

Leibniz and the Sorites

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Abstract

The sorites paradox receives its most sophisticated early modern discussion in Leibniz's writings. In an important early document Leibniz holds that vague terms have sharp boundaries of application, but soon thereafter he comes to adopt a form of nihilism about vagueness; and it later proves to be his settled view that vagueness results from semantical indeterminacy. The reason for this change of mind is unclear, and Leibniz does not appear to have any grounds for it. I suggest that his various treatments of the sorites do not spring from a single integrated view of vagueness, and that his early position reflects a mercenary interest in the sorites paradox—an interest to use the sorites to reach a conclusion in metaphysics rather than to examine vagueness as a subject to be understood in its own right. The later nihilist stance reflects Leibniz's own (if undefended) attitude towards vagueness.

A term that is *vague* in the philosopher's sense—such as 'rich', 'poor', 'bald', 'heap'—characteristically admits of borderline cases, lacks a clearly defined extension, and is susceptible to the *sorites* paradox or "paradox of the heap."¹ Take 'poor' for example. Someone with only a small amount of money may appear to be neither poor nor not poor, but on the borderline, even in the estimation of a competent language user fully informed about the amount of money the person has, the relative distributions of wealth in the community, and so on. The term 'poor' imposes no clear boundary between the poor and the not poor—it fails to single out a least number n such that anyone with at least n pennies is not poor while anyone with fewer than n pennies is poor—with cases seeming instead to fall into a spectrum across which poverty shades off gradually. And of course 'poor' can figure as a predicate for a sorites argument in the classical way; a standard version of the sorites for 'poor' might run as follows.

Someone with zero pennies is poor.

For any n , if someone with n pennies is poor, then someone with $n+1$ pennies is poor.

So, someone with one billion pennies is poor.

The conclusion is absurd, for someone with one billion pennies obviously would *not* be poor. Yet the premises appear to be true and the argument valid. In this case the sorites is set out as an instance of mathematical induction with base step ' $F(0)$ ' and induction premise 'For any n , if $F(n)$, then $F(n+1)$ ', which together license an inference to 'For every n , $F(n)$ ' or any specific instance of it. Simpler forms of inference can be marshaled to the same end. A series of particular conditionals 'if $F(0)$, then $F(1)$ ', 'if $F(1)$, then $F(2)$ ', etc., will allow the paradox to be run entirely by *modus ponens*, for example. The vagueness of the term 'poor'—like that of any vague term—yields interpretations for ' F ' on which it appears that a false conclusion can be deduced from true premises by means of logically valid inferences. Other variations on the sorites are possible, and examples of vague interpretations of ' F ' suitable to the argument can be multiplied indefinitely. But the general point can be put in a few words: vagueness leads to paradox.

In early modern philosophy it is Leibniz above all who takes interest in the topic of vagueness.² Yet about the sorites paradox Leibniz displays some ambivalence. "This was a sophism of the Stoics," he notes in a 1676 discussion of the argument, "which they called the Sorites." In another draft of that sentence he would write, "This type of argument, which the ancients called the Sorites, is not entirely useless if you use it correctly."³ In the end Leibniz would strike both versions of the sentence and present a sorites argument as a *reductio* of its induction premise, advancing the argument as a proof that someone can change from being poor to not poor by the gain of a single penny. Leibniz thus holds vague terms to have sharp boundaries of application. A look across his writings after 1676, however, reveals Leibniz apparently undergoing a significant change of mind in his view of vagueness in the years that follow. In 1678 he seems to abandon his original position that vague terms have sharp boundaries of application in favor of the view that vagueness carries with it a kind of conceptual indeterminacy and that vague terms do not truly apply to things as they are in themselves; and this appears to be his settled view of vagueness thereafter. Why Leibniz changes his mind in this way is rather a mystery. He offers no evident reasons for giving up his original view nor any for embracing the later one. The answer to this puzzle, insofar as there is one (I think the answer can be at best incomplete), is to be found in *Pacidius Philalethi* of 1676, "Chrysippus' Heap" of 1678 and the *New Essays on Human Understanding* of 1704. There Leibniz's views of vagueness are best documented, and a critical review of them will first be in order. In light of that survey I shall suggest that Leibniz's discussions of vagueness do not arise from a single integrated view and that his early interest in the sorites paradox is a mercenary one—that he uses the

sorites in his early writings for certain philosophical ends without having a well-defined commitment to the view of vagueness embedded in that use.

1. 1676: The Sorites in *Pacidius Philalethi*

In the 1676 dialogue *Pacidius Philalethi*, Leibniz advances his most precise and complete discussion of the sorites paradox; indeed, it appears to be the most fully articulated statement of the sorites in early modern philosophy. To understand Leibniz's interest in the sorites here, some of the background for its appearance in the dialogue will need to be considered. The topic of the *Pacidius* is the nature of motion—its subtitle is *Prima de Motu Philosophia* or “First Philosophy of Motion”—and in it motion is provisionally defined as “change of place” (A VI,3,534). Leibniz constructs an exotic metaphysics of change according to which the occurrence of change always consists in a pair of immediately neighboring moments in time, the first containing one state *F* and the second containing the opposite state *not F* (cf. A VI,3,558).⁴ It is in the course of crafting this metaphysics of change that Leibniz invokes the sorites. He does so twice: first in application to “discrete quantity,” by which he evidently means a quantity with pre-assigned minimal units of measure, as wealth is measured by the penny; second in application to “continuous quantity,” such as distance in space or time, in which the units of measure can be taken as finely as one wishes (A VI,3,539f.). The first sorites, or the sorites applied to discrete quantity, is used to establish that the change from one state to its opposite—the change from being *F* to *not F*—can always in principle be effected by the smallest difference in measure, and thus the boundary across which change occurs is always perfectly sharp. The second sorites then “transposes” the argument from discrete to continuous quantity in order to demonstrate the existence of absolutely indivisible elements or “minima” in continuous quantities, such as moments in time, points in space, and “minimum parts” in motion. We shall return briefly to Leibniz's second sorites argument at the end of our discussion. It is primarily the first sorites that will occupy us in this paper.

It is not obvious that the change from being *F* to *not F* occurs across a sharp boundary in all cases. Leibniz is well aware that some concepts, or as he says, “notions,” seem not to determine sharp boundaries for their application, in particular the vague ones—wealth, poverty, baldness and so on—whose application seems rather to shade off gradually across a region of borderline cases. There appears to be no sharp cut-off point at which one suddenly becomes poor upon the loss of a single penny. And of course this is exactly where the sorites takes hold. In the

dialogue Leibniz's main interlocutors make his case, as the character Pacidius, in the role of Socrates, questions Charinus about the boundaries of poverty and wealth.

Pacidius: If the wealth of two people differs by only one penny, could one of them be regarded as rich without the same judgment being made about the other?

Charinus: No, I do not believe so.

Pa.: Therefore a difference of one penny does not make one rich or poor.

Ch.: I suppose not.

Pa.: Nor would the gain or loss of one penny make a rich person not rich, or a pauper not poor.

Ch.: Not at all. (A VI,3,539)

In spelling out the point that *rich* (and *not rich*) and *poor* (and *not poor*) are notions whose application cannot turn on the difference of a single penny, Pacidius elicits from Charinus the elements of the induction premise operative in familiar versions of the sorites. Leibniz's own statement follows the Stoic example of a negated conjunction: $\sim(Fn \ \& \ \sim Fn+1)$. Since Leibniz does not address the logic of the conditional here—dispute over which perhaps drove some Stoics to formulate the sorites as a negated conjunction⁵—we can safely regard the induction premise in a perhaps more intuitive form: if F_n , then $F(n+1)$. Thus in this instance the premise might be that if a person with only n pennies is poor, then a person with only $n+1$ pennies is poor. Charinus's intuitions in accepting the induction premise for *rich* and *poor* are the natural ones; surely, one supposes, it *is* possible for a person to become rich or poor, though *not* by the gain or loss of a single penny. But once those intuitions are expressed the stage now is set for Pacidius to unveil the paradox.

Pa.: Therefore no one can ever become rich from being poor, nor become poor from being rich, however many pennies are given to or taken from him.

Ch.: Why is that, may I ask?

Pa.: Suppose a penny is given to a pauper. Does he cease to be poor?

Ch.: No.

Pa.: If another penny is given to him, does he cease to be poor then?

Ch.: No more than before.

Pa.: Therefore he does not cease to be poor if a third penny is given to him?

Ch.: No.

Pa.: The same applies to any other one: for either he never ceases to be poor, or he does so by the gain of one penny. Suppose he ceases to be poor

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when he gets a thousandth penny, having already got nine hundred and ninety-nine; it is still one penny that removed his poverty.

Ch.: I can see the force of the argument, and I'm surprised I was deluded like this.

Pa.: Do you admit, then, that either nobody ever becomes rich or poor, or one can become so by the gain or loss of one penny?

Ch.: I am forced to admit this. (A VI,3,539)

Leibniz executes the sorites perfectly. Charinus's "forced admission" is only to a disjunction: either nobody ever becomes rich or poor, or else such a transition can be effected by the difference of a single penny. Even admitting to this dilemma is conceptually revisionary, since each of its horns involves a radical departure from ordinary beliefs about poverty (as would parallel conclusions of sorites arguments for baldness and so on). Yet in fact the context in which the sorites occurs makes it clear that Leibniz's position is still more fully staked out. It is the second horn of the dilemma—that one can become rich or poor by the gain or loss of a single penny—that is to be embraced. For at this point in the dialogue the example of becoming rich or poor is merely standing in for the idea of change more generally: either there is no such thing as change or else there is a sharp boundary between prior and posterior states. And the reality of change is not open for negotiation in *Pacidius Philalethi* ("Who would deny it?": A VI,3,538).

The import of Leibniz's discussion extends beyond cases of becoming, of course, to cases of *being*, as his initial example makes plain. If the wealth of two people differs by only one penny, one of them could not be regarded as rich without the same judgment being made about the other. But now in light of the sorites (the interlocutors decide), that initial, naïve belief turns out to be false. Either there is no such thing as being rich, or not rich, or else there can be as little as a single penny's difference between the two. If we are to accept that a vague notion *F* is capable of coherently "dividing reality" at all by distinguishing some things as being *F* and some others as not *F*, we shall then be forced to admit that the divisions it imposes are sharp boundaries.

Though Leibniz does not rely on the metaphor of the sharp boundary, it is a useful heuristic in discussing the sorites and, I take it, a conceptually innocent one. The literal content of the claim that a notion *F* determines a sharp boundary of application is simply that the notion conforms to standard bivalent logic and that there is no alternative to its applying to something or its not applying to it. I shall sometimes speak of a "sharp-boundaries thesis" concerning one or another class of notions, and by this I shall mean a thesis to the effect that the notions of the

given class determine sharp boundaries of application in exactly the sense described. In the present instance, one holds a sharp-boundaries thesis for vague notions by holding that for any x and any vague notion F , either F applies to x or else F does not apply to x .

The sharp-boundaries thesis for vague notions allows one to respond to the sorites paradox by denying the induction premise. It can be maintained that in the case of the notion *poor*, the induction premise is false because there is some number n such that a person with only n pennies is poor while one who has $n+1$ pennies is not. Thus it is possible for someone to become poor, or not poor, and in principle this can be achieved by the gain or loss of a single penny. Yet that response is very hard to accept as a resolution of the paradox. What numbers n and $n+1$ could these be? 17,238 and 17,239? This sounds absurd, as will any other proposed pair, and indeed the sharp-boundaries thesis itself can seem antithetical to the very idea of a vague notion. It can appear to be an exceptionally clear way of saying that vague notions are actually precise ones, and so *not* vague after all; and this perhaps accounts for the unpopularity of the sharp-boundaries thesis in analyses of vagueness past and present.

The sharp-boundaries thesis will not be so quickly dismissed, however. Once we distinguish *unclarity* as the characteristic feature of vague notions—i.e. their admitting of borderline cases in which it is *unclear* whether they apply or not, their displaying no *clear* boundaries, and so on—and set it apart from the logical or semantic property of *failing to determine sharp boundaries*, we can see that the sharp-boundaries thesis need not be antithetical to analyses of vagueness.⁶ That a notion's boundaries must be *sharp* does not automatically imply that they must also be *clear*; whether there is an entailment here is a question of substance in theories of vagueness, not a triviality. The sharp-boundaries thesis by itself will preclude only the claim that vagueness (unclarity) arises from some deficiency in the logical or semantic character of a vague term. It thus remains open as a possibility to uphold a sharp-boundaries thesis for all notions and yet to admit that some notions are vague—so long as some other diagnosis of the vagueness of those notions can be advanced.

If one is not willing to maintain that there are some numbers n and $n+1$ such that a person with n pennies is poor while a person with $n+1$ is not poor, other options for resisting the sorites argument must be pursued. Those other options, however, all boil down to one of two things. One can abandon standard logic and its commitment to the principle of bivalence (that every statement either is true or else is false⁷), at least for those pieces of language that express vague notions. The sorites

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may then be seen as involving a fallacy at some level—a logical fallacy, if not one that is detectable in standard logic. Or one can adopt some form of nihilism about vague notions and hold that such notions are logically or semantically incoherent: either their application actually does lead to contradiction, or else they are deficient in some way that precludes them from belonging to well-formed propositions in the first place. Vague notions are thus never *true* of anything.⁸ Nihilists will be able to reject one or both of the premises in a standard sorites argument, or perhaps suggest that in fact there are no coherent sorites arguments, as the sentences that appear to express them fail to *say* anything. Advocates of nonstandard logics will deny either the induction premise or the validity of the argument itself, depending upon the details of the nonstandard logic they propose.⁹

Though the sharp-boundaries view of vague terms will strike most as counterintuitive (indeed it strikes most as literally incredible), the alternatives to it ought not to be attractive either. Abandoning standard logic seems to be a cure more radical than the problem, perhaps *any* problem. Chrysippus saw this clearly, as did Tarski.¹⁰ It can also seem a hollow tactic. What puzzle cannot be “solved” by changing logic?¹¹ Nihilism too is an extreme view, since the terms in actual natural languages are almost all vague in the way that makes them susceptible to sorites arguments.¹² Nihilism about vague notions would thus lay waste to just about the whole body of human understanding. What needs to be recognized here is that every systematic response to the sorites paradox is drastic. That is why it is a paradox; we might with equal justice call it an antinomy. Still, positions will have their advocates, and nihilism and the flight from standard logic have tended to be the “solutions” of choice among philosophers writing on the sorites.

As for Leibniz, he himself gives no signs of betraying standard logic. In the opening passages of the *Pacidius*, he is clear in upholding the primacy of logic in philosophy—“Logic before everything else” (A VI,3,532)—though that declaration by itself falls short of insisting upon bivalence, since he goes on to describe his project as that of developing “Phoronomy” or the science of motion as a “Physical Logic,” just as predecessors have described geometry to be a “Mathematical Logic” (A VI,3,533), and it is at least possible to think of the project here as one that is open to developing a “Logic” that is tailored, perhaps deviantly, to its subject matter. When later he asserts the principle *tertium nullum est*,¹³ this comes closer to the point. The clearest evidence in the *Pacidius* of Leibniz’s commitment to a strong principle of bivalence surfaces in the following exchange just prior to the first sorites argument:

Pa.: Can the truth of some proposition increase or decrease over a certain stretch of time, in the same way as water gets hotter or colder by degrees?

Ch.: Certainly not. I think a proposition is either at once wholly true or wholly false. (A VI,3,538)

With this statement Leibniz expressly rules out the possibility of values other than true and false as well as the idea of “degrees of truth”; and I cannot believe that he allows for truth value “gaps” or “gluts” here either.¹⁴

In later writings Leibniz’s fidelity to the principle of bivalence is evident in any number of his discussions of logic, for instance in the brief study *Primae Veritatis* (re-titled *De Principiis Praecipue Contradictionis et Rationis Sufficientis* in the recent Akademie Edition) of 1686-7:

I assume that every judgment (that is, affirmation or negation) is either true or false, and that if the affirmation is true the negation is false. (A VI,4,804)

Likewise, in the statement of the “principle of contradiction” offered in the *New Essays* of 1704, Leibniz writes:

A proposition is either true or false. This contains two assertions: first, that truth and falsity are incompatible in a single proposition, i.e. that *a proposition cannot be both true and false at once*; and second, that the opposites [opposé] or negations of the true and the false are not compatible, or better, that *it cannot happen that a proposition is neither true nor false.* (A VI,6,362)

Thus the general cast of mind towards bivalence is clear enough through the years. Also, if in the *Pacidius* Leibniz never quite says that a standard bivalent logic is mandatory, he certainly provides his interlocutors with an ideal opportunity to acquiesce in a deviant approach to a logic for vagueness. If he were at all interested in pursuing such a programme, one would think that it would surface here. Yet there is not even the slightest hint of it to be found in the dialogue. (Nor can we plausibly suppose that abandoning bivalence just never occurred to him even as a theoretical option. The Stoics, whose writings on the sorites he knew, fiercely debated the principle.) I conclude that his engagement with the sorites proceeds in a way that leaves standard logic intact.

By contrast, Leibniz seems more open to the possibility of nihilism about vague notions, both in the *Pacidius*, where he at least raises it as a possible response to the sorites, and in subsequent writings, where (as I shall argue below) he actually commits himself to it in some form. Yet nihilism should also be seen as endorsing a sharp-boundaries thesis. The point is that the only coherent notions that can figure as constituents in true propositions are those that have sharp boundaries of

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application. One then must ask whether vague notions satisfy this requirement, and here nihilism about vagueness holds that they do not.¹⁵ I am inclined, however, to read Leibniz in the *Pacidius* as holding instead that vague notions are coherent and *do* have sharp boundaries—for instance, that there is in fact a sharp cut-off between being rich and not rich. He certainly uses sorites arguments to advance sharp-boundaries claims for the notions of change and motion, and he makes no attempt to set the class of vague notions apart for some distinct type of treatment. Still, he doesn't say that vague notions *are* in fact true of things in the world, either, or that strictly speaking there *is* such a thing as becoming rich or not rich. The text alone does not speak conclusively against a nihilist reading. Nonetheless, it seems to me that the interlocutors' silence on this score counts in favor of judging that Leibniz accepts a sharp-boundaries thesis even for vague notions. The interlocutors give no indication that they are prepared to jettison their stock of ordinary beliefs about wealth, poverty, life, death, and so on, due to the vagueness of the notions involved, and I am not prepared to suppose that Leibniz means to be doing it for them.

So it seems that in the *Pacidius* Leibniz maintains a sharp-boundaries thesis for vague notions. This puts him at the start of a satisfactory analysis of the sorites paradox, but only the start. For while he has staked out a position that will reject the induction premise, he has done nothing yet to provide an account of the *unclear-ity* of vague notions—of why it should *appear* that vague notions do not have sharp boundaries, of why vague notions should suffer borderline cases in which it is unclear whether the notion applies or not. And he has done nothing to provide an account of the *sharpness* of vague notions—of how those notions could manage to have sharp boundaries even when language users who have a complete mastery of those notions cannot tell where their boundaries fall. The basic phenomena of vagueness remain unexplained, and without an explanation of them Leibniz will only seem to have traded in one mystery for another.

Still, viewed against the backdrop of the twenty-first century taxonomy of theories of vagueness, Leibniz's view (like that of the Stoics) emerges closest to the epistemicist position.¹⁶ Epistemicism holds to the sharp-boundaries thesis even for vague notions and then analyzes the phenomena of vagueness as epistemic phenomena that arise due to *ignorance*: the boundaries of vague notions are unclear not because they are not sharp but because we do not know where they fall. It is hard to see what else there is for a sharp-boundaries theorist to say about vague notions, once those notions are admitted to be coherent. But, at least in the *Pacidius*, Leibniz does not seem to take even this first step beyond the sharp-boundaries

thesis itself.

2. 1678: The Sorites in “Chrysippus’ Heap”

If in the *Pacidius* Leibniz holds a hard line on vague notions by maintaining that they, like all coherent notions, must determine sharp boundaries, two years later his view has evolved into a perhaps more radical one. In a 1678 note entitled “Chrysippus’ Heap,” he announces:

Finally at last:

“The surveyor of your heap, Chrysippus, has been found.” For all those notions in which the heap or sorites of the Stoics applies, such as wealth, poverty, baldness, heat, cold, tepidness, white and black, big and small, taken absolutely, are vague imaginary notions, indeed false ones, that is, ones having no corresponding idea. (A VI,4,69)

This is nihilism about vagueness. In saying that vague notions are “imaginary,” and indeed “false,” and have “no corresponding idea,” Leibniz is claiming, among other things, that vague notions do not truly apply to anything. Although this is not made explicit in “Chrysippus’ Heap,” Leibniz holds a notion to have no corresponding idea when the notion itself is not truly possible but rather involves some impossibility.¹⁷ That comes out clearly in Article 25 of the *Discourse on Metaphysics*, for example, where he writes:

Now it is evident that we have no idea of a notion when it is impossible. And in the case where knowledge is only suppositive, even when we have the idea, we do not contemplate it, for such a notion is only known in a way in which we know notions involving a hidden impossibility. (A VI,4,1569).

Vague notions, by having no corresponding idea, are therefore notions that we could know (or perhaps “know”) only suppositively; they in fact involve hidden impossibilities that are revealed by the sorites paradox. And this yields nihilism about vagueness.

Taken absolutely,¹⁸ no one is rich or poor or bald, for to all these notions the sorites applies and generates a contradiction. (Taken comparatively, however, it may still be true that one person is richer or poorer or balder than another: “Concerning imaginary notions, comparative propositions, at any rate, remain true, and to these the sorites cannot be raised as an objection” (A VI,4,70).¹⁹) The sharp-boundaries thesis also continues to be in force here, in the nihilist’s sense that a notion is coherent only if it has sharp boundaries of application. But it seems now that by Leibniz’s lights vague notions do not have them. Reasoning contrary-to-

fact, he says:

If poverty were assumed to be an absolutely true notion, it ought to be defined by a particular number of pennies, because it is necessary for someone who is not poor to become poor on the removal of one penny; or he will never become poor at all. (A VI,4,69)

The notion of poverty is not defined by a particular number of pennies, however, and instead requires help from “the laws” which “come to the aid of this defect, and define a pauper as someone who does not possess some particular number of shillings” (A VI,4,69). Yet this is only a practical fix—a shift from the ordinary notion of poverty to some different, precise notion—and it does nothing to make the notion of poverty itself into a “true notion,” for the laws merely add what the notion of poverty by its very nature cannot provide: a sharp boundary. Poverty thus cannot characterize anything in the world as it is in itself.

In “Chrysippus’ Heap” Leibniz still has not formulated any very complete analysis of vagueness, but his expressed thoughts on the subject are notably richer. Using terms that will crop up repeatedly in later writings, he calls vague notions “imaginary” and suggests that they “indicate something with respect to our opinion, which varies” (A VI,4,69). Those are two strands of thought, not one, and should be pulled apart. I shall consider them in turn.

In an important passage from the same document, Leibniz elucidates ‘imaginary’ further:

I call those notions imaginary which are not in [*non sunt in*] the things outside us, but whose essence it is to appear to us. (A VI,4,70)

So vague notions are not “in” (not exemplified by) the things outside us. As was evident in the first passage quoted from this text, Leibniz classifies nonsensory vague notions like wealth and poverty together with sensory “secondary quality” notions like white and black, hot and cold. Though there is a natural connection here in that secondary qualities themselves offer good examples of vagueness, Leibniz’s point is less to draw out the vagueness of secondary qualities than to identify vague notions as being merely “in the mind,” and not “in the things outside of us.” On Leibniz’s view, all of those notions are only imaginary. A person might have a definite number of pennies or a definite number of hairs, but the further thought that the person is rich or poor or bald confuses appearance with reality. Nothing in the world as it is in itself actually answers to such categories.

Alongside this discussion of vagueness and vague notions Leibniz includes the second strand of thought according to which vague notions suffer from a sort of relativity to perspective, or to opinion, or to context.

[T]he above [vague] notions indicate something with respect to our opinion, which varies. For example, what is cold to one person seems hot to another, truly in each case, and this is even so for the same person at different times. It is the same with poverty. For someone we call poor in a certain respect, we deny to be so in some other respect. (A VI,4,69)

The suggestion seems to be that understood relative to (say) a given person's opinion at a given time, a vague notion can be said truly to apply to something; but unrelativized it cannot. No doubt it is true that vague notions can be "relative" in this way, but we should distinguish such relativity from the vagueness that makes them susceptible to sorites arguments. Something may indeed be cold to you and hot to me, and then only in some respect, but even considering matters just as they are to me it will remain unclear where the boundary falls between the hot and the not hot. Vagueness is one thing, relativity to perspective, opinion, or context is yet another (probably three others), and Leibniz is not sorting them out carefully here. This will prove to be typical of his discussions of vagueness, with the secondary strand running detectably alongside the more relevant line of thought in a number of different passages.

Finally, in "Chrysippus' Heap" Leibniz offers a few hints in the direction of an analysis of the phenomena of vagueness. The vagueness of vague notions, he seems to suggest, is due to our *doubt*.

[W]henever we begin to be in doubt, as [when we doubt whether someone becomes poor] on the removal of one penny, there ceases to be a problem, and the person we are concerned with is neither poor nor rich to me. [...] When we are in doubt whether a thing is hot or cold, we call it tepid, therefore tepid is not a definite idea outside of us, but consists in our doubt. (A VI,4,70)

The claim that there "ceases to be a problem" is too fast, since presumably doubt whether a person has become poor will itself only arise gradually, so that it is unclear which particular removal of a penny brings with it the onset of doubt. (Leibniz apparently does not recognize the difficulties of higher-order vagueness.²⁰) Still, the emphasis on doubt is worth noting. Leibniz's sharp-boundaries thesis earlier placed him nearest to epistemicism among theories of vagueness, and in light of that his remarks in this passage are rather intriguing. Perhaps the phenomena of vagueness could be analyzed as phenomena of *uncertainty* or doubt. If vague notions were still considered to determine sharp boundaries, this view would amount to a version (if only a proto-version) of epistemicism. But Leibniz appears in "Chrysippus' Heap" to have abandoned that view of vague notions in favor of nihilism, and therefore along with it any epistemicist theory of vagueness.

3. 1704: The Sorites in the *New Essays*

Leibniz's later view of vague notions is on display in the 1704 *New Essays*, and although the overall position is much like that of "Chrysippus' Heap," his thoughts about vagueness appear to have grown more nuanced. Also, as usual, his engagement with Locke's *Essay* draws out a relatively concrete discussion that helps to illuminate Leibniz's thought from fresh angles. In the central passages in which vagueness is discussed, Leibniz's concern is to reply to some of Locke's criticisms of realism about essences for species and natural kinds. Locke had suggested that the boundaries of species are fixed only by convention, and indeed that species terms do not even determine boundaries for species since different opinions about the associated "essences" were often in conflict. In reply, Leibniz writes:

Ordinarily, these boundaries [*ces limits*] of species are fixed by the nature of things—for instance the line between man and beast, between stabbing and slashing. I do admit though that there are some notions which involve a truly arbitrary element: for example, determining a one-foot length, for since a straight line is uniform and indefinite, nature does not indicate any boundaries on it. There are also vague and imperfect essences, where individual opinion comes in—as in the question of how few hairs a man can have without being bald. This was one of the sophisms which the ancients used for putting pressure on an adversary, 'until he falls, tricked by the argument of the vanishing heap'.²¹ But the right reply is that nature has not determined this notion, and that opinion plays a part; that there are people whose being bald or not bald is open to question; and that there are ambiguous cases whom some would regard as bald and others would not—as in your remark that a horse which counts as small in Holland will be deemed large in Wales. Something of the kind can occur even with simple ideas, for, as I have just remarked, the outer limits of colors are doubtful. (A VI,6,302.)

Again the topics of opinion and context-dependence are threaded into the discussion of vagueness. But the central line of thought remains clear. Vague notions, unlike those belonging to true species or kinds, do not have their boundaries fixed by the nature of things. Rather, they express only "imperfect essences" and suffer from indeterminacy, like the notion of a straight line that is "uniform and indefinite" and in which no boundaries are indicated; for nature has not determined the boundaries in such notions. This is a much more explicit statement of the indeterminacy of vague notions than appears in earlier writings, and we do not find the same weight on the claim that vague notions are "false" and "correspond to no

idea.” Leibniz’s words here perhaps marks a partial shift away from the nihilist stance of 1678, and we might wonder whether in the *New Essays* he is no longer maintaining the idea that the indeterminacy of vague notions implies that they contain a “hidden impossibility” that comes to be revealed by the sorites arguments. If his focus on indeterminacy has allowed nihilism to slip from view, that would be an understandable change of consciousness. There is a holy grail of sorts for philosophers of logic concerning vagueness: an indeterminist position that vindicates the intuition that vague terms do not have sharp boundaries but which neither collapses into nihilism nor abandons classical logic. The idea that vagueness can be understood as a form of indeterminacy, and without difficult consequences, is perhaps intrinsically tempting.²² But presumably for this grail to be achieved, there will have to be some profound insight in semantics that yields such an indeterminist position; and so far no such semantics is in sight. Every known semantical treatment of vagueness either denies that sentences involving vague terms are true or rejects certain features of standard bivalent logic. (Supervaluationist semantics, for example does *both*: truth itself is reserved only for “precisifications” of vague sentences, and various classically valid forms of inference come out invalid. The general principle of bivalence also fails.²³) For this reason it seems that for Leibniz, whose commitment to bivalence is unequivocal in the *New Essays*, the indeterminist position must amount to nihilism, even if the distinctively *nihilist* element in nihilism—the denial that vague notions are ever true of anything—is no longer so salient in his remarks. That is how I interpret his philosophical commitments concerning vagueness at this point.

In the *New Essays*, the role of opinion may also be more deeply entwined in Leibniz’s analysis than before. It is certainly plausible to read him as saying that a borderline case of baldness is only a matter of opinion, and that different opinions are equally good.²⁴ If that is his view, it is too facile. In borderline cases it is not typically true that individuals seek to draw different boundaries and then advance positively disagreeing opinions about whether a borderline bald person is bald or instead not bald. Rather, the characteristic trouble is that individual opinions even taken individually do not seem to rule clearly one way or the other on borderline cases. Individual opinion is as vague as public opinion. Though your standards for baldness may differ from my own, each of us still has only a vague standard by which to make judgments, one against which certain cases will not be clearly decided and will seem to remain entirely “open to question.”

A little later in the *New Essays* vagueness returns to the stage, again in the company of the dispute over realism about the essences of species. This time Locke’s

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criticism has proceeded from the purported case of “monsters” that seem to ride across lines of classification, raising the possibility that creatures might fall into a sort of qualitative continuum that offered no natural joints at which species boundaries could be assigned. Philaethes speaks for Locke, and Theophile for Leibniz:

Ph.: But in all seriousness it is plain that species cannot always be assigned fixed boundaries.

Th.: I have already granted you that: when we are considering fictions and how things could be, there might be insensible transitions from one species to another, and telling them apart might sometimes be rather like the problem of deciding how many hairs a man must have if he is not to be bald. This indeterminacy would hold even if we were perfectly acquainted with the inner natures of the creatures in question. But I do not see that this prevents things from having real essences independently of our understanding, or us from knowing them. It is true that the names and the boundaries of species would sometimes be like the names of measures and weights, where there are fixed boundaries only in so far as we choose them. However, in the ordinary course of events we have nothing like that to fear, because species which are too alike are seldom found together. (A VI,6,321)

In the face of his concession that telling apart species in the case of “insensible transitions” from one to another might be like the problem of deciding “how many hairs a man must have if he is not to be bald,” Leibniz’s commitment to realism about essences for species seems to run contrary to his denial of realism about vagueness. If species could shade over into one another as baldness shades over into non-baldness, then why should we accept that “things have real essences independently of our understanding” in the case of species but deny that there exist such real essences to be picked out by vague notions? Leibniz would now appear to have no principle available to justify reaching different verdicts in the two cases.

Leibniz’s claim that it is the nature of the things being divided that fixes the boundaries of the notions that divide them amounts to a form of “externalism” about the content of those notions—for the world “outside of us” finally determines where the boundaries of those notions fall, and not just our knowledge or understanding of that world. And a kind of “realism” about the things being divided accompanies this externalism about content: in a particular domain, say that of men and beasts, the world is divided up independently of our actual knowledge or understanding of its divisions. Once the further step is taken to suppose that external reality can fix the boundaries for our notions even when it could be be-

yond the reach of our understanding to discern those boundaries, a very robust externalism about content and thus a very robust realism about those notions are being introduced.

In this way vagueness provides a test case for an exceptionally strong form of realism. It seems to be the very nature of vague notions that no possible improvement in our knowledge could make it clear where the boundaries fall between, say, the poor and the not poor, or the bald and the not bald. And so to hold to the existence of sharp boundaries for vague notions is to conceive of reality as being mind-independent in a very radical way: the world contains divisions that are not even in principle accessible to human understanding (nor perhaps to any understanding at all). Once Leibniz allows that the boundaries of species could be as hard to discern as a definite cut-off between the bald and the not bald, and yet allows also that those species could nonetheless be sharply bounded, just such an exceptionally strong realism has come into play. This also means that the most natural and compelling ground for denying the existence of sharp boundaries to vague notions—namely, the thought that if our vague notions *did* have sharp boundaries, we should be able to know what they are—now seems to be lost, and Leibniz appears to have no reason at all to motivate his nihilism about vague notions.

I do not know how best to unravel this difficulty for Leibniz. One could try to argue that in his most considered opinion Leibniz may be a nihilist about species as well, so that species notions and vague notions alike are not “true” notions but rather “false ones” that have “no corresponding idea.” Some other texts could be reasonably interpreted along those lines,²⁵ though to rewrite Leibniz’s various discussions of species in the *New Essays* and related texts in a way that will conform to a systematic underlying nihilism on his part would not be a pretty affair. It may be that the clearest total understanding of Leibniz on this point is one that sees him as getting into a difficulty from which he does not get out. At any rate a rescue effort is not going to be part of the present essay, so I shall be content to leave this as an unsolved problem.

What matters is to see clearly the view of vagueness that we find developing in Leibniz already in 1678 and that appears still to be the dominant view in 1704. Vague notions do not determine sharp boundaries; their “essences” are imperfect and the world outside us does not assign them boundaries either. Vagueness is at bottom a form of indeterminacy, and hence it cannot be a feature of anything as it is in itself. The essence of a vague notion is not to divide reality but rather, at best, only to appear to us. Vagueness is a fiction of the imagination.

4. 1676 Revisited: Leibniz's Use of the Paradox

The outlines of Leibniz's changing thoughts about vagueness are now fairly clear. In his 1676 discussion of the sorites he appears to uphold the sharp-boundaries thesis for vague notions. By 1678 he has abandoned that "realist" stance in favor of nihilism, denying that vague notions have sharp boundaries, and this continues to be the core of his position in 1704. Still, a question remains about the development of Leibniz's thought about vagueness. Why does he change his mind? Leibniz does not mention his reasons. Nor does any evident principle emerge in his writings that would turn him away from the sharp-boundaries thesis for vague terms. And I do not see that a shift from the sharp-boundaries thesis to nihilism is such a "natural" direction for the mind to take that no further explanation need be desired. Perhaps little more than guesswork can finally be offered here, but there are a few further considerations that might help to shed light on Leibniz's change of mind.

The suggestion that I would make is that Leibniz's treatment of the sorites argument and his view of vague notions in the *Pacidius* do not spring from an integrated view of vagueness. His use of the sorites in that 1676 dialogue is, rather, like Gödel's use of the Liar Paradox in the proof of the first incompleteness theorem. In "Über formal unentscheidbare Sätze der Principia Mathematica und verwandter System I," Gödel shows how in any sufficiently strong formal theory T a "Liar sentence" can be constructed, that is, a sentence S in the language of T that is equivalent in T to its own T -unprovability.²⁶ Provided that T is consistent, this means that S is not provable in T (and hence that S is *true* though not provable in T). In constructing his proof Gödel shows no interest in resolving the Liar as such. He is taking no stand on the correct analysis of the paradox. Rather, he recognizes the power of the Liar argument to help reach a dramatic conclusion—the first incompleteness theorem—and deploys it without examining the Liar as a paradox to be understood in its own right.²⁷ Likewise, I suspect that in the *Pacidius* Leibniz is Gödelian in his attitude towards the sorites argument. Leibniz sees in the sorites the power to reach a significant conclusion—that there are indivisible elements in reality—and uses it to achieve that end without examining vagueness as a paradox to be understood in its own right. When later he turns to consider the difficulties of vagueness in "Chrysippus' Heap," his analysis makes no effort to maintain the realist stance adopted in the arguments of the *Pacidius*, and he simply opts instead for a form of nihilism. A review of Leibniz's use of the sorites in the *Pacidius* should help bring this out.

As noted earlier, in the *Pacidius* Leibniz offers the sorites in two forms. The first

is the traditional version in which the argument is applied to “discrete quantity” to reach the dilemma that either no one ever becomes rich or poor or else one can become so by the gain or loss of a single penny. Since Leibniz evidently means to allow that one can in fact become rich or poor—that example is only going proxy for the possibility of change more generally—his resulting view appears to allow for the coherence of vague notions while still enforcing the sharp-boundaries thesis for them. But it is significant that Leibniz does not conduct any discussion of vagueness *per se* and does not offer any resolution to the paradox. What does come through in the dialogue, however, is the sense that the sorites carries an irresistible logical force to reach a conclusion. Recall how Charinus finally yields to it:

Ch.: I can see the force of the argument, and I’m surprised I was deluded like this.

Pa.: Do you admit, then, that either nobody ever becomes rich or poor, or one can become so by the gain or loss of one penny?

Ch.: I am forced to admit this. (A VI,3,539)

Leibniz’s interest in the sorites paradox appears to be in its power as a pattern of inference. He intends to harness that power to drive certain lines of argument in the *Pacidius*, using it to deliver critical elements in his account of the nature of change and some of the metaphysics that will accompany it.

The first sorites is only the opening move in his broader argument, of course, as it sets the stage for a second and somewhat different deployment of the sorites. This second sorites argument is meant, again, not to probe the paradoxical nature of vagueness, but to reach a significant conclusion about space, time and motion. It can be divided into two phases, the first of which applies the sorites to “continuous quantity” in which increments can be taken to any degree of precision.

Pa.: Let us transpose the argument from discrete to continuous quantity: for example, if a [movable] point *a* approaches a point *H*, then at a certain time it will turn from not being near to being near. Shall we not conclude by the same argument as a little while ago that either it never gets near to *H*, or it does so by the addition of one inch?²⁸

Ch.: Yes.

Pa.: But couldn’t we have substituted for the inch a hundredth or thousandth of an inch, or any other part, however small?

Ch.: We could, without affecting the force of the argument.

Pa.: Therefore we could have substituted a part smaller than any named by us?

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Ch.: Of course. (A VI,3,540)

In those lines an important feature of the sorites is being drawn out: the force of the argument is independent of the units that are selected to measure the quantity in which change is occurring. Not only is the boundary between opposite states always sharp, but its sharpness reaches any degree of precision.²⁹ The second phase of the argument will address the ancient topic of passage to the limit and, as I see it, Leibniz invokes yet another ancient paradox to serve as its engine (namely, Zeno's dichotomy). Leibniz uses the second sorites to reach the conclusion that there are indivisible or "minimal" elements of motion, and likewise minimal elements of space and time.³⁰ The associated line of argument in the second phase of the second sorites deserves close scrutiny, but that is a substantial project that I shall have to postpone to another occasion. What matters at this point is simply to observe that in engaging with the paradox Leibniz again does not seek to resolve it but rather aims to harness the force of its underlying inference pattern in order to drive his own case for a significant metaphysical conclusion—in this instance, the existence of minima in reality.

If that account of the point of Leibniz's interest in the sorites in *Pacidius Philalethi* is correct, it may be somewhat less difficult to understand why he appears to change his mind two years later and abandon the sharp-boundaries thesis for vague notions. In "Chrysippus' Heap" of 1678 Leibniz is finally taking up the topic of vagueness in its own right. Though in that text he does not rehearse the steps of the sorites argument, he does explicitly say that vague notions are those to which the sorites applies (cf. A VI,4,69). His subsequent claims that vague notions are "false," "imaginary," etc., are clearly meant to constitute a response to the paradox arising from vagueness: those notions to which the sorites applies cannot be true of anything in the world. The sorites is thus stopped at the first premise. No one is rich or poor, and so on. In this case there can be no question of his appealing to nihilism as a convenience to some further purpose, for there appears to be no further purpose to the document than to confront the difficulty posed by vague notions. And perhaps it is only here for the first time that Leibniz sets himself to decide how best to respond to the paradox. Although in the *Pacidius* Leibniz is content to accept the sharp-boundaries thesis in pursuit of other game, that bold position does not express what will be his considered view of vagueness. Thus no significant change of mind is required for Leibniz to shift in his commitments about the nature of vague notions after the *Pacidius*, for in 1676 his mind has not yet been made up.

This understanding of Leibniz will allow us to cut down the scope of the original question of why he gives up the sharp-boundaries thesis in favor of nihilism

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about vague notions. But it leaves a central point of that question still unanswered. Why does Leibniz opt for nihilism at all? Alas, no indication of his reasons for preferring nihilism to the sharp-boundaries view seems to be forthcoming in the texts, and we are left only to speculate. Perhaps, we might suggest, there is something natively psychologically attractive about the idea that vague terms are in some strong sense deficient—that they suffer from logical or semantical indeterminacy and, as Leibniz puts it, express only “imperfect essences.” Perhaps especially to a philosopher who dreamed of a logically perfect universal language (another holy grail for philosophers of logic), the difficulties that the sorites paradox poses for vague terms would appear to be obvious *prima facie* evidence of such a deficiency, and in default of any countervailing theory of vagueness the nihilist view might commend itself. This attitude appears to be what one finds in the writings of Frege and Russell,³¹ and it would perhaps be no surprise if their forerunner Leibniz should have harbored it as well. Such suggestions are not conclusive, of course, and I do not claim that they provide us with the definitive account of Leibniz’s acceptance of nihilism. At this point our thread of speculation has been run all the way out, however, and so even with a new depth of insight into his thought on the topic, we are left with the residue of a puzzle about Leibniz’s views of vagueness.³²

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Notes

¹The paradox to which the sorites owes its name: Greek *sôros* = ‘heap’. The three general features of vague terms mentioned here—admission of borderline cases, no clearly defined extension, and sorites-susceptibility—are naturally connected in an obvious way and are often taken to be equivalent. They may not in fact be perfectly equivalent; but no stand need be taken on this question here, for nothing in the present discussion depends on it. It suffices that vague terms typically dis-

play all three features.

²And nearly Leibniz alone; among the other early moderns only Gassendi seems to pay any attention to vagueness or the sorites (in his *De Origine et Varietate Logicae*). In light of this it is perhaps less surprising that apart from a few remarks in brief surveys of the history of the sorites by Williamson in *Vagueness* (London: Routledge Press, 1994) and Williamson and Sainsbury, “Sorites,” in Haldane and Wright, eds., *A Companion to the Philosophy of Language* (Cambridge: Cambridge University Press, 1997) 458-484, one finds no discussion of vagueness or the sorites paradox in the scholarly literature on early modern philosophy.

³Those remarks, struck from successive drafts of *Pacidius Philalethi*, are recorded in the editorial notes at A VI,3,539. All citations to primary texts are to the Berlin Academy edition of Leibniz’s works, *Gottfried Wilhelm Leibniz: Sämtliche Schriften und Briefe: Philosophische Schriften* (Berlin: Akademie-Verlag, 1928-99); specific references are to series, volume and page. In translating Leibniz I generally follow the translations of Richard Arthur, ed., *G.W. Leibniz: The Labyrinth of the Continuum* (New Haven: Yale University Press, 2001), Peter Remnant and Jonathan Bennett, eds., *G.W. Leibniz: New Essays on Human Understanding* (Cambridge: Cambridge University Press, 1996), and Roger Ariew and Daniel Garber, eds., *G.W. Leibniz: Philosophical Essays* (Indianapolis: Hackett Publishing Co., 1989), though I have sometimes modified their translations without comment. Inaccuracies are my own.

⁴For some discussion of Leibniz’s account of change in the *Pacidius*, see Levey, “The Interval of Motion in Leibniz’s *Pacidius Philalethi*,” *Noûs*, forthcoming.

⁵For discussion see Williamson and Sainsbury (1997, *op. cit.*); Barnes, “Medicine, Experience and Logic,” in *Science and Speculation* (Cambridge: Cambridge University Press, 1982), Barnes, Brunschwig, Burnyeat, and Schofield, eds.; and Burnyeat, “God and Heaps,” in *Language and Logos* (Cambridge: Cambridge University Press, 1982), Schofield and Nussbaum, eds.

⁶‘Unclarity’ and its cognates are intended in this paper to provide a neutral terminology in which to describe the phenomena associated with vagueness, one that does not beg the question against analyses of vagueness that would maintain the sharp-boundaries thesis for vague terms (specifically, the epistemicist analysis). I do not assume that the phenomena of vagueness are captured by any particular “logic of clarity”; nor do I assume in particular that the idea of vagueness can be fully represented simply by sentence operators corresponding to the terms ‘clearly’ and ‘unclearly’ (as has occasionally been suggested for the terms ‘definitely’ and ‘indefinitely’). Also, I should note that I do *not* use this terminology to invoke

Leibniz's epistemology of ideas—the vocabulary for which was already in place by 1676—in which ideas may be said to be either clear or obscure, and distinct or confused (cf. A VI,3,275ff.).

⁷A “weak” principle of bivalence might be that there are only two truth values, true and false, though some statements might fail to have any truth value at all. Throughout the paper, “the principle of bivalence” will be understood to mean the stronger principle that precludes truth-value gaps.

⁸Nihilism might be classified into three kinds: (1) incoherence nihilism: vague notions cannot figure as constituents in well-formed propositions because they draw no sharp boundaries; (2) inconsistency nihilism: vague notions are self-contradictory because they draw boundaries inconsistently; (3) all-or-nothing nihilism: vague notions do not “divide reality” but either vacuously apply to everything or to nothing by drawing boundaries that place everything onto just one side of the line. Perhaps no single account will be held for all vague notions. Or perhaps nihilism should be understood as maintaining three conditions on any notion: it must be able to figure in well-formed propositions, it must draw boundaries consistently, and its boundaries must be able to divide reality. And the nihilist then holds that vague notions fail to meet at least one of those conditions.

⁹For a survey of treatments of vagueness through such nonstandard logics, see Williamson (1994, *op. cit.*).

¹⁰Recall Tarski's dim view of the parallel option for resolving the Liar paradox (“this sentence is not true”): “It would be superfluous here to stress the consequences of rejecting assumption (II) [i.e. that the laws of logic hold for the language], that is, of changing our logic (supposing this were possible) even in its more elementary and fundamental parts” (from “The Semantic Conception of Truth” reprinted in Blackburn and Simmons, eds., *Truth* (Oxford: Oxford University Press, 1999), p. 125).

¹¹As Sorensen (2001, p.11, *op. cit.*) notes, although any theory can be rescued by changing logic, such maneuvering is considered deeply ad hoc in mathematics and natural science. Incoherent theories are adjusted in content to dissolve the incoherence; logic is not identified as the culprit. It does not seem that a theory of vagueness will be on higher ground here in proposing a retreat from standard logic.

¹²Syncategorematic terms and words from mathematics and logic are the best candidates for being free from vagueness; perhaps they are, though this has been denied (cf. Russell, “Vagueness,” *Australasian Journal of Philosophy and Psychology* 1 (1923): 84-92). In any event, no stand on that matter need be taken here. To highlight the extremity of nihilism, it is enough that most natural-language

categorematic terms are vague.

¹³In an earlier draft Leibniz had written the principle of the excluded third with its more traditional title: *Tertium non datur*. It is unclear whether Leibniz intends to assert the principle of bivalence or rather the law of the excluded middle (“Either p or not-p”).

¹⁴For discussion of “degrees of truth,” see Dorothy Eddington, “Vagueness by Degrees,” in Keefe and Smith, eds. *Vagueness: A Reader* (Cambridge: MIT Press, 1996). For discussion of truth value “gaps” and “gluts,” see Hyde, “From Heaps and Gaps to Heaps and Gluts,” *Mind* 106 (1997): 641-660.

¹⁵More exactly: one form of nihilism will hold this. To deny that vague notions determine sharp boundaries of application while maintaining loyalty to standard logic is sufficient for nihilism about vague notions. See note 8 above.

¹⁶Chrysippus’ famed response of gradually falling silent in his answer to successive instances of the question “Does this many grains make a heap?”, together with his strict assertion of bivalence, has been interpreted as indicating an epistemicist position. See Sorensen, *Blindspots* (Oxford: Clarendon Press, 1988) and Sorensen (2001, *op. cit.*), and Williamson (1994, *op. cit.*) for discussion and defense of epistemicism.

¹⁷For some related discussion, see Franklin Perkins, “Ideas and Self-Reflection in Leibniz,” *The Leibniz Review* 9 (1999):43-64.

¹⁸‘Taken absolutely’ is not to be understood in contrast to ‘taken relative to ...’, as is made clear later in the passage (see above). Moreover, it’s a mistake to think relativized vague terms are any better off than unrelativized ones. Relativizing vague terms does nothing to eliminate their vagueness. The term ‘poor for a Roman’, for example, no more determines a clear boundary than does ‘poor’ itself, for there can always be borderline cases of poor Romans, and of poor Roman soldiers, and so on.

¹⁹Note that Leibniz is not offering this as a solution to the sorites paradox; his point is simply that the comparative terms themselves are not necessarily undone by the vagueness of their counterpart absolute terms.

²⁰On higher-order vagueness, see Russell (1923, *op. cit.*), and Kit Fine, “Vagueness, Truth and Logic,” *Synthese* 30 (1975): 265-300.

²¹Horace, *Epitres*, II,I,v.47.

²²A sociological fact from philosophy can attest to the pull of the idea that vagueness is due to semantical indeterminacy: Of the main candidates to be the source of the phenomena of vagueness, indeterminacy in our linguistic practices (and in the associated semantical properties of our language) has been the most popular among

contemporary writers, with indeterminacy in the world coming in a distant second. Ignorance of the sharp boundaries of our language and world—epistemicism—is the least popular, having very few advocates to date.

²³For some discussion of supervaluationism, see Kit Fine (1975, *op. cit.*), Michael Dummett, “Wang’s Paradox,” *Synthese* 30 (1975): 301-324, and Williamson (1994, *op. cit.*)

²⁴See Williamson and Sainsbury (1997, *op. cit.*).

²⁵Presumably on this reading Leibniz defends the idea of real essences against Locke’s critique even though he doesn’t accept a realism about species because his theory of individual substances is an essentialist one, and so he is committed to real essences anyway. This gets Leibniz out of inconsistency only at the price of changing the score on Locke. Still, in the central passage in the *New Essays*, Leibniz indeed shifts from speaking of insensible transitions between *species* to real essences of *things*, before returning to talk of species (cf. A VI,6,321 above). So perhaps that is what goes on there. And realists about species essences who wish to reject realism about vagueness will be left searching for a principle to distinguish them if they accept the possibility of insensible transitions between species.

²⁶First published in 1931 in *Montshefte für Mathematik und Physik*, v. 38, 173-198. Reprinted in English as *On formally Undecidable Propositions of Principia Mathematica and Related Systems* (New York: Basic Books, Inc, 1962), translated by B. Meltzer.

²⁷Boolos likewise presents a distinct proof of the incompleteness theorem that relies not on the Liar but on Berry’s paradox (“the least integer not nameable in fewer than nineteen syllables”). Cf. George Boolos, “A New Proof of the Gödel Incompleteness Theorem.” *Notices of the American Mathematical Society* 36 (1989): 388-390 and 676; reprinted in *Logic, Logic and Logic* (Harvard Press, 1996), 383-388. Boolos reports that the interest of the new proof is that it “provides a *different sort of reason* for the incompleteness of algorithms” (1996, p.388).

²⁸Leibniz includes a diagram with several labeled points; for clarity in the present discussion I have omitted a few references to labeled points.

²⁹James Cargile imagines a similar example: a rapid-fire series of photographs is taken of a tadpole turning into a frog; the sorites will force us to admit that for some consecutive pair of pictures, p_n and p_{n+1} , the creature depicted in p_n is a tadpole but the one depicted in p_{n+1} is not, no matter how close in time those pictures were taken. “And this argument,” Cargile notes, “is obviously independent of the speed of the camera, which could be taken right up to the theoretic limit” (p. 89 of “The Sorites Paradox,” *British Journal for the Philosophy of Sci-*

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ence 20 (1969), 193-202; reprinted in R. Keefe and P. Smith (1996, *op. cit.*), 88-98; page reference is to the reprint).

³⁰Leibniz's conclusion, in fact, is exactly parallel to that of the first sorites: "either there is no way for something to become near properly and of its own accord, or something turns from being near to being not-near by the addition or subtraction of a minimum, so that there are minima in reality" (A VI,3,540).

³¹See Frege "Comments on Sense and Meaning" and "The Argument for My Stricter Canons of Definition," both in *Posthumous Writings* (Oxford: Basil Blackwell, 1979), Long and White, eds. And see Russell (1923, *op. cit.*) and "Lectures on Logical Atomism" in *The Collected Papers of Bertrand Russell*, Vol. 8 (London: Allen & Unwin, 1983), Eames and Slater, eds.

³²My thanks to Christie Thomas, Bob Fogelin, Walter Sinnott-Armstrong, Glenn Hartz and a referee for helpful comments and suggestions.