G. W. Leibniz, *The Labyrinth of the Continuum: Writings on the Continuum Problem, 1672-1686.* Translated, Edited, and with an Introduction by Richard T. W. Arthur. New Haven: Yale University Press, 2001. Pp. lxxxviii + 484.

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This book consists of four parts: 1) an introduction of about 60 pages, mainly focused on the problem of the continuum in Leibniz; 2) a selection of Leibniz's papers according to the critical edition prepared in Berlin (and Münster), with English translation; 3) two appendices with excerpts in English from Leibniz's papers and from texts of other authors on the continuum; 4) a rich collection of notes and a Latin-English glossary (with an index and English-Latin glossary). In what follows I will make some short remarks about each of these parts.

I

The introduction develops the central ideas of Arthur's seminal paper: *Russell's Conundrum. On the Relation of Leibniz's Monads to the Continuum* (in J. R. Brown and J. Mittelstrass, eds., *An Intimate Relation. Studies in the History and Philosophy of Science*, Reidel, Boston-Dordrecht-London, 1989, pp. 171-201). In respect to this paper, here Arthur substantiates with more evidence his interpretation and gives a clear and very convincing account of the development of Leibniz's doctrine about the continuum. The final result is an essay which represents a turning point not only in the studies on Leibniz's philosophy of mathematics but also in the studies on Leibniz's metaphysics.

As is well known, in the 17th Century (but the same holds for ancient and medieval times) the problem of the continuum was tackled without distinguishing the mathematical from the physical issue. Roughly speaking, the mathematical problem was to explain how the continuity of geometrical figures originates (a right line or a curve, for instance); whereas in the physical case the problem consisted in giving an account of continuity in physical bodies (the surface of a table, for example). From the mathematical (geometrical) point of view, a major challenge was that of justifying how simple, non-extended points could give rise to continuous extended lines and figures; from the physical point of view, the problem was how physical objects with continuous parts could originate from mere aggregation of separated, individual atoms (or from the smallest parts of

matter). Leibniz's mature solution to the problem—on Arthur's reconstruction—is based on a strong analogy between the mathematical and the physical case. Concerning geometry, Leibniz thinks that points are not the component parts of the line: a segment or a line is an aggregate of points, not something *composed* of them (as, for instance, a house is composed of building blocks). A segment may be cut into other segments which constitute its *parts*, and each part may be cut again into other parts to infinity, without arriving at a point. In the same way, Leibniz believes that, if we cut bodies into parts, and these parts into parts again and so on, we never reach atoms of any sort (pp. lvii-lx). Thus Leibniz admits the existence of points, *denying* that they may be parts of geometrical figures; and in the same way he admits (better: he *postulates*) the existence of atoms, *denying* that they may be parts of physical bodies.

Leibniz's atoms, as is well known, are *soul-like* entities, incorporeal beings—the individual substances named by him "monads"—and it is their not being corporeal that prevents them from composing physical bodies. That points cannot be part of the line is first of all a consequence of the fact that point and line are not homogeneous things: a point is extended, whereas a line is not. Moreover, resting on an argument which was well known at the time, Leibniz shows that, if points were parts of the line, then the principle that the whole is greater than its proper part would be violated and a paradox would obtain. To see that, suppose that each point of the side of a square is put in a one-to-one correspondence with each point of the diagonal of the same square: then the number of points belonging to the side is the same as that of the points belonging to the diagonal. But if we project the side onto the diagonal, we may easily see that it is shorter than this latter: thus side and diagonal are composed of the same number of points but, at the same time, the first is shorter than the latter. Therefore, to avoid this paradoxical consequence, Leibniz denies that points *compose* the line (pp. lxii ff.).

Thus, assuming this kind of parallelism between points and individual substances, as a line is composed of smaller lines or segments, bodies are composed not of monads, but of other smaller bodies; and the smaller bodies are composed of even smaller bodies, *ad infinitum*. Here the two central notions are, respectively, that of *aggregate* and that of *corporeal substance*. The totality of parts which compose a given body constitutes an aggregate and each part, in its turn, is an aggregate of *corporeal substances*. A corporeal substance is a complex constituted by an aggregate (of other corporeal substances) and a dominant, simple monad. Speaking of corporeal substances, Leibniz says that they are made of a simple monad which *dominates* the associated body, and that the aggregate constituting

the body receives a true unity from the dominant monad. This unity, however, means simply that the body obeys the dominant monad and that this latter exerts its control over the colony of subordinated corporeal substances. "Unity" in this case does not allude to any kind of physical link among the constituent parts of the body. Each part of a body is an aggregate, and aggregates do not have a true unity: they are not a whole in a proper sense. Thus Leibniz claims: 1) that each body (and each proper part of a body) is *actually divided* at each instant into an infinity of parts; 2) that there does not exist a smallest part of an aggregate because, taking a part as small as one wants, there is always another part even smaller than it.

On Arthur's reading, the notion of *corporeal substance* assumes in Leibniz's mature philosophy a privileged role which is not so easily recognized in the present literature. On many occasions Leibniz claims that physical bodies are simply aggregates like heaps of stones, lacking in intrinsic unity and that they receive unity from the perceiving subject. This claim is usually accompanied by the austere statement that there is nothing in the world other than simple monads and aggregates. On other occasions, however, Leibniz recognizes the difference between organic and inorganic bodies, and characterizes the organic ones as endowed with *intrinsic* unity. What possess this intrinsic unity are bodies of *corporeal substances*. Thus the first claim authorizes a kind of combinatorial view of the physical word: there are simple monads from the aggregation of which all bodies are produced; bodies are mere phenomena and the only real entities in the world are simple monads. As we read in a text written after 1695:

> [...] simple substances seem to be the only true substances, whereas all other things—i.e. the aggregates—if we want to speak accurately, exist only νόμῳ, to use the expression of Democritus, that is on the basis of our opinion (LH IV 3, Bl. 2 v).

In contrast to the first, the second claim seems to concede more to a traditionally Aristotelian perspective, conceiving substance as composed of form (the dominant monad) and matter (the body), where the body receives its unity from the monad dominating it. Many scholars think of these two views as conflicting, and a lot of work has been done attempting to introduce some coherence into Leibniz's metaphysics of substance. In the introduction Arthur does not devote much attention to this question: here he simply assumes that the two views are not conflicting. However, in a paper published in this review (*Infinite Aggregates and Phenomenal Wholes: Leibniz's Theory of Substance as a Solution to the Continuum Problem*, Vol. 8, Dec. 1998, pp. 25-45) Arthur explains at length the

reasons for his "compatibilist" assumption. To express these reasons very briefly, in Arthur's own words:

> [...] Leibniz's commitment to the reality of corporeal substances is steadfast. He neither abandons them later in life, nor is he less than earnest in maintaining that they are true unities. Although "ultimately" all God needs to posit are the simple substances from which compound substances result, once he has created simple substances he has *ipso facto* created corporeal substances too. And since each corporeal substance is formed into a true unity by its dominant monad, it is a substance, and not a mere phenomenon. (p. 27)

Thus, Arthur interprets Leibniz's assertion that there is nothing in the world other than simple monads and aggregates as compatible with the view that there are corporeal substances, because corporeal substances, in the last analysis, are made of simple monads and aggregates of monads. It seems to me, however, that this solution underestimates the fact that Leibniz, as we have seen, states that *all aggregates* receive their unity from our perception, maintaining at the same time that corporeal substances have a true unity *independently* of our perception. If simple substances are "the only true substances" and "all other things—i.e. the aggregates—[...] exist only νόμῳ," this seems to be in overt conflict with the assertion that corporeal substances are a kind of aggregate which *do not* exist on the basis of our opinion only.

A possible solution to this antinomy may be that Leibniz uses the word *unity* referring to two different kinds of unities. That all aggregates receive their unity from our perception may be true of all bodies, including bodies of corporeal substances: this is not incompatible with the claim that corporeal substances (i.e. organisms) enjoy a supplementary unity, induced by the functional de-pendency on the dominant monad. The first kind of *unity* is the unity that all bodies—organic and inorganic—receive from our perception, whereas the second kind is the unity which organic bodies have in themselves, independently of our perceiving activity. Thus, on this interpretation, even the body of a corporeal substance lacks of a certain degree of unity which will be supplied by our knowing activity.

II

A relevant consequence of the picture Arthur gives of Leibniz's ontology is that the entire fabric of the world, comprised of all inorganic bodies, is built on *organic* parts (i. e. parts composed by corporeal substances). A second consequence is

that what appears as a continuous body is simply an aggregate of contiguous parts: *physical continuity* is just an appearance due to our way of perceiving and knowing concrete things. And it is on the basis of this appearance that we human beings conceive *mathematical continuity* (p. lxxiv).

Leibniz distinguishes sharply between what is *ideal* and what is *real*. To the realm of the *ideal* belong, for instance, the mathematical concepts of circumference, that of the *infinitesimal* and that of *continuity*: all these notions refer to mere ideal entities which cannot be found among the real, existing things. They are *abstractions* got by means of the activity of the understanding and with the help of the faculty of *imagination*. At first we perceive as continuous the surface of a body, thus getting the concept of physical continuity; and then, building on this concept, we are able to work out the concept of *mathematical* continuity. In contrast to what appears, real bodies are not continuous: they are subject to instantaneous motions which fragment them into an infinity of parts; and these motions prevent them from giving rise to something uniform and truly continuous. As Arthur observes:

> Concerning the actually infinite division of matter, three considerations seem particularly pertinent. First, there is the infinitude of the division itself. Given Leibniz's rejection of infinite (cardinal) number, this implies that there is no assignable number of parts. If every body or part of matter is further divided, then no body can be considered a completed collection of its parts. Second, since the division is without a lower bound—i.e. there are no minima and no atoms—every body is an aggregate of parts that are themselves bodies. Thus every part is an aggregate, but no part is a unit, because what is divided is not truly one. Consequently, matter has a merely accidental unity. Third, because the divisions of matter are determined by the instantaneous motions of its parts, and because these are constantly changing due to the actions of the plenum, no part will remain exactly the same for longer than a moment, and the aggregate of parts will change from a one instant to another. So matter has a merely accidental unity through time as well; and the same applies to universal space, or the aggregate of places [pp. lxi-lxii].

The analogy between the point and the line on one hand and the individual substance and the body on the other, surely casts light on Leibniz's solution to the problem of the continuum, but it takes for granted the existence of substances. As emerges from Arthur's introduction, to have an account of the reasons which drove Leibniz to assume the existence of substances one has to turn to physics and, in particular, to dynamics. The notion of *force* here plays a major role:

Leibniz realizes that the mechanistic doctrine dominating at the time and centered on the concepts of motion, figure and matter is insufficient to explain the nature of force. Force as such cannot be reduced to the classical notions of mechanistic philosophy: it is something primitive which witnesses the existence of an immaterial entity situated, as it were, *behind* the appearances of figure, matter and motion. This view of the mature Leibniz is rooted, as Arthur shows, in the period between the years 1668-1671, when the idea takes shape "that mechanism requires a foundation in individual substances whose principle of activity is mind" (p. xxviii).

Leibniz's view of physical world is strongly influenced by Cartesian philosophy. At the beginning of his philosophical career, as Arthur emphasizes, Leibniz refuses to accept the existence of the void, and thinks of matter as a uniform fluid mass in which motion induces various forms, giving rise to the physical objects. Progressing further in the study of mechanics, Leibniz recognizes that motion in itself is not sufficient to explain how different portions of matter may cohere, thus constituting a body. Therefore, during the years 1675-76 he devotes a great deal of his intellectual effort to giving an account of physical cohesion (pp. 38 ff). From this period onwards, the problem of cohesion, that of the nature of force and that of the physical continuum coalesce, determining a riddle of questions which find their proper answer in the new concept of substance. Arthur locates the emergence of the Leibnizian concept of substance in the *Pacidius Philalethi* (October-November 1676) (pp. lxxvii - lix), where Leibniz for the first time presents an argument which we encounter in his letters to Arnauld and in several other texts. The argument is summarized as follows by Arthur: "if matter consists only in bulk, then there is no unit of matter, but only an infinite regress of parts, with no undivided wholes. Again, if matter consists only in bulk, then each of these parts will be ephemeral: the parts out of which it is in turn constituted last only for an instant, due to the changing motions that define them" (p. lxvii). To hold matter together, to avoid it dissolving into "a powder of points", something non-material is needed—something which may gives true unity and identity to the body. Thus evoked, the notion of substance is soon strongly connected with that of force when Leibniz discovers that what is conserved in motion, is the quantity of force, not the quantity of motion as believed by Descartes and the Cartesians (p. lxxxiv). Resting on this discovery, Leibniz attributes to each body an innate force, proportional to mv^2 and claims that "insofar as it is a temporal continuant, [it] conserves the same quantity of force overall, and this is merely redistributed among its parts on collision with another body,

producing the effects of cohesion and elasticity. [...] When another body collides with it, this provides a mere *occasion* for a body to undergo a redistribution of its internal motions, in keeping with the conservation of its total force" (*ibidem*). Considering the case of motion, Arthur argues that, for Leibniz, it is substance, "not anything phenomenal, that consists in an endeavour or instantaneous tendency to change its relations to all other things according to a certain laws contained in it. [...] Thus, even if continuity cannot be found in the phenomena of motions, all such motions presuppose a continuity of activity that is resolvable into instantaneous changes of state or endeavours or 'appetitions'" (p. lxxxvi).

In conclusion, Leibniz finally emerges from the labyrinth of the continuum recouping "the idea of a continuity of change and duration with the idea of a unity of substance which is continuously acting, and which therefore has a state and an action or endeavour to change this state at every instant" (*ibidem*).

Even accepting Arthur's interpretation, well argued and with its large basis of solid evidence, it seems to me that facing Leibniz's account of matter one still has some reason to be perplexed. The most relevant source of perplexity being the ontological status of matter understood as a uniform fluid mass. In a well known passage of the *New Essays* Leibniz describes matter in a particularly vivid way:

> Rather, we should think of space as full of matter which is inherently fluid, capable of every sort of division and indeed actually divided and subdivided to infinity; but with this difference, that how it is divisible and divided varies from place to place, because of variations in the extent to which the movements in it run the same way. That is what brings it about that matter has everywhere some degree of rigidity as well as of fluidity, and that no body is either hard or fluid in the ultimate degree—we find in it no invincibly hard atoms and no mass which is entirely unresistant to division (AVI, 6, pp. 59-60; *New Essays on Human Understanding*, transl. and edited by P. Remnant and J. Bennett, Cambridge UP, Cambridge, 1981).

I suppose that 'matter' here means secondary matter, i. e. a huge mass of aggregates of corporeal substances. However, at the basis of corporeal substances there are simple substances; and simple substances are some kind of forces. Thus the problem arises how secondary matter can emerge from simple monads —i.e. from something immaterial as forces. As I understand Leibniz, what contributes in an essential way to the constitution of secondary matter is *primary* matter: and the concept of primary matter is strongly connected with our perceiving activity. Therefore, it seems that the Leibnizian concept of matter—

like other concepts of Leibniz's philosophy—has an ambiguous ontological status: it is presented as referring to something objective on one hand and has a phenomenal character on the other.

Arthur, however, does not limit himself to giving an account of Leibniz's final thesis on the continuum problem: he reconstructs in great detail all the Leibnizian attempts to solve the problem during the Paris period and beyond, till the year of the *Discourse on Metaphysics*. Of the several suggestions of the introduction, I have found particularly remarkable that concerning the concept of 'fold', traced back to a probable reading of Francis Bacon on Leibniz's part (p. lix); and that relative to the importance of Gassendi for the formation of the doctrine of *monads*. As Arthur observes: "[...] Gassendi conceived his atoms as elements of matter that God has not only rendered indivisible, but also, as is often forgotten, endowed with active force" (p. xxviii).

III

The original texts of Leibniz are reproduced with some minor misprints which do not influence the English translation. The following is a list of the misprints I have run across on a cursory reading:

p. 2, 10th row from bottom: "cohaerent" instead of "cohaerunt"

p. 30, 9th row from top: "est exquirere quomodo fiat" instead of "quomodo flat".

p. 32, last row: "se penetrare" instead of "se penetrate".

p. 38, 5th row from bottom: "cum alio" instead of "cum allo".

p. 46, 10th row from top: "An videtur rationi consentaneum esse Atomos" instead of "An videtur rationi consentaneum Atomos"; 16th row from top: "multitudo" instead of "multitudino"; 6th row from bottom: "dissolvi" instead of "dissolve"; 5th row from bottom: "Ens" instead of "End".

p. 48, second row from top: "sensuum" instead of "sensum"; 8th row from bottom: "rigorosissime" instead of "rigorissime".

p. 50, 8th row from top: "Hypothesin" instead of "Hypothesian"; 13th row from bottom: "gaudiis" instead of "guadiis".

p. 54, 16th row from top: "ad ostendendum" instead of "ad ostendum"; 3th row from bottom: "separationi" instead of "seperationi".

p. 70, 14th row from bottom: "per aliquam rectam" instead of "per allquam rectam".

p. 88, last row: "illa fictitia" instead of "ilia fictitia".

p. 92, Leibniz's marginal note: "coincidentibus" instead of "coincentibus".

p. 98, 15th row from top: "rei constitutio" instead of "rei constitution".
p. 100, 7th row from top: "Imo sequeretur" instead of "imo sequeretur".
p. 118, 18th row from top: "homogeneo" instead of "homgeneo".
p. 120, 15th row from bottom: "at potest cogitatio" instead of "et potest cogitatio"; third row from bottom: "figura" instead of "figure".
p. 132, 9th row from top: "historia civili" instead of "historian, civili".
p. 158, 12th row from bottom: "quasdam interspersas" instead of "quasdam intersperses"; second row from bottom: "pro quietibus interspersis" instead of "pro quietibus intersperses".
p. 162, 11th row from top: "quietes interspersas" instead of "quietes intersperses"; 13th and 16th row from top: "interspersa" instead of "inetrsperse".
p. 206, 11th row from bottom: "lineas ordinarias" instead of "lineas ordinaries".
p. 230, 16th row from top: "dubitare incipimus" instead of "dubitare incipmus".
p. 240, 10th row from bottom: "immediate, quae" instead of "immidiate, quae".
p. 250, 12th row from the bottom: "scilicet" instead of "siclicet".

The translation—as far as I can judge—is very good. Arthur was confronted with a difficult task, because most part of the texts he translates are written in a hard Latin, full of technical words from the XVII Century scientific vocabulary, and afflicted in many points by an intricate style and a heavy syntax. As is well known, Leibniz claimed that human beings cannot think without having recourse to signs—i.e. without the help of a spoken or written language. Looking at his manuscripts, one can hardly resist the impression that this claim was probably grounded on personal experience. The exceptional mass of unpublished manuscripts which Leibniz left behind during the years may be explained only by postulating an incapacity on his part to think without writing. This intimate relationship between thinking and writing results in texts which possess a great complexity and which allow the reader direct access to Leibniz's most private way of developing his thoughts. In many cases, however, this advantage is balanced by the difficulty of penetrating into the labyrinthine prose of a thinker who struggles with great energy to find his own solution to hard philosophical problems.

Reading Arthur's translation one can only be impressed by its faithfulness to the original and—at the same time—by its clarity. The most intricate passages are adapted to the syntactic structure of contemporary English; very long sentences are wisely divided into shorter ones, and all this gives rise to a text which maintains intact a characteristic "Leibnizian flavour". There is a point where Arthur's translation does not fit the original (if I interpret correctly what

Leibniz says). On p. 91 the Latin text runs:

> Ergo imago illa perfectum exprimit Circulum. Est hic quaedam difficultas mira et subtilis. Etsi enim falsa sit imago, in se tamen Ens est verum; adeoque sequitur in mente esse circulum perfectum, vel potius esse imaginem realem.

Arthur's translation is:

> Then the image presented to the mind is a perfect circle. Here there is a surprising and subtle difficulty. For even if the image is false, the entity in it is nevertheless true; and so it follows that in the mind there is a perfect circle, or rather, there is a real image.

Leibniz here is contrasting the image of the circle as it may be perceived by men, with the *circle in itself*—i. e. with the perfect circle as we consider it, for instance, in a geometrical proof. Thus, the sentence: "For even if the image is false, the entity in it is nevertheless true" has to be corrected changing "in it" to "in itself": "For even if the image is false, the entity in *itself* is nevertheless true". What is true is the circle in itself, not the circle which is contained in the image (as the expression "in it" seems to suggest).

The texts included in this collection are all of great significance for understanding the development of Leibniz's thought from the Paris period to its maturity. Amongst them the paper on the infinite numbers (April 1676, pp. 83-100) and the long dialogue *Pacidius Philalethi* (October-November 1676; pp. 128-221, translated here in its entirety), are very important for understanding the strong link connecting mathematics, physics (dynamics) and metaphysics in Leibniz's philosophy. Another very interesting text dated 15 April 1676 is reproduced only partially (pp. 117-18): unfortunately it has been shortened by an entire page (p. 511 of the Academy edition) which was worth publishing. In the missing page there is the first draft of *On the Method of Distinguishing Real from imaginary Phenomena*, which helps greatly to explain Leibniz's attitude towards realism. As the missing part clearly shows, just in April 1676 Leibniz reached the conclusion that what distinguishes reality from dreams is the regularity and order of what we perceive. Moreover, here Leibniz seems to admit the possibility of worlds not only with different laws of motion, but with a different space too, in respect of the actual world. It is true, however, that these issues are not directly connected with the topic of the continuum, and this may have determined the decision of not translating the page in question.

As I have remarked above, the book is enriched by two appendices and a glossary. The first Appendix includes the translation of selected excerpts from passages on the continuum from Leibniz's works before 1672. The second one

comprises the translation of texts on the continuum from Aristotle, Galilei, Descartes, Hobbes and Gassendi. Even in this case the translation is very good and the choice of texts very appropriate. The same holds for the glossary, which puts at the reader's disposal an extraordinarily useful tool.

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