Wolfgang Lenzen, *Calculus Universalis: Studien zur Logik von G. W. Leibniz.* Paderborn: Mentis, 2004. Pp.

Reviewed by Massimo Mugnai, Scuola Normale Superiore, Pisa

This book is a collection of essays published by the author in the long run of about 20 years and is centered on the reconstruction of Leibniz's logical calculi. All the essays have been revised for the present edition and some of them constituted the background for Lenzen's first monograph on Leibniz's logic (*Das System der Leibnizschen Logik*, Berlin-New York, De Gruyter, 1990). A feature common to all these essays is the vindication of the relevance and originality of Leibniz's logical achievements. Lenzen manifests strong dissatisfaction with the evaluations of Leibniz's logic previously offered by interpreters like Louis Couturat, Clarence I. Lewis, Karl Dürr, William and Martha Kneale, and states that till now Leibniz's results in the field of logic have been widely underestimated (p. 22).[1]

The book contains a careful and detailed examination of almost all Leibniz's papers on the logical calculus and it is based on the knowledge of a wide range of texts unknown (or only partially known) to previous interpreters. Lenzen's acquaintance with the entire corpus of Leibniz's logical texts (including a number of relevant manuscripts) is impressive. Some chapters of the book in particular contain very solid and useful logical analyses. Chapter 7, for instance, includes the most profound account of Leibniz's theory of negation I ever read. Chapter 8 presents in a very clear way Leibniz's attempt to reduce traditional syllogistic to a calculus based on logical inclusion between terms. Chapter 14 is devoted to Leibniz's a priori proof of the existence of God and presents the first edition of an important manuscript on the proof. On chapters 3 and 5 a series of convincing reasons are given to argue that Leibniz's concept of *ens* does not have to be considered a constant in the logical calculus. In brief: this work discusses a wide range of topics in such a clear and learned way that it will surely become a reference book for scholars interested in the study of Leibniz's logical papers in the forthcoming years.

At the same time, however, Lenzen's attempt to credit Leibniz with a place 'higher' than that usually assigned to him in the histories of logic, seems to me quite weak. Lenzen is very fond of the category of 'anticipation'. Thus Leibniz 'anticipated' the "essential principles of the logic of quantification" (pp. 21; 99); the distinction between quantification on a domain of possible objects on the one hand and quantification on the domain of real existing objects on the other (an idea" which is now well known in the field of quantified modal logic) (p. 21); Leibniz's reduction of deontic modalities to the alethic ones was 're-discovered' in recent times (ibidem), etc. Clearly in contrast with the received view, Lenzen attempts to show that Leibniz has to be reckoned the first founder of the so-called Boolean algebra; that Leibniz was right assuming that the reciprocity-principle holds between intensional and extensional approach to the logical calculus; that Leibniz was a forerunner of the modern theory of quantification. Each of these claims, however, seems to me questionable. Let me consider, first, the statement that Leibniz had found a logical calculus analogous to the Boolean algebra.

As is well known, and as Lenzen openly recognizes, a specific treatment of *disjunction* is conspicuously absent from Leibniz's logical calculi (pp. 178; 183). Even though, in the *Mathesis universalis*, Leibniz proposes the use of the symbol 'v', i.e. the first letter of the Latin word 'vel', to express the non-exclusive disjunction, he does not develop a proper calculus based on it.[2] Of course, given conjunction and negation (complement), it is always possible to introduce the disjunction by means of De Morgan's laws, but precisely this point raises a specific problem: Leibniz doesn't make any substantial use of De Morgan's laws either in the *General Inquiries* or in other logical essays. To the best of my knowledge, Leibniz explicitly refers to De Morgan's laws only in few texts and in a very cursory way. Amongst these texts, the most interesting one is surely a marginal remark inspired by a work of Joachim Jungius. I here give the text of Leibniz's remark (the numbers between brackets are added by me):[3]

[1] et A et B
[2] neque A neque B
[3] vel A vel B hoc est non et A et B
[4] sive A sive B hoc est non neque A neque B
non et A et B
[5] aut A aut B, hoc est — simul
non neque A neque B

This text was published for the first time in 1903 by Couturat, who amended it inserting a 'non' before "et A et B" ([1]: "<non> et A et B").[4] My impression (shared with the Editors of the *Vorausedition*), however, is that this insertion is not necessary. The passage from Jungius which prompts Leibniz's marginal remarks concerns the problem of finding a symbolism adequate to render the Latin expression *not A and not B*. In a quite natural way, Leibniz here begins considering the *conjunction* of two expressions (terms or sentences) *A* and *B*, and then analyses various ways of combining conjunction and negation. There is no reason to suppose that [1] above has to be corrected to "non et A et B".

Commenting on this same text, Lenzen accepts Couturat's correction and attributes to Leibniz one more mistake. Lenzen's conjecture is that Leibniz considers the expression *non et A et B* as logically equivalent to *A vel B*, where *vel* has the same meaning of the non-exclusive *or* in English. A consequence of this interpretation is that one has to assume that Leibniz has mistakenly written "non et A et B" in the manuscript, instead of "non et non A et non B". And this is precisely what Lenzen suggests (p. 27). Even in this case, however, there is a more natural interpretation which avoids any attribution of a mistake to Leibniz and which, therefore, on the basis of a principle of economy (and of charity), has to be preferred. If we take at face value what Leibniz writes at point [3], it becomes obvious that in the passage in question Leibniz does not use *vel* to denote the non-exclusive disjunction, but to express the logical connective corresponding to incompatibility or to what is known as *Sheffer's stroke* (*vel A vel B* is *false* only in the case that *A* and *B* are both true).[5] This solution has the advantage of being consistent with some uses of the Latin word *vel* and with the general context of the entire passage.

On line [2] Leibniz introduces the *joint denial* (*not A and not B*); on line [4] he denies the joint denial: *non* (*not A and not B*) to explain the meaning of the expression "...sive...sive..." which, in ordinary Latin, may have (and in this case has) the same meaning as the *non exclusive* 'or' in English (corresponding to the ordinary meaning of *vel* in Latin); on line [5], finally, he defines the exclusive use of disjunction (Latin *aut*) by means of the conjunction between *non* (*A and B*) and *A or B*. Clearly, on line [4] Leibniz is making recourse to one of the so-called De Morgan's laws.

Medieval logicians were well acquainted with these laws and, given Leibniz's knowledge of scholastic and late scholastic logic, it would be very surprising if he would not have recognized them. Thus, the above passage constitutes no particular wonder in itself. What is surprising is that, if Leibniz had a "complete axiomatization of the Boole-Schröder algebra of sets", as Lenzen claims, he did not show any awareness of the principle of duality between conjunction and disjunction for the logical calculus. Lenzen acknowledges that this may constitute a problem for his interpretation, but he seems not to be much worried about it.

Lenzen extracts from Leibniz's logical writings a set of axioms and rules of inference which give rise to a basic system *L1*, a system which is deductively equivalent to a Boolean Algebra. About this system, Lenzen makes a remark which is revealing of his attitude as an interpreter of Leibniz's logic: from the fact that Leibniz did not find some valid principles that characterize *L1*, it does not follow that his logical system has to be considered incomplete. What really matters, in this case, is that these principles are derivable from the axiomatic base of the system (p. 58). As remarked above, however, and as Lenzen himself openly admits, Leibniz hasn't the faintest idea of the principle of duality which logically connects conjunction and disjunction; moreover, Leibniz has even 'forgotten' (Lenzen's word (p. 69, footnote 8)) to formulate explicitly the associative property of logical product which figures amongst the axioms of *L1*. Therefore, when Lenzen repeats in his book that "already in 1686, in the *General Inquiries*, Leibniz had found a complete axiomatization of the (so-called 'Boolean') set-theoretical Algebra, so that [...] he has to be considered as the founding father of modern logic" (p. 182), this seems quite an exaggeration.

Another central claim of Lenzen's interpretation of Leibniz's logical calculi is that the reciprocity principle which describes the relationship between intension and extension is sound (pp. 23-46). As is well known, one of Leibniz's most basic convictions is that the copula expresses a *relation* – in particular the relation of *inherence* (latin: *inesse*) – linking together subject and predicate. To explain the nature of this relation it is necessary to take into account that *subject* and *predicate* are names for concepts *and* classes. As Leibniz recognizes: "words indicate the things as well as the ideas": and the inherence-relation behaves differently according to the interpretation at issue.[6] If we consider *subject* and *predicate* as names for *concepts*, then any sentence of the form "A is B" means that the concept named by 'B' *inheres* in the concept named by 'A'; whereas if *subject* and *predicate* are names for classes, the same sentence means that the class of individuals named by 'A' inheres in the class named by 'B'. Thus Leibniz distinguishes two different ways of considering the relationship between subject and predicate in a sentence: the *intensional* way on the one hand and the *extensional* one on the other. As the following well known passage shows, Leibniz attributes the intensional approach to Aristotle, whereas he considers the extensional one more common:

> The common manner of statement concerns individuals, whereas Aristotle's refers rather to ideas or universals. For when I say *Every man is an animal* I mean that all the men are included amongst all the animals; but at the same time I mean that the idea of animal is included in the idea of man. 'Animal' comprises more individuals than 'man' does, but 'man' comprises more ideas or more attributes: one has more instances, the other more degrees of reality; one has the greater extension, the other the greater intension.[7]

Of the two different approaches – intensional and extensional – Leibniz says notoriously that, for the logical calculus he prefers the intensional one. The main reason for that is that the intensional approach does not depend on the existence of individuals: "I have preferred to consider universal concepts, i. e. ideas, and their combinations" – so writes Leibniz – "as they do not depend on the existence of individuals."[8]

The preference accorded by Leibniz to the intensional point of view does not exclude, however, the extensional approach from the calculus. As a matter of fact, Leibniz develops logical essays based on intension *and* on extension as well.[9]

Leibniz's firm conviction, indeed, was that a fundamental reciprocity subsists between extension and intension. In the *Elements of calculus* for example, immediately after having introduced the intensional point of view, Leibniz writes that "with suitable symbols, we could prove all the rules of logic by a calculus somewhat different from the present one – that is simply by a kind of inversion of it."[10] In another text, written ten years later, Leibniz explains with more details what kind of *inversion* he has in mind:

> 'Being quadrilateral' is in 'parallelogram', and 'being a parallelogram' is in 'rectangle' (i. e. a figure every angle of which is a right angle). Therefore 'being quadrilateral' is in 'rectangle'. These can be inverted, if instead of concepts considered in themselves we consider the individuals [*singularia*] comprehended under a concept; A can be a rectangle, B a parallelogram, C a quadrilateral. For all rectangles are comprehended in the number of parallelograms, and all parallelograms in the number of quadrilaterals; therefore all rectangles are contained in quadrilaterals. In the same way, all men are contained in all animals, and all animals in all corporeal substances; therefore all men are contained in corporeal substances. On the other hand, the concept of corporeal substance is in the concept of animal and the concept of animal is in the concept of man; for being a man contains being an animal.[11]

If we represent the intensional containment with the symbol '$\geq$' and the extensional one with the usual set-theoretical symbol for inclusion '$\subseteq$', we may express the reciprocity principle as follows:

(Princ.*): $(A) \geq (B)$ iff $(A) \subseteq (B)$.

Lenzen characterizes the extension of a given concept *A* by means of the usual comprehension principle:

Ext. $A = \{x | A(x)\}$;

whereas he characterizes the intension of *A* as:

Int. $A = \{\varphi | \forall x(A(x) \longrightarrow \varphi(x)\}$.

According to Int. *A*, the intension of a given concept *A* is the set of all properties φ possessed by all individuals which possess the property corresponding to *A* (pp. 37-38). Then, Lenzen works out two different interpretations, extensional *and* intensional, of the basic axioms of the Leibnizian logic *L1*. As a final move, he attempts to show that "every extensional interpretation of *L1* may be associated in a canonical way to an intensional one which satisfies the same sentences of the former", and vice versa. (p. 42) From this it follows "that any sentence of the Leibnizian logical language *L1* is extensionally valid if and only if it is intensionally valid" and, therefore, the reciprocity principle (Princ.*) holds (pp. 42-43).

To show that Leibniz was right stating the reciprocity principle, Lenzen resorts to the algebraic notion of a *field of sets* and develops a sophisticated proof which is presented in detail in the first Appendix (chapter 15, pp. 343-46) of the book. It seems to me, however, that in this proof Lenzen neglects the crucial question concerning the nature of intensional negation. As far as intensional negation does not enter into play, (Princ.*) holds without problems, whereas if negation is admitted as an ingredient of the logical system, then some troubles arise. Dummett was the first to call attention to the following question.[12] Consider the sentence "no man is a donkey": on the basis of the extensional (i.e. set-theoretical) approach it is easily expressed asserting that the intersection between the class of men and that of donkeys is the null class; but how to express the same sentence from the intensional point of view? Even though the two sets are extensionally disjoint, they may have – and in fact *they have* – a lot of *intensions* (concepts) in common. The question raised by Dummett is directly connected with the more fundamental one: whereas on the extensional point of view *negation* is expressed by means of the set-theoretical *complement*, what properly corresponds to the complement in the *intensional* approach?

To answer this question, Lenzen takes advantage of a solution suggested by Dummett himself. Under the assuming that the intension of a given concept *A* is the set of all properties which are shared by all the (possible) individuals that have the property corresponding to *A*, the intension of *not-A* will be the set of all properties which are common to all the individuals that *do not have* the property corresponding to *A*. In other words, to determine the intension of the negation of a given property *A*, one has first to consider all the (possible) individuals which constitute the (extensional) complement of *A*, and then to 'put together' the properties that are common to all these individuals. But, it seems to me that the individuals belonging to the complement of A will have in common only properties which convey very poor information about the nature of the *intension* of *not-A*. Using Lenzen's symbolism, and denoting as '-A' the intension corresponding to 'not-A', we have:

$$\text{Int. } {-A} = \{\varphi \mid \forall x(\text{not } A(x) \rightarrow \varphi(x)\}.$$

Surely, amongst the φs which characterize *-A* there will be the property of not being *A*, that of being identical with itself, and a few other properties as, for example, that of being something. These properties, however, are quite trivial and I wonder if they are able to characterize in a perspicuous way the intensional content of *-A*.[13]

Moreover, there is a sense in which Lenzen's (and Dummett's) definition of the *intension* of a given property is clearly non-Leibnizian. As we have seen, Lenzen defines the *intension* of a property *A* referring to the individuals which share *A*. For Leibniz, however, intensions have to be *independent* of the individuals (even of the merely possible ones) because intensions, playing an essential role in determining the exact nature of individuals, have an ontological and even temporal priority in respect to these latter. In Leibniz's philosophy, individuals are instances (actual and possible, respectively) of complete concepts; and complete concepts are the result of the combination of intensions in God's understanding. Without complete concepts, i.e. without intensions, no individual can be conceived. Thus, making recourse to the individuals to define intensions is a curious inversion of the ontological and metaphysical order devised by Leibniz.[14]

Strongly connected with the two previous topics (respectively: of the algebra of logic and of the reciprocity between extension and intension), is Lenzen's attempt to attribute to Leibniz a kind of 'implicit' use 'in disguise' of the set theoretical disjunction. Lenzen sees Leibniz's concept of *common* intensional part as an intensional correlate of a logical operator which, from the extensional point of view, behaves as a disjunction (in fact, he says, *it is* the disjunction, p. 315). And by means of the common part in Leibniz's sense (and of negation, of course) Lenzen attempts to show that Leibniz, at least implicitly uses De Morgan's laws as well. Lenzen's claim may be summarized as follows.

Suppose, as Leibniz does, that two concepts A and B are given, and that A results from putting together two other concepts, say P and M, whereas B results from the conceptual sum of the two concepts M and N. This situation can be illustrated by the following equalities (where the juxtaposition of letters expresses the conceptual sum):

(1) $A = PM$

(2) $B = MN$.

Moreover, suppose that P and N are completely disjoint. Clearly A and B share the same conceptual part M. Lenzen calls this part (with the German word) *Kommune* (translating the Latin word *commune* employed by Leibniz) (p. 315), and represents the *Kommune* between A and B as "AvB". The use of 'v', as Lenzen points out, is not fortuitous; he represents the *Kommune* with the symbol usually employed to denote the classical *extensional* disjunction, because the *Kommune* really plays, in Leibniz's logical essays, *under the extensional reading*, the role of the disjunction: "The '*Kommune* between A and B' is reducible to the negation of the conjunction of the negative concepts *not-A* and *not-B*, i.e. to the *disjunction* of the concepts A and B" (315). Thus, Leibniz would have used the disjunction, even though unaware of this very fact: as the celebrated character of Molière, he would have spoken prose all his life without knowing this.

I suspect, however, that the extensional (Boolean) disjunction has a very weak link with the Leibnizian *Kommune*. To see why, let me dwell on some logical properties of the *Kommune* in Leibniz's sense. First of all, consider the following equality obtained expressing the *Kommune* by means of 'v' (*A* and *B* are names of concepts and *M* denotes their common part, as above):

(5) $(A \text{ v } B) = M$.

Remembering that for Leibniz the intensional relation of inherence holds when the concept of the predicate inheres in the concept of the subject, and that, as we read in the *General Inquiries*, if *A est B* and *B est A*, then A = *B*, we may think of (5) as composed of two parts (using '≥' to express the relation of intensional containment, as above):

(6) $(A \text{ v } B) \geq M$

(7) $\mathrm{M} \geq (A \text{ v } B)$.

(6) asserts that *M* is included in the conceptual part common to both, *A* and *B*, whereas (7) states, conversely, that the conceptual part common to both, *A* and *B*, is included in *M*. From (6) and (7) together follows (5); and all these last three formulae are obviously true.

If we interpret (6) and (7) from the extensional point of view, according to the reciprocity principle ((Princ.*) above), we have:

(8) $(A \cup B) \subseteq M$

(9) $\mathrm{M} \subseteq (A \cup B)$.

Of these two formulae, (8) states that the union of the extensions of *A* and *B* (i.e. the set-theoretical union of *A* and *B*) is included in the extensional counterpart of the concept *M*, common to both, *A* and *B*. To see that (8) is right, suppose, for instance, that *A* and *B* are, respectively, the concept of *man* and the concept of *dog*, and that *M* is the concept of *animal*, common to both. Passing from the intensional point of view to the extensional one, we have that the extensions of *man* and *dog* are included in that of *animal*: in this case, it is true that the set-theoretical union of man and dog is included in the extension of animal. (9), on its part, states that the extensional counterpart of *M* is included in the set-theoretical union of *A* and *B*, but this seems to be false.[15] To continue with the previous example, (9) says that the extension of *animal* is included in the set-theoretical union of *man* and *dog*, which is clearly untrue. Lenzen properly speaks of the *Kommune* between *A* and *B* as the 'greatest' concept *C* which is included in both, *A* and *B*, but I find difficult to understand what 'greatest' means in this case. If it means the 'poorest' concept in intensional content, then its extensional counterpart is the richest in items, and I wonder how (9) can be right. If it means a concept with the richest intensional content, being common to two other concepts, it will have lesser intensional content than the others two. But it is impossible that its extensional counterpart would be lesser in extension than the union of the extensions of the other two concepts. Therefore, the Leibnizian *Kommune* cannot have the set-theoretical union as its extensional counterpart.

Another thesis which sounds implausible and which is one of the main tenets of the book concerns the theory of quantification (pp. 99-131). Undoubtedly Leibniz, attempting to express in form of algebraic equivalences the four categorical sentences of the Aristotelian-scholastic tradition, employs variables in a way strongly reminiscent of modern (post-Fregean) quantification. Lenzen, however, is not satisfied with this statement and writes that Leibniz "anticipated" the essential principles of the (modern) logic of quantification, so that he has to be considered "if not the founder, at least the forerunner of it" (p. 99). To recall briefly the issue, let me first quote a well known passage from the *General inquiries*: "An *affirmative proposition* is *A is B*, or, *A contains B*, or, as Aristotle says, *B is in A* (that is, directly). That is, if we substitute a value for *A*, *A coincides with BY* will appear. For example, *man is an animal*, i.e. *man is the same as a...animal* (namely, man is the same as a rational animal). For by the sign *Y* I mean something undetermined, so that *BY* is the same as *some B*, or *a...animal* (where *rational* is understood, provided that we know what is to be understood), or, *some animal*. So *A is B* is the same as *A is coincident with some B*, or, *A* = *BY*."[16]. Here Leibniz employs the letter *Y* to transform the universal affirmative sentence "All A are B" in an equation of the algebraic type. In the *General inquiries* and in other logical papers, Leibniz generalizes this idea, making a systematic use of what he names *indefinite letters*: i.e. letters of the Latin alphabet (mostly the last ones) employed to express the quantity of the terms (classes) involved in the logical calculus. Whereas in the passage just quoted Leibniz uses letters as something like *particular* quantifiers, there are even passages in which he employs these letters with a function which reminds one of *universal* quantifiers.[17] Therefore one seems to be authorized to call these letters (as Lenzen does) *quantifiers in disguise* (*verkappte Quantoren*: p. 99). As Lenzen is forced to admit, however, from these texts does not emerge any clear awareness, on Leibniz's part, of the 'true' nature of the letters which behave like quantifiers.

As is clear from the quoted passage, however, Leibniz's indefinite letters usually do not range over a domain of individuals: they are concepts (genus, species) or classes of individuals employed to express whether a given genus or class is taken partially or in its totality. Here, to make recourse to Lenzen's category of 'anticipation', Leibniz at most anticipates George Boole's attempt to express in form of equations the universal and particular sentences of the traditional Aristotelian logic. Therefore, on the basis of this evidence, it seems quite difficult to understand how Leibniz could have "anticipated" the essential principles of the (modern) logic of quantification, as Lenzen claims. Leibniz employed *undefined letters* to denote undetermined concepts (intensions) or undetermined classes of objects (extensions) and we may interpret this as a first step towards a theory of quantification, but such an interpretation is possible only because we *already know* the theory of quantification in its present form.

Finally, a last critical remark on Lenzen's claim that Leibniz 'anticipates' by more than two hundred years C. I. Lewis's definition of strict implication (p. 303). To determine the exact meaning of a conditional when it appears in a text of logic written before the first decade of the past century is not an easy task. There is some evidence of a dispute about the nature of the conditionals in antiquity: the logicians belonging to the Megarian-stoic tradition distinguished the (extensional) conditional proposed by Filo from the conditional favored by Chrysippus (a kind of *strict* conditional) and from Diodorus' conditional (related to time). In medieval times there was an analogous dispute concerning the right nature of the conditional, but a clear-cut distinction of the material (Filonian) conditional from other kinds of conditionals is not easy to find. To have this distinction in clear form we have to wait for Charles Sanders Peirce and Hugh McColl's work in the second half of nineteenth century. The main reason for this is that in natural languages (in the everyday, ordinary uses of the language, at least), the so-called strict conditional has a kind of preeminence on the 'material' one. Thus, it is no wonder that logicians of the past, before the 'matematization' of logic, have tacitly assumed it as *the* conditional *par excellence*. If I say that a conditional cannot be true if its antecedent is true and its consequent false, I may use the word 'cannot' in its full modal meaning, but I can even use it simply as equivalent to "it is not the case that". Thus, when Albert of Saxony, for instance, writes that "for the truth of a conditional proposition it is required that the antecedent cannot be true without the consequent being true as well",[18] it is impossible, without further reading, to establish if he is giving here the truth conditions of a material or of a strict conditional. The same holds for the text which Lenzen quotes (p. 295) as an evidence for Leibniz acceptance of the strict conditional: "[...] If *L* is true, then it follows that *M* is true: the sense [of this sentence] is that one cannot suppose that *L* is true and *M* false". Leibniz never distinguished between different kind of conditionals, and if it emerges that in the logical calculus he mostly employs the strict conditional, surely he was not aware of this. On the contrary, it would have been very surprising and, for many reasons, truly revolutionary, if Leibniz would have theorized, in his logical calculi, the preeminence of the *material conditional*.

The above criticism, however, is not intended to undermine the importance of a work which deeply investigates Leibniz's logical papers. Since Couturat's seminal work, other monographs have been published which have significantly improved our knowledge of Leibniz's logic.[19] Surely, Lenzen's book – setting aside the aspiration of making the historical relevance of Leibniz's achievements in the field of logic even greater – belongs to these works.

Massimo Mugnai
Scuola Normale Superiore, Pisa
Piazza dei Cavalieri
7-56100 Pisa, Italy
m.mugnai@sns.it

Abbreviation

P = Leibniz, *Logical Papers. A Selection, Translated and Edited with an Introduction by* G. H. R. Parkinson, Oxford, Clarendon Press, 1966.

Notes

[1] Cf. C; Karl Dürr, *Neue Beleuchtung einer Theorie von Leibniz*, Otto Reichl Verlag, Darmstadt (Abhandlungen der Leibniz-Gesellschaft, hrsg. Von P. Ritter, Band II), 1930; Raili Kauppi, *Über die Leibnizsche Logik*, "Acta Philosophica Fennica", XII, Helsinki, 1961; William & Martha Kneale, *The Development of Logic*, Oxford, 1962; Clarence I. Lewis, *A Survey of Symbolic Logic*, University of California Press, 1918.
[2] Cf. G. W. Leibniz, *Mathematische Schriften,* hrsg. von C. I. Gerhardt, Berlin-Halle, 1849-63, voll. 1-7, vol. 7, p. 57.
[3] G. W. Leibniz, *Vorausedition zur Reihe VI – Philosophische Schriften – in der Ausgabe der Akademie der Wissenschaften der DDR – Bearbeitet von der Leibniz-Forschungsstelle der Universität Münster*, Faszikel 7, 1988, p. 1632.
[4] C, p. 425.
[5] Cf. Herbert B. Enderton, *A Mathematical Introduction to Logic*, New York, Academic Press, 1972, p. 51.
[6] G. W. Leibniz, *New Essays on Human Understanding*, transl. and edited by P. Remnant and J. Bennett, Cambridge, 1981, p. 287.
[7] G. W. Leibniz, *New Essays on Human Understanding*, transl. and edited by P. Remnant and J. Bennett, Cambridge, 1981, p. 487.
[8] P, p. 20 (A VI, 4A, p. 200).
[9] Cf., for instance, C, p. 292 ff.
[10] P, p. 20 (A VI, 4A, p. 200).
[11] P, p. 136 (A VI, 4A, pp. 838-39)
[12] Michael Dummett, Review of Nicholas Rescher, "Leibniz 's Interpretation of his Logical Calculus", in *Journal of Symbolic Logic*, 21, 1956, pp. 197-99.

[13] Thomas Liske has discussed at length the problem of expressing intensional negation in Leibniz's logic (Michael-Thomas Liske, "Ist eine reine Inhaltslogik möglich? Zu Leibniz's Begriffstheorie", in *Studia Leibnitiana*, XXVI, 1, 1994, pp. 31-55). Criticizing Lenzen's approach, Liske has developed very interesting considerations which are worth being summarized here. Liske moves from a genuine Leibnizian insight: each concept contains, embodied in itself, as it were, a kind of logical repulsion with respect to other concepts. Consider, for example, a genus and its subordinated species: the species are mutually exclusive in the sense that the same individual may not belong to two (or more) of them. If the individual *i* is a man, for example, then it cannot be a donkey or a fish or…etc. - which means that *i* is at the same time *non-donkey* and *non-fish*, and… etc. This corresponds to Leibniz's claim that the concept of *man*, for instance, includes (or contains) the concept of *non-donkey*, and that of *non-stone* etc. Thus, that men are not donkeys may be expressed – on the intensional reading – saying that *man* is *non-donkey* – i. e. that the intension *non-donkey* belongs to or inheres in or is contained in the concept of *man*. That a concept inheres in another – Liske remarks – does not mean necessarily that the first is a "real component" of the second: "negations of concepts are contained in the content of a positive concept, without necessarily constituting it" (p. 39).

Yet the problem remains what the expression *non-donkey* properly means – in general: what is the conceptual content of a negated concept? Of course, we cannot say that the negation of *donkey* is the set of all concepts which are not the concept of a donkey, because this is a contradictory set. It seems more promising to take advantage of the way Leibniz characterises the negation of a given concept A in the *General Inquiries* (§§. 76; 104-105): *Non-A est non (AB)*. According to Liske, the intension of the negation of a concept A has to be understood as the negation of all species which are subordinated to it "till to the individual concepts of the most specific kind or of the *species infima*" (p. 48). This amounts to an intensional negation which behaves as the symmetrical counterpart of the extensional one: the intension or conceptual content of *non-man* is the negation of all conceptual complexes got by further specifying or determining the concept of *man*, up to the individual concepts of Peter and of Paul etc. subordinated to it. Even this idea of intensional negation, however, is not without problems. First of all – as Liske emphasized – it takes for granted that we know what it really means to deny an individual concept. But what is worse – it seems to me – is that, assuming that the negation of a concept is the negation of all genus and species which are subordinated to it, amounts to assuming as given the logical meaning of *negation*. If I say that to deny a given concept A is to deny its subordinated species and genera, to understand this I have first to know what it really means to *deny a species or a genus*.

[14] On the relationships between extensional and intensional approach in Leibniz, cf. Chris Swoyer's excellent paper "Leibniz on Intension and Extension", in *Nous*, 29, 1, 1995, pp. 96-114. On Leibniz'calculus in general cf. of the same author, the illuminating "Leibniz's Calculus of Real Addition", in *Studia Leibnitiana*, 26, 1994, pp. 1-30. These two papers are, in my opinion, amongst the best essays written on Leibniz's logic in the last decade.

[15] Prof. Sergio Bernini (Department of Philosophy, Florence) called this fact to my attention.

[16] P, p. 56 (A VI, 4A, p. 751).

[17] Cf. A VI, 4A, p. 771.

[18] Albertus de Saxonia, *Perutilis Logica*, Venice, 1522 (Hildesheim, Olms, 1974), p. 19 verso.

[19] Among them, besides the texts quoted above, are worth mentioning: Nicholas Rescher, "Leibniz's interpretation of his Logical Calculi", in *The Journal of Symbolic Logic*, 19, 1954, pp. 1-13; Hans Burkhardt, *Logik und Semiotik in der Philosophie von Leibniz* (Munich: Philosophia, 1980); Franz' Schupp's introductory essays, respectively, to Leibniz's *General Inquiries* (G. W. Leibniz, *Generales inquisitiones*, Meiner, Hamburg, 1982) and to the logical calculus (G. W. Leibniz, *Die Grundlagen des logischen Kalküls*, Hamburg, Meiner, 2000).