as early as 1668, arguments whose connection with the problem of the composition of the continuum is made very clear in the papers of 1676.

Before proceeding to the historical part of my argument, though, I want to make one or two clarifications. Leibniz's axiom that "nothing that is aggregated can be a real being" entails that even a pair of substances is not a real being. But this does not make it a mere phenomenon. So we should distinguish two senses in which a body may be called phenomenal. The first might be called the Democritean sense:[9] only true unities are substances, so that all aggregates are *entia per accidens*, and are wholes by convention only. In this sense, a herd of sheep, or even a pair of monads, is not a real being. But this means that the whole formed from them is not real, not that the individuals from which it is formed are phenomenal. Body in this sense of a being by aggregation (let's denote it 'body$_2$') is called by Leibniz "secondary matter".[10] By contrast, insofar as bodies are conceived as purely geometrical or material[11]—as what Leibniz calls "primary matter"—there are no true units from which they might be aggregated at all. "Matter itself, per se, or bulk, which you may call primary matter, is not a substance, and not even an aggregate of substances, but something incomplete" (to Johann Bernoulli, Aug or Sept 1698; GM.iii.537). Leibniz often talks of "pure phenomena" in this connection, although in his rigorous terminology such a purely mathematical body ('body_1') is an *ideal* entity.

It should be stressed that Leibniz is not committed to phenomenalism in the second sense. His position is distinguished from the idealism of Berkeley precisely by the claim that bodies are not mere phenomena, but aggregates of real unities.[12] Nevertheless, bodies are phenomena in a more acute sense for him than for Democritus, in that they are *infinite* aggregates. Crucial here is Leibniz's denial of infinite number. For this means, as we shall see, that bodies are not aggregates in the sense of completed collections; they are not composed out of the ultimate unities into which they can be resolved. For no matter how large a number of unextended monads one adds together, one will not get an extended body; but an infinitude of monads is presupposed by the merest body.

Lastly, all of this concerns body in its spatial aspect, or taken *synchronically*, in Adams's apt terminology (*Leibniz*, 231). But bodies will also be phenomenal *diachronically*, or taken over time. For a substance, according to Leibniz's understanding of it, is something that remains the same over time. However, the very same argument that establishes the infinite division of matter also demonstrates that body is not the same over time. For if each body or part of matter is individuated by its motion, and the motion it acquires at each instant as a result of the different momentaneous endeavours acting on it differs from one instant to the next, then body is not the same from one instant to the next. Therefore, insofar as it is purely material, it is not a substance but an *ens per accidens*, an accidental unity of the different material forms it takes on at these different times. Whatever unity it has is either conferred on it by a mind recognizing it as the same according to some convention, or is imparted to it by some non-material principle which somehow unites its different forms into the same individual over time.

II

Let me now try to illuminate Leibniz's understanding of infinite aggregates by turning to the historical genesis of this doctrine in his unpublished papers of 1676 and the few years following. To this end I shall need first to situate the arguments of these papers by considering some of the earlier development of his views. This can be motivated quite neatly by considering how the logic of the above argument might be resisted.

Clearly the analysis depends completely on the infinite dividedness premise, and on Leibniz's interpretation of infinite aggregates as being incomplete: without the first, a body might be composed of a finite number of substances, and without the second, it might be understood as a completed infinite collection. This infinite collection, in turn, could be understood as composing either a continuous body, or a merely discrete one. This gives us three ways in which one might challenge the infinite division thesis as interpreted above:

(i) there might simply be a limit on the dividedness of matter: body would then consist in atoms or collections of atoms.

(ii) the continuum might be infinitely divided into indivisibles: body would then be composed of its constituent indivisibles.

(iii) body might be divided into infinitely many minima, in such a way that no true whole results.

Interestingly, versions of all three of these options are vigorously explored by Leibniz before he reaches his mature solution. Let us consider them in order.

The first option, atomism, was that proposed by the dissident Cartesian, Gerauld de Cordemoy, whom Leibniz often mentioned in connection with his "aggregates" argument.[13] In company with Leibniz (and with a long tradition from Parmenides through Democritus, Aristotle and Plotinus), Cordemoy held that substance must be unchangeable, partless, indivisible and truly one; but, unlike Leibniz, his atoms are purely material. In fact they are simply Cartesian extended substances, except for the fact that they are indivisible.

Now Leibniz, who calls Cordemoy a "semi-Gassendist", had himself subscribed to Gassendian atomism in his youth. But early in his intellectual development, he had developed an argument based on the principle of individuation by which he sought to show that it is impossible for a purely material body to be recognized as the same over time.[14] He brings this argument to bear against Cordemoy in his reading notes of 1685 on the latter's *On the Distinction between Body and Mind.* Suppose, Leibniz says, that two triangular atoms compose to form a square body; and suppose there is another square corporeal substance, or atom, equal to the first body. "In what respect, I ask, do these two extended things differ? Certainly no difference can be conceived in them as they are now, unless we suppose something in bodies besides extension; rather they are distinguished solely by memory of their former conditions, and there is nothing of this kind in [Cordemoy's] bodies" (Ve 157: 696-7).

There is a simple solution to this objection, and that is to grant that there is something more to body than mere bulk, namely a kind of mental principle, by means of which an atom would "remember its former conditions". It is no coincidence that Leibniz had posited precisely such a mental principle as individuating each of his corporeal substances or atoms in the late 1660s.

In a series of works whose general title is the *Catholic Demonstrations*, Leibniz argues that the principle of activity and unity in human bodies is the mind, but that an analogous mental principle is required in non-human bodies too. He identifies this as the *concurrent divine mind*, that is, as an Idea peculiar to each body, but existing in the Universal Mind, or God (A VI.i 508-512). Of particular interest to us here is Leibniz's argument for God in part I, the *Confession of Nature against the Atheists* of 1668, from the inadequacy of mechanism. Everything should, he concedes, be derived as far as possible from the nature of body and its primary qualities: magnitude, shape and motion (L 110). But the origins of none of these can be found without supposing an incorporeal principle. In the case of motion, the argument is essentially Aristotle's argument for the prime mover. But a similar infinite regress argument can be applied to magnitude and shape: either these are due to an incorporeal principle, or have their origin in motion. But the latter alternative presupposes bodies with their own magnitudes and shapes due to prior motions of yet other bodies, and so on to infinity. Thus on this basis "no full reason for a body's shape will ever be given" (111). Considerations of the harmony among the magnitudes, shapes and motions now serve to establish that the incorporeal principle must be the universal mind, or God (112).

Leibniz also applies this reasoning to typical mechanical acounts of cohesion of matter, such as the atomistic ones in terms of hooks and crooks. For what explains the cohesion of the atoms, and their hooks and crooks themselves? Are we to explain this in terms of hooks and crooks of smaller particles within? Without recourse to a cause lying outside the regress, he claims, no account can be given for the cohesion or firmness of the "ultimate corpuscles" one assumes.[15] "So in providing a reason for atoms, it is right that we should have recourse to God, who is responsible for the firmness in these ultimate foundations of things. And I'm surprised that neither *Gassendi* nor anyone else among the very acute philosophers of our age has noticed this splendid occasion for demonstrating Divine Existence" (A VI.i 492).

A little afterwards, Leibniz also locates the principle of motion in God, analyzing motion as the creation of each body at each assignable instant, with the body being at rest and therefore non-existent at each intervening, unassignable time.[16] Thus his position in the late 60s is radically discontinuist: atoms, whose cohesion is due to God, and whose motion depends on continual divine re-creation.[17]

By 1670, however, this theory has collapsed, partly as a result of internal difficulties, but also under the influence of Hobbes, whom Leibniz had studied assiduously at this time. In his new theory published in the *Theoria Motus Abstracta* in 1671, motion is restored as continuous by an analysis into *conatuses* or endeavours, infinitely small "beginnings of motion"; and the basis of cohesion is explained in terms of this theory. For a continuous body is actually infinitely divided into points or indivisibles (defined as "unassignable", or parts smaller than any that can be given, as distinct from the partless points of Euclid), with each point of a body proportional to the endeavour of the body at that point at that instant. One body coheres with an another if it has an endeavour to penetrate it, so that a cohering whole can be formed from a continuous string of indivisibles with different endeavours, as in a spinning corpuscle. With this explanation of cohesion, Leibniz is able to eschew atoms in favour of the theory of *bullae*, hollow but extremely hard spinning corpuscles that, unlike classical atoms, are not everlasting. Their spinning motion explains their "equatorial" cohesion, and their cohesion along the meridians is accounted for in terms of a principle of minimization of disturbance of motion.

This then represents a version of the second option outlined above: (ii) the continuum is infinitely divided into indivisibles, out of which a unified body is composed. But Leibniz is far from abandoning his minds. Indeed, he hopes to erect a science of mind on the correspondence of minds with indivisibles. For one of the main properties of mind in Leibniz's view is its indestructibility, and a point, being indivisible, cannot be destroyed: so "it will follow that mind can no more be destroyed than a point" (A II.i 114). Indivisibles, moreover, are individuated by endeavours, and Leibniz, inverting Hobbes, construes endeavour as thought, thus opening the door to "the true distinction between bodies and minds". For "every body is a momentaneous mind, i.e., a mind lacking *recollection*, since it does not retain its own endeavour and a contrary one together for longer than a moment." There is much else of great interest in this early theory, but I won't dwell any further on it here.

For by late 1671 Leibniz has abandoned the idea that these unassignable points are in fact indivisible. Perhaps as a result, in the period 1672-76 he reverts to atomism: the minds in bodies are again located in true atoms. His motivation for atoms from the principle of individuation is alluded to in notes written on March 18, 1676, where he mentions how matter's continually becoming something different can be proved from this principle.[18] This occurs in the process of a proof that "space is a whole or entity *per accidens*" because of the changing of its parts from one instant to the next, implying that matter as a whole must be too (392). But later in the paper Leibniz alludes to "other grounds" by which he has "established that there is some portion of matter that is solid and unbreakable", so that, given this and the impossibility of accounting for its connection by matter and motion alone, the portion must be constituted as "single and indissectible, i.e., an atom" by its possessing a single mind. The "other grounds" appear to be those he had given in "Secrets of the Sublime" the previous February, to which I will now turn.

There Leibniz had argued that since "the liquid matter of the surrounding plenum will immediately endeavour to dissolve" any body contained in it, it follows that if such a body is not an atom, or perfect solid, it will soon be dissipated (Aiii60: 473). This is because every body accepts every endeavour that impinges upon it. On the other hand, if such a perfect solid is at rest in a liquid, then the argument "from a solid in a liquid" entails that—even if the liquid consists in minute solid particles borne by an even more aetherial "perfect liquid"—the perfectly liquid matter will have to be infinitely divided. This alludes to Descartes' argument in his *Principles*, II, 33-35, mentioned above.

Here, as elsewhere, Leibniz interprets this actually infinite division as issuing in an actual infinity of "perfect points", rather than—as Descartes maintained—"indefinite particles". By "perfect point" he means "a body smaller than any that can be assigned", so that perfectly fluid matter will be "nothing but a multiplicity of infinitely many [such] points" (473). But, as noted, he has come to doubt the idea that these perfect points, each distinguished by its own endeavour, can be interpreted as indivisible elements of the continuum. He therefore concludes here that "a perfect fluid is not a continuum, but discrete, i.e., a multiplicity of points" (473). Accordingly, "matter is a discrete entity, not a continuous one; it is only contiguous, and is united by motion or by a mind of some sort" (474). Thus Leibniz is here entertaining an interpretation of infinite division of the third kind envisaged above: (iii) body might be divided into infinitely many minima, in such a way that no truly continuous whole results.

III

Now Leibniz is not whole-heartedly committed to this interpretation of the actually infinite division of body as being a division into minima. One can already see this in "Secrets". After reaffiriming his belief that the infinite division of matter would issue in an infinity of points, Leibniz immediately identifies a couple of consequences of this that demand further investigation: it would follow that "any part of matter is commensurable with any other", and from this, that the area of a circle would be commensurable with its diameter (474). He sets himself the task of investigating whether the first consequence "truly follows", and whether the second inference "is a good one"; a task which, as we shall see, he will take up in earnest some two months later.

But he also urges himself to consider an alternative interpretation: "whether, on the other hand, there does not follow in a liquid a subdivision that is now greater, now less, in accordance with the various motions of a solid in it." At first Leibniz equates this alternative with a division into "mathematical points", as opposed to "metaphysical points or minima". One might, he allows, take these to be "Cavalierian indivisibles". But against this, "if it can be shown that a liquid is divided to a greater or lesser degree, it will follow that it is not resolved into indivisibles." Thus he considers instead whether the infinitesimals and infinities of his calculus can be interpreted as actual. Perhaps, by a rigorous unravelling of the labyrinth of the continuum, it could "be demonstrated that there is something infinitely small, yet not indivisible. If such a thing exists, there follow some wonderful consequences concerning the infinite: namely, if we imagine creatures of another world that is infinitely small, we will be infinite in comparison with them" (475). "Since we see the hypothesis of infinites and the infinitely small is splendidly consistent and successful in geometry, this also increases the likelihood that they really exist."

Thus in February 1676 Leibniz is actively entertaining two interpretations of the composition of matter—a discrete multiplicity of minima, and a continuum composed of actual infinitesimals. Crucially, both depend on an interpretation of actually infinite division as *categorematic*, i.e., as issuing in an *infinite number* of parts: "Another way of establishing that there is necessarily an infinite number is when a liquid is actually divided into parts infinite in number: if this is impossible, it will follow that a liquid is impossible." But the paper ends with a proof that "there is no such thing as an infinite number" (477), thus throwing everything into doubt. The argument is that in a series of finite numbers each exceeding the last by one, "the number of numbers will equal the greatest number," which is finite by hypothesis. In fact, as Leibniz himself immediately realizes, "this proves only that such a series is unbounded" (477, fn. 2), that there is no last or greatest number. But this does not in itself resolve the issue of how to interpret the multiplicity of its terms.

When Leibniz returns to this proof at the end of the paper "Infinite Numbers", written in early April, he will drive it to a radical conclusion. It is true that there is no last number in an infinite series. But from this it follows that "it must be said that an infinity of things is not one whole, or that there is no aggregate of them" (Aiii69: 503). For, he argues, if you object "that in an unbounded series there exists no last finite number, although there can exist an infinite one, I reply: not even this can exist , if there is no last number" (504). But this "extremely subtle consideration" occurs as the culmination of a long argument. In order to see how he comes to deny the reality of infinite aggregates, we need to take into account some of the far-reaching changes that have occurred in Leibniz's position in the two months since writing "Secrets of the Sublime".

First, Leibniz no longer takes the success of the calculus to legitimate an interpretation of infinitesimals as actual unassignable points. Indeed, now he confidently defines a point as "that whose part is nothing, an extremum; for we have already shown that there is nothing else unassignable" (Aiii 69: 498). What proof Leibniz is alluding to here is unclear, although by the end of March he had proved to his own satisfaction that the differentials of his calculus are not infinitely small, but "nothing" (Aiii52: 434). And in "On Motion and Matter" (written just prior to "Infinite Numbers"), the fact that a curve is composed of an infinity of such differentials, alongside other considerations, had led him to suggest that curves exist only as "general entities".[19] Thus by the beginning of April, the questions he had set himself in "Secrets" concerning the commensurability of infinite aggregates of points and the squaring of the circle had taken on a greater urgency, and in "Infinite Numbers" he sets about trying to resolve them.

Can (the area of) a circle be understood as an infinite aggregate of differentials, and if so, will it not be commensurable with the square? In a difficult argument whose details need not detain us here, Leibniz establishes that if a circle exists and is squarable, this means it should be expressible as an infinite sum of infinitesimal areas $y_1 dx$, whose ratio $y_1 : y_2$ to the elements of area of the corresponding square $y_2 dx$ must be rational—or, he corrects himself, must be "as infinite number to infinite number", i.e., as the irrational root of an algebraic equation (496-7). Conversely, if it can be shown that the diameter of the circle and the side of the square have no common measure, not even an infinitely small one, then a quadrature of the circle of this kind is impossible, and it will follow "that the magnitude of the circle cannot be expressed by an equation of any degree" (498)—or as we would say, it would prove that π is a transcendental number.

Leibniz appears to take it for granted that no infinitely small common measure can be found; and that, this being so, a circle, as an infinite sum of infinitesimals, cannot "exist". At any rate, without pausing for a new paragraph he immediately concludes that "A circle, as a polygon greater than any assignable (as if that were possible), is a fictive entity, and so are other things of that kind" (498), and proceeds to offer an approach to defining a circle as an infinite polygon that is very suggestive of the ε–δ definition of a limit. To attribute some property to the circle, he says, is to attribute it to a polygon in such a way that "there is some polygon in which the error is less than any assigned amount *a*, and another polygon in which the error is less than any other definite assigned amount *b*. However, there will not be a polygon in which this error is less than all assignable amounts *a* and *b* at the same time, even if it can be said that polygons somehow approach such an entity in order" (498). Thus a circle as an infinite-sided polygon does not exist, but is a limiting case to which finite polygons, increasing according to some law, approach arbitrarily closely.

A similar syncategorematic construal allows Leibniz to give a definition of the sum of a converging infinite series that anticipates closely the modern textbook definition in terms of the limit of a sequence of partial sums (503): "whenever it is said that a certain infinite series of numbers has a sum, I am of the opinion that all that is being said is that any finite series with the same rule has a sum, and that the error always diminishes as the series increases, so that it becomes as small as we would like."

It is this new understanding of the infinite that leads Leibniz to the conclusion noted above concerning the unreality of infinite aggregates. "For numbers do not *in themselves* go absolutely to infinity, since then there would be a greatest number. But they do go to infinity when applied to a certain space or unbounded line divided into parts" (503). Thus there is no infinite number, even though there are infinitely many divisions in matter, just as there is no last term in an infinite series. That is, Leibniz has finally eschewed the inference from "spheres smaller than spheres to infinity" to the existence of "spheres smaller than any assignable"—the "minima" of "Secrets"—in favor of a new syncategorematic understanding of infinite division. He makes this explicit in a piece written only a few days later: "being divided without end is different from being divided into minima, in that [in such an unending division] there will be no last part, just as in an unbounded line there is no last point" (A VI.iii 71: 513). Thus Leibniz comes to reject the "finest division, or division into minima" that constitutes a perfect fluid. This view is forcefully presented in the dialogue *Pacidius Philalethi* of November of the same year (1676), where he rejects the whole ontology of perfect solids (atoms) moving in perfect fluids, in favor of one where every body "is everywhere flexible, but not without a certain and everywhere unequal resistance." Such a body

> still has cohering parts, although these are opened up and folded together in various ways. Accordingly the division of the continuum must not be considered to be like the division of sand into grains, but like that of a sheet of paper or tunic into folds. And so although there occur some folds smaller than others infinite in number, a body is never thereby dissolved into points or minima. (Aiii78: 555).

Now if matter has varying degrees of resistance to division, a given body can respond to the actions of the plenum by differing internal divisions, manifested as elasticity. But with no atoms to act as units, no portion of matter will retain its identity over time. It is possible that as a result Leibniz flirts with phenomenalism in this period, and some have found independent evidence to suggest this. However, once Leibniz has discovered the conservation of force (*vis viva*) in 1678, he can ascribe the unity of the body through such changes to a force within, which remains constant though differently distributed among the infinity of parts. This eases the way for his rehabilitation of substantial forms in 1679. I cannot do justice to the complexities of that topic here, of course. Suffice to say that by 1680 Leibniz is already offering these souls/forms/primitive forces as a solution to the problem of infinite aggregates: "unless there were a soul, i.e. a kind of form, a body would not be an entity, since no part of it can be assigned which would not again consist of further parts, and so nothing could be assigned in body which could be called *this something, or some one thing*"(Ve144: 651).

This is the argument for the need for synchronic unity. There is, however, a closely related argument for diachronic unity that follows from the conception of curves outlined in "On Motion and Matter" and "Infinite Numbers". For it is also a consequence of "the fact that no body is so very small that it is not actually divided into parts excited by different motions" that "no precisely straight line, circle, or any other assignable shape of any body, is found in nature" (*A Specimen of Discoveries*, 1686; Ve125: 488). As he explains in another piece of the same year, although one can draw an imaginary line at each instant, "that line will endure in the same parts only for this instant, because each part has a motion different from every other....Thus there is no body that has any shape for a definite time, however short it might be" (Ve321: 1478). So "shape involves something imaginary, and no other sword can sever the knots we tie for ourselves by a poor understanding of the composition of the continuum" (Ve125: 488).

In the late seventies and early eighties Leibniz will develop arguments to show that, like shape, motion and extension too are merely "phenomena, rather than true attributes of things, which contain a certain absolute nature having no relation to us" (Ve82: 294). But in "Infinite Numbers" there is an argument for the phenomenality of the fourth defining characteristic of bodies —magnitude. And this leads directly to the phenomenality of infinite wholes because of the intimate connection between the notions of "whole" and "magnitude". If a curve is regarded as consisting of infinitely many infinitely small straight sides, then no matter how many sides are taken, they never add up to the whole; rather, as in the sum of a continuous series, the sum is a kind of limit to which their continuous aggregation attains. Thus magnitude itself must not be understood as the multiplicity of parts of a thing (as Leibniz had previously defined it), but needs redefining as "the constitution of a thing by knowledge of which it can be regarded as a whole" (Aiii69: 503). Given the interdependence of the notions of 'magnitude' and 'whole', however, it follows that a whole, too must be redefined: "a whole is not what has parts, just what can have parts." And this leads Leibniz to doubt "whether what is really divided, that is, an aggregate, can be called one." It just "seems to be" one, he writes, "because names are invented for it" (503).

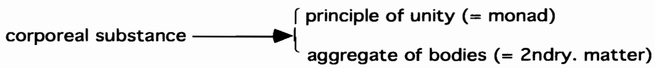
Here his argument connects back up with the argument from the Principle of Individuation discussed above. For "a whole exists when many things can come to be out of one"; yet in order for something to become something else, "something must remain which pertains to it rather than to the other thing." And this thing that remains need not be matter—in fact, as we have seen, it cannot be, since matter considered as the aggregate of its parts does not remain the same thing from one moment to the next. Rather, Leibniz argues, "it can be mind itself, understanding a certain relation" (503). In this way, Leibniz claims, provided there is a law underpinning these transitions from one aggregated whole to another new one produced from it, "continuous motion is imitated in a way, just as polygons imitate the circle. And hence one may be said to come out of the other, by a similar abuse, as it were, of the imagination."

Clearly Leibniz still has some distance to go before we reach his mature philosophy of substance. Among other things, the principles of unity he posits in bodies need to evolve from "minds" vaguely conceived into simple substances proper, whose perceptual states emanate according to a law unique to each one; and he needs to discover the conservation of force, enabling him to restore the continuity of change by positing as metaphysical foundation an appetition or primitive active force that takes a substance through its continuous series of states. Nevertheless, many of the chief tenets motivating the mature solution are already in place: There is no infinite number, and infinitesimals are fictions. Thus body as a continuous entity can be regarded as composed of infinitely small parts; but body in this sense, $body_1$, is ideal, and its parts fictions. $Body_2$, on the other hand, is not continuous, but infinitely divided into further parts. But although it is an infinite multiplicity of these parts, these do not make up a true whole. Body will be distinguished from space by the existence of minds (later souls) in it; however, the self-identity of a body will not be definable in terms of the sameness of shape and size through time, but by some other principle that maintains the unity of the body notwithstanding its infinite division.

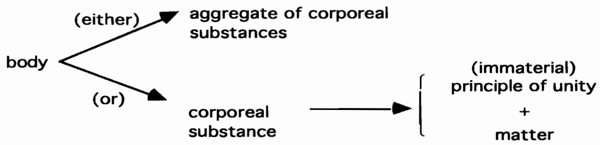
IV

Returning to the problem of interpretation with which I began, it should be clear how I think it is resolved: bodies are phenomena because they are mere *entia per accidens* both synchronically and diachronically. Considered at a given time, a body (*qua* extended entity) is always further divided by differing motions within. It is infinitely divided, and thus always an aggregate of other bodies. Moreover, because there is no infinite number (i.e., no infinite cardinal, no infinite sets), bodies as such do not constitute a complete entity or true whole. Over time, a body does not stay the same from one moment to the next; so again, considered apart from its principle of unity, it is phenomenal.

But I think the same considerations throw light on the second problem of interpretation identified in my introduction, concerning the reality of corporeal substances. To see this, it should be recognized that Leibniz has two ways of talking about the composition of corporeal substance, as Sleigh has lucidly observed (*Leibniz and Arnauld*, 98-99). Considered abstractly, a corporeal substance consists in a primitive active force and a primitive passive force, or "primary matter", neither of which is further decomposable, and this is perhaps what has led some to see the corporeal substance theory as distinct from the monadic. But Leibniz also considers corporeal substance concretely; in this sense, it is an aggregate that is unified by a dominant entelechy or monad. This monad has a body that is itself an aggregate of bodies, each one of them either a corporeal substance or an aggregate



of corporeal substances. Thus



But as we saw above, body is analysed as follows:

On my interpretation, then, Leibniz's analysis of body, and of corporeal substance, is a *quasi-recursive* one. A body is either a corporeal substance or an aggregate of corporeal substances; corporeal substance, in turn, is an aggregate of bodies united by a principle of unity. Each of the bodies is again either a corporeal substance or an aggregate of them, and so forth. But the analysis is not truly recursive because of Leibniz's subtle understanding of the infinite as unbounded yet actual.[20]

Thus the infinitude of the analysis of matter is understood to mean that however far we divide it, we find either living beings, or bodies that are aggregates of living beings. This is a syncategorematic construal, analogous to Leibniz's take on the sum of an infinite series: the error in approximating it by a finite number of terms can be made arbitrarily small. But the series has an actual infinity of terms, and likewise the division of body is actually infinite, even if only God can comprehend this. Therefore, since the material aspect of body is always further reduced, and the immaterial principles of unity (simple substances, monads) never are, it follows that when the resolution is taken to infinity, all that are left are monads.[21]

So there are organic bodies all the way down, and corporeal substances all the way down, despite the fact that the material aspect of body is always further resolvable into unities whose principles are immaterial. Yet if the analysis of bodies into their constituents is taken to infinity, in the end—and this must mean in a God's-eye-view, since it is only in God that the infinite is complete[22]—there is no need to posit anything other than immaterial principles of unity; even though these are never "bare" or lacking an accompanying body, and however small a body presupposes an infinity of them. It is this analysis, I believe, that forms the backbone of Leibniz's metaphysical position, and—although, to be sure, there are endless nuances and changes in the corresponding synthetic accounts he gives of how simple substances, forces and phenomena are related—it is this analysis that accounts for the unity of his position from the 1680s till the end of his life. I could quote numerous passages in support of this, but the following from about 1712 seems particularly apt, so I shall conclude with it:

> A substance is either simple, such as a soul, which has no parts, or is composite, such as an animal, which consists of a soul and an organic body. But an organic body, like every other body, is merely an aggregate either of animals or other things which are living and therefore organic, or finally of small objects or masses; but these are also finally resolved into living things, from which it is evident that ... in the analysis of substances, what ultimately exist are simple substances—namely souls, or if you prefer a more general term, *monads*, which are without parts. For even though every simple substance has an organic body which corresponds to it—otherwise it would not have any kind of orderly relation to other things in the universe, nor would it be acted upon in an orderly way—it is nonetheless in itself without parts. And because an organic body, or any other body whatsoever, can again be resolved into substances endowed with organic bodies, it is evident that in the end there are simple substances alone, and that in them are the sources of all things and the modifications that happen to things.
> —"Metaphysical Consequences of the Principle of Reason"[23]

Received September 4, 1998

Richard Arthur
Philosophy Department
Middlebury College
Middlebury, Vermont 05753 USA
arthur@panther.middlebury.edu

Notes

[1]A version of this paper was read to the Leibniz Society of North America in Philadelphia in December 1997, and expanded versions were read to the Philosophy Departments at the University of Guelph in March 1998, and the Università di Firenze in April 1998. I am indebted to my audiences at all three for their perceptive comments; as well as to Glenn Hartz for his helpful comments on the original draft.

[2]I have in mind especially the recent books on Leibniz's metaphysics by Catherine Wilson, *Leibniz's Metaphysics* (Princeton: Princeton UP, 1989); Robert Sleigh, Jr., *Leibniz and Arnauld* (New Haven: Yale UP, 1990); Donald Rutherford, *Leibniz and the Rational Order of Nature* (New York: Cambridge UP, 1995); and Robert Adams, *Leibniz: Determinist, Theist, Idealist* (New York: Oxford UP, 1994); and Daniel Garber's classic article, "Leibniz and the Foundations of Physics: The Middle Years", pp. 27-130 in Kathleen Okruhlik and J. R. Brown, *The Natural Philosophy of Leibniz*, (Dordrecht: D. Reidel, 1985).

[3]For a vigorous championing of this position see Catherine Wilson's *Leibniz's Metaphysics*, as well as her response to Jan Cover's 1993 review in *Leibniz Society Review*, 4, 1994, pp. 5-8.

[4]I have given a preliminary account of Leibniz's path through this conceptual maze in the introductory essay to my forthcoming Yale Leibniz volume, *G. W. Leibniz: the Labyrinth of the Continuum* (Arthur 1999). Other works commonly cited here are abbreviated as follows: **A**: cited by series, volume and page, e.g. A VI.ii 229, except in the case of Series VI, volume iii, where the series number is omitted and the piece number is given: e.g.Aiii4: 96. **Ve**: Gottfried Wilhelm Leibniz: *Vorausedition zur Reihe VI*, ed. Akademie der Wissenschaften der DDR (Münster: Leibniz Forschungsstelle, 1982-); cited by piece number (e.g. Ve81), by page number (e.g. Ve 223), or both (e.g. Ve447: 2047). All translations are my own.

[5]For a persuasive defence of the reality of corporeal substances, see Glenn Hartz's "Why Corporeal Substances Keep Popping Up in Leibniz's Later Philosophy," *British Journal for the History of Philosophy*, 6 (2) June 1998, 193-207.

[6]Although I claim that this reason for bodies being infinite aggregates is "quite simple", note that it immediately renders problematic Adams' reading of bodies as aggregates of monads directly. For bodies are certainly not actually divided into monads; Leibniz will say rather that monads are "presupposed" by the fact that

bodies are actually infinitely divided.

[7]See Philip Beeley, *Kontinuität und Mechanismus* (Stuttgart: Franz Steiner, 1996), ch. 3, esp. 56ff.

[8]This is worth quoting in full: "But actually no entity that is really one is composed of a plurality of parts, and every substance is indivisible, and those things that have parts are not entities, but merely phenomena" (Ve284: 1253).

[9]It may seem a little odd to assimilate Leibniz, a harsh critic of materialism, to one of its principal expounders; but Leibniz himself often invoked Democritus on this point; see e.g., his letter to To De Volder, 1703/6/20 (G.ii.252).

[10]Leibniz occasionally refers to *entia per aggregatione* as "semi-entities" (not to be confused with semi-entities by division, like Monty Python's "Eric-the-half-a-bee").

[11]Leibniz, in common with his contemporaries, held matter to be geometrically defined; although after 1670, following Hobbes, he put a lot more weight on motion or endeavour as a defining characteristic of bodies. See especially Ak5 and section 3 of my introduction to Arthur 1999.

[12]For discussion of the difference between Leibniz's and Berkeley's phenomenalism, see Margaret Wilson, "The Phenomenalisms of Leibniz and Berkeley", pp. 3-22 in Ernest Sosa, ed., *Essays on the Philosophy of George Berkeley* (Dordrecht: D. Reidel, 1987); and Adams, *Leibniz*, pp. 224-228. Translations of the relevant texts can be found in the Ariew and Garber, *G. W. Leibniz: Philosophical Essays* (Indianapolis: Hackett, 1989) pp. 306-7.

[13]Gerauld de Cordemoy, *Six Discours sur la distinction et l'union du corps et de l'ame*, 1668; later published in a Latin edition in 1679 as the first of "Two Treatises on Physics", *Tractatus Physici duo. I. De Corporis et Mentis Distinctione*, which Leibniz excerpts and annotates in 1685.

[14]See the dissertation he wrote in Leipzig in 1663, *De Principio Individui*, sections 15, 25 (A VI.i 15, 18).

[15]A VI.i 492; L 111. This becomes one of Leibniz's favourite arguments, repeated almost whenever he discusses the problem of cohesion (see A VI.ii 250-1, 365 (1671), Aiii4: 96 (1672/3), Ve145: 654 (1677), Ve125: 493-4 (1686), all given in translation in Arthur 1999.

[16]See Leibniz's second letter to Thomasius, 1669 (Ak II.i 23-24; L 102).

[17]The question of Leibniz's atomism, though, is vexed, since he continued to uphold the infinite division of matter whilst embracing atoms. I explore this conundrum in an as yet unpublished paper, "The Enigma of Leibniz's Atomism".

[18]"Matter too is perpetually becoming one thing after another, since it exists only in relation, as I have shown elsewhere from the principle of the individuation of all things" (Aiii36: 392). Leibniz makes this argument explicit two weeks later with the important "Meditation on the Principle of the Individual" of April 1 (Aiii67: 490-491; G. H. R. Parkinson, ed., *De Summa Rerum*, Yale UP: New Haven, 1992, 50-53), where he proves the necessity of minds with almost exactly the same argument as that in his later notes on Cordemoy given above.

[19]Cf. "as long as there is no method for directly establishing the quadratures of curved lines, there will be a strong suspicion that none exist" (Ak68: 492); "Assuming there are no curved lines, what is said about them will be the properties of polygons. And some particular circle will be nothing but a general entity" (494).

[20]Here I am indebted to my audience in Florence for correcting my previous claim that the analysis is truly recursive.

[21]This, of course, is only a sketch. A more complete account of Leibniz's idea of corporeal substance would need to explain how its reality consists in a power of acting (this is its monad, or principle of unity), and a power of being acted upon (this is its matter, or principle of multiplicity). Without a body, a substance would not be acted upon, it could not sense or have a relation to other things in the universe. There would, in fact, be only one (purely mental) substance, representing all others in its states. Thus there are no naked monads, just as there are no monadless bodies.

[22]The "in the end" is Leibniz's phrase: see the passage below. In quoting the same passage in his excellent article mentioned in fn 4, Glenn Hartz writes "The entire matter turns on what 'in the end' comes to. Does this mean that in the end there *aren't* corporeal substances because all there are are simple ones? Or does it mean that there still *are* corporeal substances?" (p. 204). I believe Leibniz's position is that God needs to posit only simple substances, albeit an actual infinity of them; but that composite or corporeal substances are a necessary and immediate result of his having posited this infinite multiplicity of simple ones—they do not need to be separately posited.

[23]C, 11-16; *Leibniz: Philosophical Writings*, ed. G. H. R. Parkinson, p. 175, translation slightly modified.