(LH XXXV, I, 14, bl. 57)

Ars Representatoria

Difficile est aliquid toto suo genere novum dare. Hanc tamen artem talem esse negari non potest. Ea tanto praestantior est Algebra, quanto majus est figuras et motus fideliter et nativa facie menti referre quàm solos numeros seu magnitudines.

Ductu igitur hujus artis et ope notarum simplicissimarum omnem figuram, motum, machinam, totam denique naturam quatenus mechanicè explicata habetur, in mente possumus depingere, sine figuris et modulis, idque tam exactè quam figuris et modulis fieri posset. Imò multò exactius et utilius. Nam possumus ita paucis et exiguis lineis repraesentare, quod alioqui repraesentadum esset multis et magnis figuris. Et cum figurae atque – cum ad solida veniendum est – moduli multo tempore ne dicam expensis indigeant, et motus per solas figuras sine modulis difficillimè repraesententur, possumus hac arte intra horam plures figuras et earum transmutationes sive motus describere, quàm figurae delineari possent intra septimanam, et moduli fabricari intra annum. Praesertim cum ex multis figuris et modulis pleraeque initio inquirendi descriptae aut fabricatae comperiantur inutiles, tempusque et sumtus perdantur; hac verò delineatione et fabricatione mentali sive repraesentatoria, facile multae figurae multique motus exhibentur brevi tempore et nullo sumtu.

At inquies idem facere possum imaginatione, ita sane aliqua ex parte. Sed ad hoc ipsum inventa est haec ars ut sublevet et perficiat imaginationem; **sublevet** ne forti illa ad figuras attentione quae sanè in figuris et motibus implicatis

Ars Representatoria [1691]

It is difficult to offer something completely unprecedented in its kind. Yet, it cannot be denied that this art is unprecedented. It is as much superior to Algebra as the accurate and faithful referring of figures and motions to mind is superior to that of just numbers or magnitudes.

By one stroke of this art and the help of very simple notes, we can depict every figure, motion, machine, and even the whole of nature as long as it has been mechanically explained, in our minds, without needing figures or models—and all this will be done as exactly as if by figures and models. Actually, more exactly and more usefully. In this way in fact we can represent through very few little lines that which would otherwise require many big figures. And while both figures and – when it comes to solids – models require a long time, not to speak of the expense, and motions can hardly be represented by figures without resorting to models, on the contrary thanks to this art we can describe a greater number of figures and their transformations or motions in just an hour than figures could be drawn in a week or models be constructed in a year. Most of all, since many of the figures and models we might have described at the beginning of our research will prove useless in the end, we will have wasted time and money; on the contrary, through a mental or representational drawing and construction, many figures and motions will be easily shown in a short time and at no expense.

You will object, however, that you can do the same thing by imagination, certainly to a point. But this is precisely why this art has been invented—to relieve and perfect imagination.[1] To **relieve** it, in order to avoid the imagination's being too strained by intense attention to figures, which certainly must be applied in

[1] Cf. *De Analysi Situs* (circa 1693) : «I like to call it *analysis situs*, because it explains situation directly and immediately, so that, even if the figures are not drawn, they are portrayed to the mind through symbols; and, whatever the empirical imagination understands from the figures, this calculus derives by exact calculation from the symbols. All other matters which the power of imagination cannot penetrate will also follow from it. Therefore this calculus of situation which I propose will contain a supplement to sensory imagination and perfect it, as it were. It will have applications hitherto unknown not only in geometry but also in the invention of machines and in the descriptions of the mechanisms of nature» (GM V, pp. 182-83; tr. Loemker). See also the letter to Huygens of 29 December 1691 (GM II, p. 123; A III, 5, n. 53, p. 239). It is worth noting that Leibniz used similar words addressing Huygens as early as 1679 (GM II, pp. 17-25; A III, 2, n. 347, pp. 851-60).

persequendis maxima esse debet, nimis fatigetur; **perficiat** vero, quoniam imaginatio nostra non simul multa considerare potest, praesertim cum ea quae non simul videri possunt, sunt simul cogitanda, ubi phantasmata mentem deserint; hinc fit, ut antequam ad postrema perveniat, prima effluxerint; certè distinctè multa considerare; et per multas trasmutationes ire, nihilque negligere aut transsilire, difficillimum, nec nisi à magnis artificibus in iis rebus in quibus jam multum habent usum fieri potest; quamquam et ipsi saepe peccent & haereant in difficilioribus.

Verum quod maximum est, praestat haec ars aliquid quod sit super omnem imaginationem, non tantùm enim exhibet figuras sed et explicat atque exprimit, intimas earum naturas seu causas, adeoque rem reducit ad perfectam analysin | et ex hac rursus resultantem synthesin.

Eaque ratione fit ut possimus omnes ordine modos enumerare efficiendi propositas figuras motusque; eligereque aptissima in rem praesentem, et determinare quod est impossibile, prorsus ut Algebra facit in numeris. Algebra hoc habet sanè egregium, quod eius ope etiam figurarum et motuum problemata possunt solvi, sed quia omnia a situ transferuntur ad magnitudines earumque habitudines seu ad calculum numerorum. Hinc ubi semel à figura rem transtulimus ad Algebram, jam imaginationem planè deserit mens, et quasi per metaphysica vagatur, dimensiones scilicet altiores quam ut in figuris dentur; unde ad exitum quidem pervenitur, sed miris ambagibus, à rei ipsius contemplatione remotis; omni cogitatione in symbola conversa.

Sed in novo hoc genere calculi repraesentatoris perpetuò imaginatio mentem vel calamum comitatur, ut quicquid calamus designat, id continuè phantasia possit imaginari. Et animus numquam ita symbolis adhaeret, ut rei ipsius considerationem deserere cogatur, ac proinde quaevis lineola est theorema aliquod vel proprietas ipsius figurae vel motus. (Ita tamen ut non sit necesse – nisi velimus – actu ipso calculum imaginatione sequi). Hinc etiam calculo ipso adhibito et constituto sequetur mirifica confirmatio atque confortatio pursuing the figures and implicated motions; and to **perfect** it, on the other hand, because our imagination cannot take a vast plurality of things together into consideration, especially when things that cannot be simultaneously seen need however to be simultaneously thought of, but then the images have already deserted our mind; before the last of them arrive, in fact, the first ones have already slipped away. It would be very difficult for our imagination to take distinctly many things into consideration, or go through many transformations, without overlooking or skipping anything. Nor can it be done unless by resorting to great artifices in those things in which they are already usually employed; although even such artifices often fail and leave us when the proceedings are more difficult.

What is most important, this art grants something that goes beyond all imagination, and in fact not only does it show figures but it also explains and expresses their inner nature and causes, and thus it reduces the thing to a perfect analysis and from it back to the resulting synthesis.[2]

For this reason we are enabled to enumerate in an orderly manner all the ways of obtaining the figures and motions we want; and to choose among them the most suitable to the present case, and determine the impossible ones, in exactly the same way in which Algebra does with numbers. Algebra surely has this advantage, that thanks to it the problems concerning figures and motions can also be solved, but because all of them have been shifted from situations to magnitudes and their properties, that is to say to numerical calculus. Therefore once we have shifted from figures to Algebra, our mind completely deserts imagination, and it almost wanders through metaphysical regions, that is, through higher dimensions than those to be found in figures; hence we do arrive at a result but at the price of many intricate twist and turns, estranged as we are from the contemplation of the real thing, all thought having being reduced to symbols.

On the contrary, in this new kind of representational calculus all the time our imagination goes along with both our mind and pen, so much so that phantasy can continuously imagine whatever the pen is drawing. Furthermore, our mind never has to stick to symbols as strictly as to be compelled to abandon the consideration of the thing itself, and thus any small mark whatsoever is a theorem or a property of the figure or its motion. In such a way that we do not need – unless we want to – to follow our calculus with the very act of imagination. Furthermore, once we have resorted to and established this calculus, a wonderful confirmation and en-

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[2] Cf. *De Analysi Situs*: «Yet this kind of analysis [the ancient geometry] does not reduce the matter to a calculation, nor it is carried through to the first principles and elements of situation, as it is necessary for a perfect analysis» (GM V, p. 179; tr. Loemker).

imaginationis, ut possimus in postremum etiam difficillimas res et maxime compositas consequi, idque sine calamo et charta aut figuris, habito semel hoc filo Ariadnaeo mentis in imaginando quod iste calculus nobis ostendet.

Ac jam ostendi in tentamine quodam huius artis, quomodo sphaerica et plana superficies, itemque circumferentia circuli et linea recta per notas repraesententur exactissimè, ita ut eorum quasi imago cum notis menti subjiciatur (expressione tamen, non phantasiam tantùm, iuvante, sed et menti definitiones praebente). Hinc sequitur omnes alias lineas et superficies eodem modo posse exhiberi, quia per has possunt determinari.

Ultima analysis omnium erit in situs seu respectus punctorum. Videndum quomodo cognosci possit punctum quaesitum habere plures valores possibiles. Non semper eundum ad ultimam analysin, sed quo licet manendum in mediis, seu in ipsis motuum genesium repraesentationibus; ut in calculis ita et [hic] semper determinanda puncta per locorum intersectiones. Cum magni sit usus calculus Trigonometricus res ita instituenda est ut quaelibet problemata ad ipsum commodè referantur, est enim algebraico commodior in praxi.

dorsement of the imagination will follow, so that in the end we will be able to achieve even the most difficult and complicated results, and this will happen without any pen and paper or figure, once and for all endowed as we are with this Ariadnean thread of the mind in imagining what this calculus is going to show us.[3]

And through a few essays of this art I have already shown how a spherical and a plane surface, as well as a circumference and a straight line are represented by characters with the utmost exactness, in such a way that their quasi-images along with the characters are made subject to the mind. This kind of expression, however, does not only help phantasy, but it also offers definitions to the mind. Furthermore, all other lines and surfaces can be exhibited in the same way, because they can be determined through the above-mentioned surfaces and lines.

The ultimate analysis will concern the situation or consideration of points. It remains to be seen how to know that a required point has a plurality of possible values. We do not always need to go to the ultimate analysis, but we can stop midway, that is to say in the representations of the geneses of motions; here too, just as in calculating, points must always be determined through the intersections of the loci.[4] As the Trigonometrical calculus is very useful, this new discipline has to be established in such a way that any problem whatsoever may be easily reduced to it; for in practice this calculus is easier than the algebraic one.

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[3] For a similar phrasing in an early text (1679?) see the conclusion of the *Inventorium Mathematicum* (GM VII, p. 17).

[4] For some developments on the punctual determination of a locus, and the concept of *semideterminatio*, see the *Specimen Geometriae Luciferae*, GM VII, pp. 261-62.