Leibniz on Infinite Number, Infinite Wholes, and the Whole World: A Reply to Gregory Brown[1]

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I

Reductio arguments are notoriously inconclusive, a fact which no doubt contributes to their great fecundity. For once a contradiction has been proved, it is open to interpretation which premise should be given up. Indeed, it is often a matter of great creativity to identify what *can* be consistently given up. A case in point is a traditional paradox of the infinite provided by Galileo Galilei in his *Two New Sciences*, which has since come to be known as Galileo's Paradox. It concerns the set of all (natural) numbers, N: since to every number there is a corresponding square, there are as many squares as numbers. But since there are non-squares between the squares (2, 3, 5 etc.), "all numbers, comprising the squares and the non-squares, are greater than the squares alone" (EN 78)[2], i.e. there must be fewer numbers in the set of all squares S than in N. Thus N is both equal to and greater than S. This is a contradiction, so one of the premises must be given up: the question is, which one?

In his consideration of the paradox in 1676, Gottfried Leibniz concluded:

> Hence it follows that in the infinite the whole is not greater than the part, which is the opinion of Galileo and Gregory of St. Vincent, and which I cannot accept; or that infinity itself is nothing, i.e. that it is not one and not a whole.[3]

Here he identifies two candidates for rejection: (W) that in the infinite the whole is greater than the part, and (C) that an infinite collection (such as the set of all numbers) is a whole or unity. The 'whole' in question here is the set of all natural numbers N; the 'part' is any proper subset of N. Leibniz upholds W, and this leads him to reject C. Cantor upholds C, and this leads him to reject W. In fact, after Bolzano, Dedekind and Cantor, ~W came to be regarded as the defining characteristic of infinite collections, as Nicholas Rescher noted over thirty years ago.[4] But here I wish to draw attention to the logical symmetry: ~C is a conclusion for Leibniz, with W as premise. ~W is a conclusion for Bolzano and Cantor, with C as premise.

Now Cantor's huge success in developing a theory of the transfinite on the basis of swallowing what Leibniz could not—that an infinite collection could be equal to its proper subset—has led some commentators (beginning with Cantor) to rue the fact that Leibniz did not go further in Cantor's direction, especially given his espousal of the actual infinite. Says Rescher: "Subsequent developments in mathematics—the theory of transfinite numbers, point-set topology, measure theory—have shown that Leibniz's method of attack was poor. Indeed, Galileo had already handled the problem more satisfactorily ..."[5]

In assessing this kind of judgement I believe it is necessary to keep several points in mind. The success of Cantor's theory counts against Leibniz's choice, since the theory of the infinite Leibniz based on that choice has undergone no comparable modern development. But it does nothing to show whether Leibniz's theory is inconsistent. Given the other premises assumed in Galileo's Paradox, W ∅ ~C (Leibniz); and this is logically equivalent to C ∅ ~W (Cantor). Of course, if one assumes C—or anything logically equivalent to it—this will automatically generate a contradiction when combined with Leibniz's theory of the infinite (which is based on W). So my first point is a logical one:

1. When judging the consistency of Leibniz's theory of the infinite, one cannot assume C, or that Cantor's account of the actual infinite is true. Nor can one prove it unsound by appealing to C, since that will beg the question.

Second, with regard to Rescher's remark about Leibniz's "poor method of attack":

2. Even if, judged by modern standards, Leibniz's theory be deemed false or inconsistent (I do not cede that it is), Leibniz might still have had good historical reasons for adopting the response to Galileo's Paradox that he did.

Third, a corollary of point 1;

3. Even if Cantor's theory is consistent, this does not show that Leibniz's theory is inconsistent. This must be judged on the latter's own terms.

Perhaps, pace Cantor, there is more than one consistent theory of the actual infinite. Indeed, as I shall argue, I believe Leibniz does have a consistent account of the infinite as a syncategorematic but actual infinite. Lastly,

4. In making judgements about Leibniz's metaphysics, it will be better historiographic practice to try to understand Leibniz's positions through his own philosophy of the infinite than through Cantor's.

In this connection, it seems to me that Leibniz's theory of the infinite allows one to understand why bodies, which are (according to him) infinite aggregates of their parts, are not infinite collections of them; and why the universe, understood as the collection of all unities, is not itself a unity.

These considerations, and particularly those mentioned last, are what led me to respond[6] to Gregory Brown's criticisms[7] of an argument by Laurence Carlin[8] in this journal. This prompted a detailed reply from Brown,[9] to which I shall now try to respond with relative brevity.

II

Re point 1: Brown characterizes Leibniz's rejection of infinite number as "unsound" (pp. 23, 24, 34, 40, 41, 47); "Leibniz ... held that infinite number and infinite wholes are generally contradictory. But again, he was mistaken about that premise" (p. 30). "[H]e was wrong ... [to think] that he had established that the very notion of infinite number was contradictory"; "Cantor and Frege were able to establish that $\aleph_0$, for example, is no more contradictory than the number 5" (p. 28). Brown's point is that if Leibniz were correct in assuming that infinite number is generally contradictory, Cantor would not have been able to erect a successful theory on that basis. Therefore Leibniz was wrong, and his argument unsound. But Leibniz does not *assume* that infinite number is contradictory: it follows from his assumption of W. So Brown's argument (like those of Cantor, Russell and Rescher before it) reduces to this: if with Cantor one assumes C, that an infinite collection (such as the set of all numbers) is a whole or unity, then one can establish a consistent theory of infinite number; therefore Leibniz's argument against it is unsound. But is this not to argue in effect that since C entails ~W, it is unsound to infer ~C from W (as in *point 1* above)?

Of course, the argument works both ways: failing any *independent* argument for W, Leibniz cannot claim to have demonstrated in all generality that C is false, and this is no doubt the charitable reading of Brown's contentions against him on this score. Still, it remains the case that to say that Leibniz's argument is unsound on the basis of the success of Cantor's theory is to assume the truth of C, and thus to beg the question (unless one has an independent argument for C, which Cantor does not).

Re point 2: But hasn't Leibniz's position been discredited? Wasn't Galileo's approach in abandoning W superior, as Rescher is not alone to have alleged? In my opinion there are two problems with this kind of assessment. First, it is whiggish. The rejection of W is a necessary first step to the theory of the transfinite, and to the point-set topology and measure theory erected on that foundation. But to regard W as superior for this reason is to assume that the worth of a historical position should be assessed in terms of whether it leads to a position now generally regarded as correct. Second, it does not seem a very accurate statement of the historical situation here. Galileo did not reject W in order to apply a Cantorian (or Bolzanian) criterion of equality of size of infinite sets in terms of 1-1 correspondence. In fact he rejected *both* that the whole is greater than the part, *and* that it is equal to the part. He wrote:

I do not see how it is possible to come to any decision other than to say that all

> the numbers are infinite, the squares are infinite, and their roots are infinite; the multiplicity of squares is neither less than that of all the numbers, nor is the latter greater than the former. And in final conclusion, the attributes of equal, greater, and less have no place in infinities, but only in bounded quantities (EN 79).

That is, Galileo embraced the actual infinite and rejected the idea that infinite multiplicities are susceptible to quantity, i.e. can be ranked according to size. (This is the complement of his thesis about the actually infinitely small, which is also not quantifiable: the infinitely many indivisibles in the continuum, Galileo calls *parti non quante*.) In my earlier paper I provided historical reasons why the option of taking 1-1 correspondence as definitive of equality of infinite sets of indivisibles (as does Cavalieri), runs into difficulties in the case of geometric quantities; difficulties that can only be avoided once one has the appropriate theory of *measure*.[10] Galileo does not avoid these difficulties very successfully by insisting that greater, less and equal do not apply to the infinite. In this context, I claimed, Leibniz had "good reasons" for holding onto W instead (1999, p. 109). This was a historical point. I was not arguing that Leibniz's strategy is viable from a Cantorian standpoint, as Brown seems to suggest in his reply when he argues that modern considerations about measure show that Leibniz did not have good reasons for concluding from his Diagonal Paradox that a line cannot be composed from an infinite number of points.[11]

Re point 3: Brown writes:

> [A]bsent a sound argument to the effect that infinite number is generally contradictory, we may reasonably say that if the world contains an *actual* infinity of creatures, as Leibniz does, then the cardinality of the set of creatures is an infinite number (p. 31).

Here, as remarked on earlier, Leibniz's argument against infinite number is characterized as "unsound" because his conclusion (~C) is incompatible with Cantor's premise (C); we are therefore urged to interpret the "actual infinite" in Leibniz in Cantor's terms. But this begs the question of whether Leibniz's actually infinite division can be understood consistently through his own philosophy of the infinite. Brown argues that it cannot. Picking up on arguments of Benardete and Levey, he argues that whilst Leibniz is committed to a potential infinite in his mathematics, this is at odds with his espousal of the actual infinite in his metaphysics.

This is a crucial point. For if Leibniz's construal of the infinite is not consistent even on its own terms, and Cantor's is, then Brown will certainly be correct in his judgement that Leibniz would have done better to embrace infinite wholes and thus infinite number (Brown 1998, p. 121).

Leibniz's construal of infinite number can be found in various papers and letters written over a span of some 40 years. Here are two characteristic formulations:

> When it is said that there is an infinity of terms, it is not being said that there is some specific number of them, but that there are more than any specific number.[12] Accurately speaking, instead of an infinite number, we ought to say that there are more than any number can express.[13]

These formulations have much in common with the doctrine of the infinite as *syncategorematic*, advocated by William of Ockham. Ockham held that there are actually infinitely many parts of the continuum, but that this was to be understood in the sense that "there are not so many parts finite in number that there are not more" (*partes non tot finitas numero quin plures*, or *non sunt tot quin sint plura*).[14] This is contrasted with the *categorematic* sense of infinity, according to which to say that there are infinitely many parts is to say that there is a number of parts greater than any finite number, i.e. that there is an infinite number of parts. This distinction can be expressed using predicate calculus. Thus if x and y are numbers of parts, then to assert an infinity of parts *syncategorematically* is to say $(x)(\exists y)y > x$. But to assert their infinity *categorematically* would be to assert that there exists some one number of parts y which is greater than any finite number x, i.e. that $(\exists y)(x)y > x$. Thus the first way of expressing it does not commit you to infinite number, since x and y may both be finite. But the categorematic expression commits you to the existence of a number greater than all finite numbers.

Let us compare this with what Brown says of Leibniz's actually infinite division:

> even if there is no last division in a body, or no least part of a body, it does not follow that the number of parts in a body *actually* divided to infinity cannot reasonably be said to be infinite. (p. 47)

How would Leibniz have responded to this? I believe he would have granted Brown that, if a body is *actually* divided to infinity, then it must indeed have an *actual infinity* of parts. But it does not therefore follow that it has an *infinite number* of parts. Indeed, to reason from $(x)(\exists y)y > x$ to $(\exists y)(x)y > x$ is to commit a logical fallacy (the "quantifier shift fallacy"). So when Brown states that "infinite number, as I have already argued, could reasonably be thought to be implied by the existence of bodies actually divided to infinity" (p. 32) and "a body *actually* divided to infinity can reasonably be said to imply the existence of an infinite cardinal number" (p. 33), it must be replied that no such implication holds on the assumption of a syncategorematic actual infinite. The number of parts can "reasonably be said to be infinite" only if one adopts Cantor's rival view.

This may not constitute a full reply to the case Samuel Levey has made for a tension between constructivist tendencies in Leibniz's philosophy of mathematics and his commitment to the actually infinite,[15] which Brown cites at length in his paper. But I think it establishes the consistency of the core idea of an infinity that is actual, but syncategorematically understood, and which involves no commitment to infinite number. Nevertheless, one might still question, as does Brown, whether Leibniz's syncategorematic infinite is adequate to his metaphysics of actually infinite division. So let me now turn to a consideration of the applicability of Leibniz's construal of infinite number to his metaphysics of matter.

III

Re point 4: In many places Leibniz gives explicit reasoning for his conclusion that "Created things are actually infinite", but none so explicit as in a piece beginning with these words which he wrote in the 1680s, as I have explained elsewhere:[16]

> Created things are actually infinite. For any body whatever is actually divided into several parts, since any body whatever is acted upon by other bodies. And any part whatever of a body is a body by the very definition of body. So bodies are actually infinite, i.e. more bodies can be found than there are unities in any given number (A VI iv N266: 1393; LLC 235).

This, I submit, is the same syncategorematic (yet actual) infinite that is involved in Leibniz's definition of infinite number: "there are more than any number can express". Leibniz is not asserting bodies' infinity categorematically, that there is a number of bodies **I**, where **I** is greater than any finite number; but syncategorematically: for any finite number **F**, there are more bodies than this. Thus Leibniz can reasonably claim that there are actually infinitely many bodies without committing himself to infinite cardinals.

But it will be worth teasing out his argument a little more, to see the connection with the issue of *wholes*, which is central to my dispute with Brown. According to Leibniz's conception, the parts of matter are instituted and individuated by their differing motions. Each body or part of matter would be one moving with a motion in common. But this does not rule out various parts internal to that body having their own common motions, which effectively divide the body within. Moreover, a body that is divided into parts can be considered as an *aggregate* of those parts. Accordingly we have the following four premises:

1) Any body whatever is actually divided into several parts.
2) Each such body is the *aggregate* of the parts into which it is divided.
3) Any part whatever of a body is a body.

4) Each part of a given body is the result of a division of that body or of a part of that body.

From 1) and 2) it follows that every body is an aggregate of parts; and from 3), that

5) Every body is an aggregate of other bodies.

Incidentally, if one accepts the further premise that

6) What is aggregated cannot be a substantial unity.

this immediately yields one of Leibniz's chief doctrines, that

7) A body cannot be a substantial unity.

This does not, of course, entail that a body cannot be an aggregate of substantial unities. But I will not pursue that further just yet. To resume: we have the result that every body is an aggregate of parts, each of which is an aggregate of further parts, and so on to infinity. That is, by recursion,

8) Every body is actually infinitely divided, or, is an aggregate of an actual infinity of parts.

It is also the case that for Leibniz any aggregate of things that can be considered together at the same time can be considered a whole, as he says in a passage quoted by Brown.[17] That is, since

9) A whole is the aggregate of its parts.

it follows that

10) Every body is an infinite whole.

Here it is important to note that 'whole' in this sense of the aggregate of things that can be considered at the same time does not necessarily carry the connotation of "collection" of its parts. Indeed, in this case of a body as an infinite aggregate it does not. It is a whole such that any part of it whatever is an aggregate of further parts. Although each part can be specified, there is "no end" of parts. There is, correspondingly, no number of all its parts.[18] Thus, as I explained in my previous paper, body may be understood as a whole in the distributive mode, but not in the collective mode. *Each* division is actual and determinate, but there is no such thing as *all* the divisions, understood collectively.[19] Although bodies are infinite aggregates of their parts, they are not infinite collections of them. They are not, Leibniz would say, "true wholes", a true whole being the collection of all its parts. Any such collection will have a determinate number of parts. Thus, assuming

11) A true whole is the collection of all its parts.[20]

we have, given Leibniz's rejection of infinite number,

12) No infinite aggregate is a true whole.[21]

A body, then, is actually infinite, but not a true whole, not complete. Each of its parts is actual and perfectly determinate, but there is no complete collection of them. As I have argued elsewhere (and in detail in the paper cited in note 16), I believe this was one of Leibniz's leading arguments for the phenomenality of bodies, and for the necessity of substantial unities. If material bodies cannot be true wholes, then neither will any aggregates of them be; hence the need for something substantial and non-material to act as a principle of unity for body.

This distinction between collective and distributive wholes is relevant to Brown's view of the gap between Carlin's views and my defence of them. He claims that while I deny that "a body of finite extensive magnitude can be ... a *whole,*" Carlin argues that such a body "*is* a whole precisely because it is an arithmetical unity" (45). But to me this seems to trade on an equivocation on these two meanings of whole. I claim that for Leibniz a body is a whole in distributive mode, although, unlike the universe, its parts can sum to an arithmetical unity. But neither it nor the universe are wholes in the sense of infinite collections of their parts.[22]

To reiterate: What is under dispute is whether an infinite aggregate can be considered as a totality, whether it can be treated as a 'one' or collective whole. Leibniz rejects the idea that it can—I think, consistently. Cantor's theory of the transfinite, on the other hand, rests firmly on the idea that (at least some) infinite multiplicities can be treated as sets or consistent totalities. For according to Cantor's second generating principle of real whole numbers, there is a least ordinal after the whole sequence of natural numbers, and ω is the name he gives to this, the first transfinite ordinal. (On one interpretation, ω simply is the infinite set of all its predecessors, the natural numbers in their standard ordering. $\aleph_0$ is the cardinality of this set.)

Thus when we come to the question of whether one can reasonably say that there is an actually infinite number of divisions (Brown, p. 47), the answer is yes: Cantor's theory shows that this can be done. On that theory, if the division is of order-type ω, the (cardinal) number of divisions will be $\aleph_0$. But the point was whether we are obliged to interpret Leibnizian actually infinite division à la Cantor, and I believe the argument of this section shows that we are not.

IV

Now let me turn to the question of whether the universe could have a soul. In my previous paper I argued that Leibniz was correct in contending that, according to his principles, the world or infinite accumulation of substances is no more one or a whole than infinite number itself. We can see how this follows here. For if we take (9) "A whole is the aggregate of its parts", together with (W) "the whole is greater than the part", (and assume that if A is greater than B, A cannot be equal to B), we obtain

(13) No whole is equal to a part of itself.

Now suppose the universe U is the aggregate of all wholes. If U is itself a whole, it will be included in itself. Therefore it will be equal to a part of itself, contrary to (13). Thus the universe, conceived as the aggregate of all wholes, cannot be a whole.[23]

An analogous Leibnizian refutation of infinite number might run as follows:

> Suppose N is the number of all numbers, regarded as a whole. If N is itself a number, it will be included in itself. Therefore, it will be equal to a part of itself, contrary to (13). Therefore there is no number of all numbers N, if N is regarded as a whole.[24]

Now to such reasoning Brown objects, with some justice, that the issue is not whether the universe is a *whole*, but whether it can be a *corporeal substance* (p. 45). After all, (and as premises 6 and 9 together entail), substances are not wholes composed of parts. One might perhaps reply to this on Leibniz's behalf that if the universe is not *one entity*, then it cannot be a substance. But, as Brown has urged, such an argument appears to be too strong: if the universe is disqualified from being a substantial unity on the grounds that an infinite aggregate of substances is not one entity, then this should apply just as surely to any corporeal substance, whose organic body is also an infinite aggregate of substances. This, as Brown reminds us, was Carlin's initial point: "Why should we admit that infinite aggregates, like the world, cannot admit of a soul? After all, organic bodies just are an accumulation of infinitely many substances, yet he clearly thought they had souls..." (Carlin, 1997, p. 7; Brown, 2000, p. 32).

In my previous paper I sketched a defence on Leibniz's behalf (1999, 112), using an analogy with infinite series to support Carlin's argument that (i) the universe, being actually infinite in magnitude, would not possess an arithmetical unity, and (ii) "only things which have arithmetical unity can be endowed with souls" (Carlin, p. 12). I suggested that the universe, conceived as an infinite addition of bodies not decreasing in size, would involve infinite quantity, like the sum of a diverging infinite series; whereas body, in containing smaller and smaller parts within, would be analogous to a convergent infinite series, summing to a finite quantity. Thus the argument would turn on the impossibility of infinite number only indirectly, insofar as infinite magnitude involves infinite number and finite magnitude does not. The infinite aggregate of parts of a finite organic body, on the other hand, could be understood as a distributive whole: Leibniz would be no more committed to construing an organic body as an infinite collection of parts, than to construing a converging infinite series as an infinite collection of all its terms.

To this Brown has countered that Leibniz has an "operational" approach to infinite series, in which their sum "is to be explained in terms of a *potentially* infinite sequence of partial sums" (p. 26). But on this construal even the partial sums of a divergent infinite series would only ever be finite. So the analogy fails: "on the operational approach, neither a convergent nor a divergent series implies infinite number" (p. 46). Moreover, if Leibniz were relying on this analogy to argue, à la Aristotle, that a universe of actually infinite magnitude is impossible, then he would be guilty of letting his mathematical constructivism "spill over disastrously into his philosophy of matter," as Levey has put it (1999, p. 155), since his premise is that the universe is an *actual* infinity of substances.

As should be clear from my arguments above, however, I believe Leibniz advocated not a potential infinite, but an actual infinite syncategorematically understood. On this reading, an infinite series has an actual infinity of terms in the sense that no matter how many terms are taken, there are actually more; but it has no number of terms which collectively add to its sum. To say that an infinite series has a sum is to say that (S) there is a number such that, for any specifiable error, some finite series with the same rule and first term will sum to that number within the specified error.[25] Similarly, the universe may be said to be infinite in magnitude in the (syncategorematic) sense that, no matter how large a magnitude one takes, its magnitude exceeds this, but not in the (categorematic) sense that it has a magnitude greater than any finite magnitude. Consistently with this, as Carlin pointed out (p. 11), Leibniz claims that "it is of the essence of number, of a line, and of a whole to be limited" (to Des Bosses, G.II.304). Thus a converging infinite series can be regarded as a whole, since it (more accurately, the sequence of its partial sums) is limited by a finite number. But a diverging series cannot, since it is not limited by any finite number. If one thinks that this begs the question, let the sequence of partial sums of a diverging series be limited by a (categorematically) infinite number: then, by rule (S), this would have to be regarded as its sum. Analogously, "although the world is infinite in magnitude, it is not one whole" (G.II.304), since it is not limited. Or, if it is to be regarded as a whole and therefore limited, it would have to be limited by a categorematically infinite magnitude, since only a magnitude greater than any finite sum of magnitudes could stand as a limit to a sequence of arbitrarily increasing magnitudes; but this would involve the categorematic infinite; therefore not.

In sum, I believe Leibniz has a defensible position regarding the soul of the universe and the body of a finite corporeal substance, although his expression of it is not always very felicitous. The body of the universe, conceived as a whole, would involve, *per impossibile*, infinite number, which finite body would not. The infinite parts of a finite body, on the other hand, although not possessing a true unity, could appear as one thing to perception; and as Carlin says, it seems to be a tacit premise for Leibniz that only such a phenomenal body is a candidate for being the body of a real substance.

In one sense Brown and I are not very far apart here, as he has pointed out himself. For he agrees that only a phenomenal body could be the body of a substance, but argues that Leibniz should have argued against the world soul on that basis alone. For Leibniz, he thinks, has a defensible argument against the world's having a phenomenal body that is independent of his denial of infinite number, and depends only on its being unlimited (1998, p. 124).[26] But this premise is not independent of Carlin's if one grants Leibniz's axiom that "it is of the essence ... of a whole to be limited"; and to regard the unlimited body of the universe as one or a whole is, as I have argued, to commit oneself to infinite number.[27]

That said, I nevertheless agree with Brown that when Leibniz argues, as he sometimes does, that the universe, understood as an infinite aggregate of substances, cannot be a substance, he begs the question at issue. And when he describes organic body as an "infinite accumulation of substances" he makes matters even worse. For by his principles, body can be an infinite aggregate, if this is understood as a whole in distributive mode, but not an infinite collection, since there is none. Likewise the universe can be the aggregate of all substances, if this "all" is understood distributively, but it cannot be an infinite collection. Nor, being of infinite extension, could its body be considered a whole without that implying a contradiction.

V

In conclusion, if one treats Leibniz's philosophy of the infinite from a point of view that presupposes the truth of the Cantorian theory of the transfinite one will find much to object to. One will find, as Brown explains in abundant detail, a premature rejection of the idea of an infinite set, a false assumption that the unboundedness of the infinite rules out its being treatable as a determinate whole, a failure to appreciate that if the measure of a set of points A is greater than that of a set of points B, this does not entail that the number of points in A is greater than that in B, and so forth. But if one instead treats Leibniz's construal of the infinite on its own terms, as I was urging in my previous paper, one finds no such inconsistency. Not only does one find a philosophy of the infinite compatible with and illuminative of Leibniz's deep metaphysics, including his rejection of the world soul, but also a significant but neglected third alternative in the philosophy of mathematics to the transfinite and the potential infinite, in the doctrine of the infinite as actual but syncategorematic.[28] The urge to "Cantorize" Leibniz should be resisted!

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Notes

[1] I am very grateful to Sam Levey and Greg Brown for their comments on a previous draft of this paper. GB's generous and detailed response helped me to avoid missing the point in places, to engage his arguments more directly, and to acknowledge some errors; but in rewriting, I have abandoned some of the passages of that draft that gained from SL's always sage advice.

[2] Passages from *Two New Sciences* are keyed to the Edizione Nazionale of Galileo's *Opere,* ed. Antonio Favaro (Florence, 1898; hereafter EN). The translations are from Appendix 2 of the work cited in the next footnote.

[3] *Leibniz: The Labyrinth of the Continuum*, ed. and trans. Richard T. W. Arthur (New Haven: Yale University Press, 2001; hereafter LLC), p. 9.

[4] "The property that 'the whole is not greater than the part' to which Leibniz objects in infinite collections came to be their defining characteristic." Nicholas Rescher, *The Philosophy of Leibniz* (Englewood Cliffs, NJ: Prentice Hall, 1967), p. 106, n. 71.

[5] *Op. cit.* p. 111.

[6] "Infinite Number and the World Soul: in Defence of Carlin and Leibniz," *Leibniz Review* **9**, 1999, pp. 105-116.

[7] Gregory Brown, " Who's Afraid of Infinite Numbers? Leibniz and the World Soul," *Leibniz Society Review* **8**, 1998, pp. 113-125.

[8] Laurence Carlin, "Infinite Accumulations and Pantheistic Implications: Leibniz and the 'Anima Mundi'," *Leibniz Society Review* **7**, 1997, pp. 1-24.

[9] Gregory Brown, "Leibniz on Wholes, Unities and Infinite Number," *Leibniz Review* **10**, 2000, pp. 21-51.

[10] "Here if one seeks to avoid the contradiction by taking the 1-1 correspondence

between the points in the side and the diagonal to establish their equality in number, then, *in default of a theory of measure*, it appears the indivisibles of unequal lines must be of unequal magnitudes" (1999, p. 108; emphasis added).

[11] "For the part-whole axiom may be retained as long as it is understood that 'equality' can be defined in different ways for different purposes, and not all legitimate construals of 'equality' need satisfy the part-whole axiom... Thus the Diagonal Paradox gives no good reason for thinking that lines cannot be composed of an infinite number of points" (p. 23).

[12] Leibniz, 1698, GM.iii.566; Sam Levey's translation.

[13] Leibniz to Des Bosses, 11 March 1706, G.ii.304-05.

[14] See Philip Beeley, *Kontinuität und Mechanismus* (Stuttgart: Steiner, 1996), 56-64, esp. p. 59.

[15] See in particular Samuel Levey, "Leibniz's Constructivism and Infinitely Folded Matter", pp. 134-162, *New Essays on the Rationalists*, ed. Rocco Gennaro and Charles Huenemann (New York: Oxford University Press, 1999).

[16] "Infinite Aggregates and Phenomenal Wholes: Leibniz's Theory of Substance as a Solution to the Continuum Problem," *Leibniz Society Review* **8**, 1998, pp. 25-45: p. 27.

[17] *Definitiones notionum metaphysicarum atque logicarum*, A VI, iv, 627; this piece appears in my translation volume (cited in note 3 above), as "On Part, Whole, Transformation and Change" (LLC 271).

[18] This is the sense in which my claim in 1999 should be understood, that "no last term in an infinite series" is equivalent *for Leibniz* to "no totality of parts in an infinite division" (p. 111).

[19] Brown makes much of the fact that "*every part* of matter is actually divided" so that "*all* the divisions must *actually be given at once*, even if there is no last division" (p. 38). As I have argued, Leibniz would accept this if "all" and "every" are interpreted distributively; this does not, *pace* Brown, commit him to "an infinite *number* of parts in matter" (p. 41).

[20] Thus Brown was quite correct to challenge any hint in my previous paper that "true wholes" could be identified with substances, which have no parts (Brown 1999, pp. 41-42).

[21] Cf. Leibniz, *New Essays*, p. 159: The idea of an infinite number is absurd, "not because we cannot have an idea of the infinite, but because an infinite cannot be a true whole".

[22] Cf. Carlin: for Leibniz "there is no actually existing infinite, *if* that is understood as a genuine whole consisting in infinitely many parts" (1997, p. 10). Brown

is right, though, that I have overestimated my agreement with Carlin, extrapolating too quickly from this point of concurrence. In particular, putting aside any remaining differences about unities, I do not agree with Carlin's claim that an actual infinite could not be susceptible to quantity, nor his identification of the syncategorematic with the potential infinite and his relegation of it to the ideal realm (pp. 9-10).

[23] This argument derives from a similar one given by Rudy Rucker in *Infinity and the Mind* (New York: Bantam, 1983, p. 51), who proves, on the assumption that no physical thing may be a part or component of itself, that "the Cosmos is not a thing, but only a Many that can never be a One".

[24] Compare with the following proof offered by Leibniz in 1672: "Or perhaps we should say, distinguishing among infinites, that the most infinite, ôr all the numbers, is something that implies a contradiction, for if it were a whole it could be understood as made up of all the numbers continuing to infinity, and would be much greater than all the numbers, that is, greater than the greatest number" (A VI iii 168; LLC 9).

[25] Cf. Leibniz's "[to say] that a certain infinite series of numbers has a sum ... [is to say] that any finite series with the same rule has a sum, and that the error always diminishes as the series increases, so that it becomes as small as we would like" (A VI.iii 503; LLC 99).

[26] This is that "since there is nothing, no bodies, outside of the universe, the universe cannot be said to stand in any spatial relationship to other bodies. Hence it cannot be a body, properly speaking, and thus ... cannot be the body of God" (Brown, 1998, p. 124)

[27] Whether such a restatement of his original argument would now be acceptable to Carlin, I should add, I am not in a position to say.

[28] I have given a more extensive treatment of the relationship between the views of Leibniz and Cantor on the actual infinite in an as-yet-unpublished dialogue, "Leibniz in Cantor's Paradise: A Dialogue on the Actual Infinite".