Leibniz on Geometry: Two Unpublished Texts with Translation and Commentary

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Leibniz pursued the study of his new geometry, the *analysis situs*, throughout his lifetime. The following essays, the *Ars Representatoria* and the text *Uniformis locus...*, both go back to the early nineties of the seventeenth century. Although most of the *analysis situs* texts are still unpublished—which makes it very hard to see how this discipline might have evolved during the years—there seems to be no doubt that the beginning of 1690s marked a breakthrough in Leibniz's studies.

The first noteworthy essays on the new Leibnizean geometry go back to 1679.[1] Here the especially relevant symbolic interest links them, as a particular *specimen*, to the great development attained in those years by the *characteristica universalis*. Such a tendency will somehow regress during the next decade, in which Leibniz will be less busy elaborating on symbolism and more engaged in fully recognizing the peculiar value of his geometrical endeavor. However, even though Leibniz had already studied Euclid during the seventies, at the time he was too pressed by his own projects and too enthusiastic about his original discoveries really to absorb ("*autrement qu'on a coustume de lire les histoires*" [2]) the full richness of the *Elements*. Thus, while his geometrical research remained indeed original, it was rather lacking in authentically scientific content. In this respect, his 1689-90 journey to Italy certainly represented a turning point in his studies on the analysis of situation. Leibniz first came into contact with the works of Italian geometers - in particular with the book *Euclide Restituto* by Vitale Giordano, the scholar who might have also prompted him to read Borelli's work. In fact, a brief yet intense epistolary exchange took place between Leibniz and Giordano. Leibniz might have noticed that the mathematicians he met in Italy, although at the top of their fame and scientific understanding, maintained respect for Euclid's geometrical formulations. Thus, he resolved to resume and expand his work on the *Elements*, recognizing that just in mastering Euclid he would possibly be able to attain the required geometrical sophistication that would free his new situational calculus from the programmatic abstractness of generic formulations or discussions about philosophical foundations.[3]

In a sense, therefore, not until the beginning of the nineties does the history of the *analysis situs* depart from the simple history of the *ars characteristica* to enter distinctively into the history of geometry. Leibniz's in-depth study of Euclid is witnessed, on his return to Hannover, by an ample unpublished text, the *Demonstrationes Euclideas*, and in the next year by the rich critical commentary on Arnauld's *Nouveaux Elémens de Géométrie*. Our *Ars Representatoria* may itself also go back to 1691, and the text *Uniformis locus*... to 1692. The paper *De Analysi Situs* that has been published by Gerhardt is dated to the following year. In the middle of the decade we finally find the great systematic exposition entitled *Specimen Geometriae Luciferae*, the masterpiece of Leibniz's geometry, which in many respects represents the precipitate of those previous studies and in any case only receives perspicuity in the light of them.[4] In the same period - that is after the Italian trip and before the *Specimen* - several letters can be mentioned in which Leibniz let some of his correspondents know about his geometrical research. Between 1690 and 1694, at least Arnauld, Jakob Bernoulli, Huygens, Mencke (the editor of the *Acta Eruditorum*), and Bodenhausen (another Italian scholar of some relevance for the history of the analysis of situation) were all told about Leibniz's project. The most attentive correspondent of Leibniz's, however, was the Marquis de l'Hospital, to whom he mentioned, in 1693, his project about a science that might express *situs* in much the same way in which algebra expresses *magnitudo*. L'Hospital showed a certain interest, answering that he was looking forward to receiving more details. When, in the following year, he wrote again to ask for them, Leibniz answered with the well-known sentence: «My metaphysics is all mathematics, so to speak, or it can become so. At the present, I dare not publish my projects on the **characteristica situs** because, unless made believable through examples of some importance, it would be regarded as just a vision. Nonetheless I see in advance that it will not fail».[5]

Exemplarily expressed, this is another feature of the major turning point affecting the 1690s studies on the *analysis situs*—while parting from logic, they become more and more involved with metaphysics. Such a tendency will go on acquiring strength to the very last years of Leibniz's life. From now on, the profound geometrical research Leibniz begins to undertake going into the foundational core of ancient geometry will provide him with an incredible arsenal of metaphysical arguments with which he can start to build his own philosophy of space and graft it on his mature monadological theory. His subsequent writings on the *analysis situs*, mostly still unpublished, will originate from the same themes he has dealt with at the beginning of the nineties and, by much elaboration, they will lead step by step towards the *Initia rerum mathematicarum metaphysica* and another cluster of similar papers on situational analysis which intertwine with the writing of the *Monadologie* and the letters to Clarke.[6]

The following texts therefore play a major role as marks of the development of Leibniz's thought on geometry. Dealing with very different and complementary topics, together they offer a good introduction to the variety of issues related to the analysis of situation during the nineties.

The *Ars Representatoria* is still a general and programmatic paper interested in problems of characteristics. Leibniz reaffirms the starting point and purpose of his *analysis situs*—meant to avoid the use of extrinsic algebraic proceedings in geometry (which many seventeenth-century mathematicians were criticizing), without however having to give up the advantages of the *characteristica speciosa* and go back to the elegant, but complicated and unproductive, synthetic geometry. It is a matter of reforming algebraic formalism and finding another one that may be *proper* to geometry. Although it can be blindly manipulated, such formalism must have this advantage on ordinary algebra—that, when interpreted, it immediately offers the geometrical object to our imagination. It is therefore a *cogitatio caeca* that promises a vast view. In any case, the *Ars representatoria* does not get entangled in technical disquisitions on formalism, but immediately stands on a superior level of investigation. There in fact Leibniz discusses the role of imagination in geometry, even though in the end he does not seem really to reconcile the two purposes of his new calculus—to relieve (*sublevare*) imagination, and to perfect (*perficere*) it. What matters here, however, is that Leibniz clearly realizes the impossibility of being content with just the intellectualistic stance of many Cartesians who wanted geometry to be a science estranged from imagination. Therefore, the discussion in the *Ars Representatoria* in many respects anticipates Leibniz's late essays on the theory of knowledge and interlocks with the theory of the *imaginatio distincta* that a little earlier Leibniz began to envision as the solution to many epistemological questions. In this sense, this paper certainly foreshadows the phenomenological and not merely logical distinction between understanding and sensibility, which will become a constitutive and crucial element in the phenomenalism of his fully mature metaphysics.[7]

Here I will also mention the problem of the determination of loci through points, which has affected both geometry and metaphysics,[8] as well as the concept, connected to the former, of the *ultima analysis* (with which the essay ends – however in a rather abrupt way). The importance of the notion of a point as well as the possibility to get to points through the geometrical analysis of a locus plays a main role in the papers on the *analysis situs* from the nineties. It is in fact since then that Leibniz becomes more and more convinced that (at an ideal level) space can actually be regarded as constituted by points, however meaning them as abstract situational elements.[9] It is only on these grounds, of course, that he will be able finally to understand space as a simple set of relations, as we see in his famous correspondence with Clarke. Such a turn in modern geometry – which on the contrary was going on, and would go on for a long time, considering points as simply abstract boundaries of the extended – seems to me to project Leibniz's research on the analysis of situation far ahead in the history of mathematics.

Uniformis locus... is more technical. Nevertheless, the geometrical issues dealt with in this text are rightly regarded as foundational ones. Leibniz's argument on the uniformity properties of the various loci is exceptionally relevant to geometry as a general theory of the invariance of figures under different kinds of transformations. In addition, the determination of *spatium ipsum* through such properties opens the way to a philosophical deduction of real phenomenal space.

The all-important concept of uniformity (in Leibnizian strict terms) first appears in this text. Such uniformity can be more or less understood as what we would today call homogeneity or, perhaps better, isotropy of space. It is the property that grants the transportation of figures through congruence, and thus free motion. What is most important is that Leibniz arrived at the analysis of this property from—besides metaphysics—his improved familiarity with the works of classical geometry, which though treating the matter at some length did not adequately underline its foundational weight. Since it can also be expressed as constancy of curvature, uniformity must have led Leibniz in the following years to his discussion of the Euclidean parallel axiom—a discussion which however does not yet appear in this text (nor, as far as I know, anywhere else before 1712).

Although it is not a discovery of the nineties, a second all-important property mentioned in this text—self-similarity—does increase its salience in this period. It is in fact the concept of similarity that becomes more important. In the subsequent *Specimen Geometriae Luciferae* it will fight against congruence for definitional and conceptual priority in Leibniz's mathematics, and then in his philosophy too. The definition of a straight line as a self-similar curve is probably the best one Leibniz has ever found, and in any case the most fruitful (Leibniz identifies a self-similar curve with the *linea simplicissima*, in which simplicity means formal identity of any of its parts). A great section of the essay is devoted to the attempt to demonstrate the definitional equivalence between this determination of a straight line and the slightly more technical one that resorts to congruence. Actually, Leibniz's demonstration here does not hit the target—in fact, it should even imply a discussion of the parallel axiom, which on the contrary is still lacking. However, this very difficulty signals how Leibniz has been applying himself to some of the deepest problems of classical geometry and how in the following years such a commitment to this line of demonstrations will bring about essential results both in the scope of geometry and in the more general one of philosophy of space.

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Notes

A	G.W. LEIBNIZ, *Sämliche Schriften und Briefe*, Darmstadt/Leipzig/Berlin 1923-.
GP	G.W. LEIBNIZ, *Die philosophischen Schriften*, edited by C.I. Gerhardt, Berlin 1875-90.
GM	G.W. LEIBNIZ, *Mathematische Schriften*, edited by C.I. Gerhardt, Berlin/Halle 1849-63.

1 A large collection of them is to be found in G.W. LEIBNIZ, *La caractéristique géométrique*, edited by J. Echeverría and M. Parmentier, Paris 1995.

2 From Leibniz's first letter to Foucher, undated but presumably written in '76, which can be read in GP I, p. 369, and in A II, 1, n. 120, p. 247.

3 The volume by V. GIORDANO, *Euclide restituto*, Roma 1686 (2nd edition augmented), and that by A. BORELLI, *Euclides Restitutus*, Pisa 1658, are both kept in Hannover and full of *marginalia* by Leibniz. The correspondence with Giordano only amounts to three letters, two by Leibniz and one by the Italian mathematician, all of them dating back to November 1689. It can be read in GM I, pp. 195-200, but the critical edition, in A III, 4, nn. 216-18, pp. 420-28, is also useful, as it offers a first draft of Leibniz's second letter that is much clearer than the copy he actually sent. Leibniz read Giordano's book while he was in Rome, and wrote to him straightaway. At the time, he was unlikely to have read the book by Borelli that Giordano had recommended, since he ignored it in his answer.

4 The full title of the rather vast writing on Euclid is itself an outline of the essay: *Demonstrationes Euclideas, ut à Clavio exhibentur revocabimus quoad opus et ratio est, ad Calculum Situs, quo melius talis Calculi Elementa constituamus*. It remains unpublished in LH XXXV, I, 3, Bl. 1-6 and ECHEVERRÍA, *La caractéristique géométrique*, p. 42, dates it 1680, against the 1690 suggestion of the *Leibniz-Archiv*; to solve the question, a complete edition of the paper would be needed. The Leibnizean *Remarques sur les Nouveaux Elémens de Géométrie* by Arnauld are to be found in LH XXXV, I, 21. They are divided into two parts, the first one made up of notes to the first five books, and the second one of extensive discussion, at times aside from the text, about the fifth book, which deals with the axioms of Euclid's Book One. Revolving around the same subject of *Ars Representatoria* but maybe of a lesser interest, there is a *Characteristica speciosa*, in LH XXXV, I, 14, Bl. 78-79, which may go back to 1695. *De Analysi Situs* is in GM V, pp. 178-83. The *Specimen* is in GM VII, pp. 260-99.

[5] The letter to Arnauld dated 23 March 1690 can be read in GP II, pp. 134-38. The letter dated 24th of September of the same year and sent to Bernoulli who, however, did not respond to Leibniz's suggestions, is in GM III, pp. 13-20; A III, 4, n. 279, pp. 571-84. Huygens answered Leibniz's letter of the 21st of September 1691 (GM II, pp. 103-108; A III, 5, n. 39, pp. 171-79) on the 16th of November (GM II, pp. 109-13; A III, 5, n. 46, pp. 196-202) confirming his (not at all unreasonable) skepticism about the possibility to exhaust all mathematics in one decade. The letter to Mencke is in A I, 8, n. 225, pp. 384-86. Leibniz's first letter to L'Hospital is in GM II, pp. 227-32, the answer and the following one by the French scholar, asking for explanations, are respectively dated 15 June 1693 (GM II, pp. 241-45) and 30 November 1694 (GM II, pp. 249-55). The quote comes from Leibniz's letter of 27 December (GM II, pp. 255-62).

[6] This link between *geometria situs* and metaphysics in the late Leibniz's thought was the subject matter of my doctoral dissertation, which has been reworked into a book, *Geometry and Monadology. Leibniz's* Analysis Situs *and Philosophy of Space*, forthcoming for Birkhäuser (Springer). In it I have also given an edition of Leibniz's late manuscripts on geometry.

[7] The expression *imaginatio distincta* first appears in *De Ortu, Progressu et Natura Algebrae, nonnullisque aliorum et propriis circa eam inventis* (ca. 1685; in GM VII, pp. 203-16). The term also recurs in Leibniz's correspondence with Bodenhausen, in a fragment (GM VII, p. 355; A III, 4, n. 242, pp. 473-77) that may be contemporaneous with the *Ars*.

[8] Leibniz devoted several geometrical papers to the problem of the determination (or "semi-determination") of loci through a given number of points, and he surely came close to the theorem concerning the number of independent points needed for the determination of an algebraic curve having any degree whatsoever, a theorem which today goes under the name of the Newtonian mathematician Stirling (who published it in 1717). The problem however also surfaces in Leibniz's philosophical writings, chiefly in his correspondence with Clarke (which heavily depends on the *analysis situs* research); cf. the *Fifth Paper* to Clarke, § 47 (GP VII, pp. 400-401). Note that the Latin word *locus*, which in translating the Leibnizean texts below I have left unaltered to follow the geometrical usage, is rendered by Leibniz with *place* in French, and it also appears as such in his correspondence with Clarke and in the *Nouveaux Essais sur l'entendement humain.*

[9] A suggestion of it already appears, for example, in the great *Dynamica* of 1690 (I, IV, 1, prop. 5; GM VI, p. 370). More definite statements can however be found in several geometrical writings dating in the same years («Spatium universum est locus omnium punctorum», from a *Scheda* of 1695, in LH XXXV, I, 14, bl. 90).