Leibniz on the Limits of Human Knowledge: With a Critical Edition of *Sur la calculabilité du nombre de toutes les connaissances possibles* and English Translation

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Of the four ancient elements, water provides Leibniz with some of his most memorable metaphors. Not only does he famously compare every piece of matter in his doctrine of monads to a pond full of fish[1], but he also describes the universe itself as being like an ocean without boundaries: all is connected with all and even the smallest motion is propagated to infinity, as a wave across water, even if it thereby becomes less and less perceptible.[2] As is well known, he also compares this with the phenomenon of concentric waves generated when a stone is thrown into water.

But it is particularly when the sciences are concerned that the water metaphors come into their own. Already in *De rationibus motus* (1669), one of his numerous preliminary studies for what eventually became the *Theoria motus abstracti* (1671), Leibniz draws a comparison between the sciences and the course of a river. He equates the source with pure sciences such as mathematics whose truths rest on what is demonstrated from definitions. It is, he suggests, a rather dry and meagre source, but from it water descends continuously into the most fertile rivers of mixed sciences like acoustics, optics, and mechanics which in turn flow out into a sea of various uses and applications.[3]

In later years, he switches his attention in this respect to the divisions which we make within the body of human knowledge and especially within the sciences themselves. Thus more than a decade after *De rationibus motus* we find him questioning the nicety of the distinction between eternal truths on the one side and knowledge gained principally through observation and experience on the other side. While he does not doubt that logic, metaphysics, pure mathematical science, and a large part of jurisprudence are demonstrated simply from the ideas of things, he now considers the mixed sciences and much of what we know about mankind, the world at large, and physical bodies to be acquired by inductions made from natural and civil history combined with abstract considerations drawn from the higher or abstract sciences. For this reason he is by this time persuaded that the distinction between intellectual and experimental knowledge and all their subdivisions have no fixed limits, just as there is no definite line separating the North

Sea from the Atlantic Ocean or the Irish Sea, but rather "it is one continuous ocean".[4]

Leibniz employs the same metaphor in one of his many drafts on the inception of Scientia generalis which he entitled *Introductio ad encyclopaediam arcanam* (1683/5) and here even goes as far as to say that it is largely unimportant in which way the different sciences are actually divided.[5] This is also the view expressed at the beginning of a draft on combinatorics and the calculability of the limits of knowledge, probably dating from the end of 1693 or the beginning of 1694, which we have edited and translated as an appendix to this note. And it also recurs in the last chapter of his *Nouveaux Essais* (1703/5), to which he pertinently gave the title "On the Division of the Sciences". In the course of describing various modes for dividing and ordering the sciences he mentions certain nominalists who according to him believed that there are as many particular sciences as there are truths, which, depending on the chosen mode of arrangement, one might subsequently unite as a whole. But then he goes on to speak of other philosophers "who compare the totality of our knowledge to an ocean, which is completely one piece and which is only divided into the Irish Sea, or the Atlantic, Ethiopian or Indian oceans by arbitrarily drawn lines".[6] Although he does not explicitly say so, this is clearly the view to which he subscribes. Thus he cites a series of examples which effectively show that one and the same truth can be ordered differently according to the concepts it contains and proceeds to liken this to the situation of a librarian who is uncertain as to where to situate a book systematically, because two or three places would be equally suitable.[7]

By emphasizing that it is we who determine the boundaries of the sciences, Leibniz also makes clear the importance of the task of the systematic ordering of human knowledge.[8] It is here where he sees one of the most significant applications of his art of combinations. The systematic order or encyclopaedia[9] he envisages will namely not only show how existing truths are interconnected but also enable us to proceed to new truths. As we see from the initial wording of *Sur la calculabilité du nombre de toutes les connaissances possibles*, the concept of encyclopaedia was very much in his mind when he first set pen to paper. Just as the compass, one of Bacon's insignia of the modern world, played a decisive role as an instrument for crossing the physical oceans, Leibniz conceived his Ars combinatoria as being able to fulfil the same purpose in relation to the metaphorical ocean of human knowledge.

For Leibniz, the art of combinations in its broader sense largely corresponds to universal character. Conceived as the fundamental method of the Scientia generalis, it will, as he writes to Oldenburg on one occasion, serve to increase the perfection of the human mind, enabling us ultimately to have "no less certainty about God and the mind itself than about geometrical figures and numbers, to overcome scientific disputes, and to have no more difficulty in making discoveries than in the construction of, say, a geometrical problem".[10] Combinatorics is thus of a much broader scope than algebra, which he indeed sees as being subordinate to it.[11] In numerous places, including *Sur la calculabilité*, Leibniz emphasizes that whereas algebra is the science of magnitude or of the equal and unequal, the art of combinations is to be regarded as the science of forms or of the similar and dissimilar.[12] In this way, it provides the foundation for the doctrine of characters, understood as signs which serve to represent the objects of thought. Indeed, as he points out to Tschirnhaus, combinatorics appears to differ little from general characteristic science, "with whose help characters suitable for algebra, music, and even for logic have been and can be contrived".[13]

An essential part of Leibniz's conception of combinatorics is that it is not restricted to synthesis or the deduction of complex concepts from simples, but involves fundamentally an analytical approach as well. This reflects the close ties which exist between the Ars combinatoria and the Ars inveniendi, which ultimately lead to their complete identification. Put simply, combinatorics is for him not just concerned with finding the number of possible variations which primitive elements can be combined to form. It is precisely for this reason that he considers the rules of combinatorics to find application not only in algebra, but also in various games and in the art of deciphering, "where it is not so much a question of composing as of resolving composites".[14] He also refers to this particular application in *Sur la calculabilité*, where in his intial wording he went on to distinguish the art of creating ciphers from that of resolving them. As we know from other sources,[15] it was particularly the latter which interested him, since he believed that the analysis of ciphers in order to discover their respective keys would throw important light on the art of discovery in general.

Already in the context of Leibniz's first work on the combinatorics, the *Dissertatio de arte combinatoria* (1666), he developed the idea that all concepts can be resolved into a small number of simple non-contradictory elements and that on the supposition that a suitable system of characters be found, the combination of these characters would allow not only all known truths to be deduced, but also new ones to be discovered.[16] But is the path of discovery one which can be followed without limit? From a metaphysical point of view, there would seem to be no grounds whatsoever for limiting the possibility of discovering new truths.

On the contrary, the infinite structures inherent in his model of nature through which it can be seen to "express everywhere the character of its creator"[17], and the fundamental conception of monads or entelechies as metaphysical entities perpetually striving towards an ever higher level of perfection quite definitely speak against such a restriction. Leibniz speaks explicitly of a continual and uninterrupted progress to greater goodness.[18] However, with a view to the reduction of all human knowledge to combinatorics, he reaches a conclusion in *Sur la calculabilité* which appears to be at odds with this. By combining the rules of combinatorics and arithmetic, he writes, the total number of possible variations which are able to be formed by the general signs, the alphabet of human thought, can be calculated. Reiterating a view that already Comenius had expressed, namely that he who understands the usage of such an alphabet will know all, Leibniz reaches the surprising conclusion that "one could calculate the number of truths of which human beings are capable" and "ascertain the size of a book which would contain all possible human knowledge".

Ascertaining the number of possible combinations is of course one of the commonest tasks in combinatorics. In *Dissertatio de arte combinatoria* Leibniz applied this for example to Latin hexameters, drawing thereby on the strong interest in combinatorial poetry at the University of Leipzig with which he became acquainted in his student days. It is therefore not entirely surprising that in the plan for a new work on the art of combinations which he drew up around September 1680 he mentions as a prospective theme the size of a book in which all possible hexameters are written down.[19] And as a further topic for consideration he notes immediately afterwards: "the size of the book in which all the truths are written down which can be grasped by human beings".[20]

As has been pointed out in recent investigations, these ideas were by no means new at the time. Among others who had considered the question of the total number of truths which are knowable, authors such as Mersenne, Guldin, and Schott can be mentioned. Particularly in the circle around Mersenne, investigations with combinatorics were undertaken similar to those later conducted by Leibniz with the aim of calculating the total number of all truths which could possibly be expressed.[21]

The ordering of all existing knowledge and its reduction to the form of a calculus based on an alphabet consisting of a limited number of primitives is still in many ways a clearly defined task. In *Sur la calculabilité*, Leibniz goes beyond this and focusses instead on the idea of calculating the number of all possible variations which could be formed using such an alphabet. These possible variations would in his view represent all the knowledge of which human beings are capable and more besides. For, alongside all the possible truths they would also encompass all the possible falsehoods and even all nonsensical expressions which can be formulated. Correspondingly, it would, he suggests, be possible to calculate the size of the book which in this very broad sense contains all possible human knowledge.

Leibniz believes that in this way it can be shown that the number of true, false, or nonsensical propositions which can be expressed, while being enormous, must nevertheless necessarily be finite. The paradox of this enormous number consists of course in the fact that an even larger number can easily be written down. It is thus similar to that of Archimedes' *Sand-Reckoner*, to which he explicitly refers. But as Leibniz is elsewhere able to demonstrate, his own paradox of the finite nature of human knowledge is of quite a different force, since the total number of possible variations far exceeds the total number of grains of sand calculated by the Syracuse mathematician.[22]

An important part of the background to *Sur la calculabilité* is represented by Leibniz's essay *De l'Horizon de la doctrine humaine* (c.1693), which as we know from a letter to Fontenelle, he sent to the sometime president of the Académie des Sciences, Jean Paul Bignon in October/November 1693. From this essay apparently only the French and Latin drafts survive and these are today to be found in the Leibniz Archives contained in the Niedersächsische Landesbibliothek in Hanover.[23] It would appear that *Sur la calculabilité* represents a more careful exposition of Leibniz's arguments for the finite nature of human knowledge, following Bignon's critique of his reasoning. It is probable that this is also the reason why Leibniz emphasizes that his investigations enable us to place ourselves in a true perspective in relation to God.[24]

Over and above the recognition of our true relation to the divinity, the central conclusion which Leibniz draws from his argumentation is that on the assumption that human nature remains unchanged necessarily a time will come when nothing can be said or written which has not been expressed already. This includes all possible books, and therefore also all possible works of literature. Drawing on one of his favourite means of exemplifying the concept of possible worlds,[25] Leibniz believes that it would then not be possible to write a novel which has not already been written or even to create a new phantasy expressible in language. The number of all possible variations respresents quite literally a totality which cannot be exceeded. But, to return to the earlier question, is this conclusion indeed irreconcilable with the fundamentals of his metaphysics? Probably not: although the conclusion is logically binding, the crucial point in his reasoning would appear to be the assumed constancy of human nature. Then is it not also fundamental to his metaphysical system that beings can transcend their present state and move on to a higher state of perfection? And is this not the reason why he writes explicitly in respect of his calculations that it is not a question of another life or of the human mind being raised to a more sublime state?[26]

Through the publication of the complete critical apparatus to the text, which is here presented for the first time, we are able to visualise the manuscript without its being present and in particular to see how Leibniz wrestled with different formulations for what he sought to express on the topic. Thus we find that he begins to describe the result of bringing together the art of combinations and consideration of language as being "the art of de[ciphering]", but then decides to use the technical term "steganography" instead, before changing this to "cryptography", which in the seventeenth century was used almost synonymously. He thereafter proceeds to explain this term, as we have already mentioned, and to describe this application of universal character as being only one of an infinity of other uses. In the end, however, he deletes these details, probably feeling that to speak of infinity in a loose manner here might be confusing, since later on in the piece the term is used in a more precise sense. We see also that the question of the conflict between the calculability of the number of possible truths and the fundamentals of his metaphysics was present in his mind right to the end. He originally concluded the piece with a remark to the effect that the calculation of the number of possible truths should not be allowed to hamper human souls in their future activities. This final line was deleted. But precisely through its deletion the line gives us insight into the thoughts which Leibniz was seeking to convey.

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Notes

[1] See for example *Monadology* §67, GP VI, 618.
[2] See for example Leibniz to the Electress Sophie, 6 February 1706, GP VII, 567; *Théodicée* §9, GP VI, 107.
[3] *De rationibus motus* §7, A VI, 2, 160.
[4] *De divisione orbis scientiarum universi*, A VI, 4, 525. "Itaque divisis scientiis in Experimentales et Intellectuales, subdivisiones reliquae non aeque fixos limites habent, non magis quam Oceanus Germanicus a mari Atlantico aut Deucaledonio ulla certa linea separatur, cum unus continuus sit oceanus".
[5] *Introductio ad encyclopaediam arcanam*, A VI, 4, 527.
[6] *Nouveaux Essais* IV, 21, §4, A VI, 6, 523: "Les Nominaux ont crû, qu'il y avoit autant de sciences particulieres que de verités, lesquelles composoient aprés des Touts, selon qu'on les arrangeoit, et d'autres comparent le corps entier de nos connoissances à un Ocean qui est tout d'une piece, et qui n'est divisé en Caledonian, Atlantique, Aethiopique, Indien, que par des lignes arbitraires". See also *Nouveaux Essais* IV, 7, §19, A VI, 6, 425.
[7] *Nouveaux Essais* IV, 21, §4, A VI, 6, 524.
[8] See *De synthesi et analysi universali seu arte inveniendi et judicandi*, A VI, 4, 544 (= Loemker, 1969, 232): "Nunc humana naturae cognitio mihi tabernae similis videtur, omnigenis mercibus instructissimae, sed ordine et repertorio carenti".
[9] See *De usu artis combinatoriae praestantissimo qui est scribere encyclopaediam*, A VI, 4, 84-85.
[10] Leibniz to Oldenburg, 28 December 1675, A II, 1, 250: "[...] fore tempus et mox fore, quo de Deo ac mente non minus certa quam de figuris ac numeris habeamus, et quo machinarum inventio non difficilior erit, quam constructio problematum Geometricorum". See also *Elementa rationis*, A VI, 4, 714-715.
[11] *De synthesi et analysi universali seu arte inveniendi et judicandi*, A VI, 4, 545 (= L, 233).
[12] See for example Leibniz to Fardella 20 August 1691, LBr 258, Bl. 28v; Leibniz to Tschirnhaus, end of May/beginning of June 1678, A III, 2, 449; *De arte inveniendi combinatoria*, A VI, 4, 332.
[13] Leibniz to Tschirnhaus, end of May/beginning of June 1678, A III, 2, 449. See also *De synthesi et analysi universali seu arte inveniendi et judicandi*, A VI, 4, 545 (= L, 233).
[14] Leibniz to Tschirnhaus, end of May/beginning of June 1678, A III, 2, 449: "Imo Combinatoria parum differe videtur, a Scientia Characteristica generali, cujus ope

characteres apti ad Algebram ad Musicam, imo et ad Logicam excogitati sunt aut excogitari possunt. Hujus scientiae etiam portio est Cryptographia, quamquam in ea non tam componere quam resolvere composita et ut ita dicam radices investigare difficile sit." See also *Initia et specimina scientiae generalis de instauratione et augmentis scientiarum*, A VI, 4, 359; *Elementa nova matheseos universalis*, A VI, 4, 514; *De arte combinatoria scribenda*, A VI, 4, 424; *De synthesi et analysi universali seu arte inveniendi et judicandi*, A VI, 4, 545 (= L, 233).

[15] See for example his letter to J. Wallis of 8 October 1697, GM IV, 42.

[16] See Leibniz for Duke Johann Friedrich, second half of October (?) 1671, A II, 1, 160; Leibniz for Gallois, end of June 1672, A II, 1, 228-229; E. Knobloch, *Die mathematischen Studien von G. W. Leibniz zur Kombinatorik, auf Grund fast ausschliesslich handschriftlicher Aufzeichnungen dargelegt und kommentiert*, Wiesbaden 1973 (= Studia Leibnitiana Supplementa 11), 56.

[17] Leibniz to the Electress Sophie, 3 October 1694, A I, 10, 61: "C'est par là que la nature porte par tout le caractere de son Createur".

[18] *Nouveaux Essais* II, 21, §36, A VI, 6, 189: "[...] un progrés continuel et non interrompu à des plus grands biens". See also for example his letter to the Electress Sophie of 13 September 1694, A I, 10, 61; *De rerum originatione radicali*, GP VII, 308; A G. Tognon, "Ad maiorem semper cultum": *Limiti e calcolabilità della perfezione dell'universo e dell'uomo nel pensiero di Leibniz*. In: *L'infinito in Leibniz. Problemi e terminologia*, ed. A. Lamarra, Rome 1990, 157-172.

[19] *De arte combinatoria scribenda*, A VI, 4, 424: "De magnitudine libri in quo omnes hexametri possibiles scripti extent".

[20] Ibid: "De libro in quo scriptae jam habeantur omnes veritates quae ab hominibus comprehendi possunt".

[21] See G. W. Leibniz, *De l'Horizon de la doctrine humaine. La restitution universelle*, ed. M. Fichant, Paris 1991, 130-135; Knobloch, *Die mathematischen Studien*, 78.

[22] Knobloch, *Die mathematischen Studien*, 89.

[23] This text (together with associated writings) has been edited and translated into French by M. Fichant in *De l'Horizon de la doctrine humaine*.

[24] See W. Hübener, *Die notwendige Grenze des Erkenntnisfortschritts als Konsequenz der Aussagenkombinatorik nach Leibniz' unveröffentlichtem Traktat "De l'Horizon de la doctrine humaine"*. In: *Akten des II. Internationalen Leibniz-Kongresses Hannover, 17.-22. Juli 1972*, Wiesbaden 1975 (= Studia Leibnitiana Supplementa 15), 55-71, 57.

[25] See for example Leibniz to Bourguet, October (?), GP III, 558; Leibniz to the

Electress Sophie, 31 October 1705, GP VII, 564. Cf. Théodicée §173, GP VI, 217.

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