Leibniz on Wholes, Unities, and Infinite Number

Gregory Brown, University of Houston

Abstract

One argument that Leibniz employed to rule out the possibility of a world soul appears to turn on the assumption that the very notion of an infinite number or of an infinite whole is inconsistent. This argument was considered in a series of three papers published in *The Leibniz Review*: in the first, by Laurence Carlin, the argument was delineated and analyzed; in the second, by myself, the argument was criticized and rejected; in the third, by Richard Arthur, an attempt was made to defend Leibniz's argument against my criticisms. In the present paper, I take up the matter again in an attempt to clarify the issues involved and to defend my original criticisms of the argument against the objections raised by Arthur.

In the 1999 issue of *The Leibniz Review*, Richard Arthur responded (Arthur, 1999) to a paper I had written (Brown, 1998) criticizing a certain argument that Leibniz had formulated to refute the possibility of a world soul. This argument, which had been delineated and analyzed in an earlier paper by Laurence Carlin (Carlin, 1997), turns upon Leibniz's rejection of infinite number and infinite wholes, on the grounds that they imply a contradiction. In the present paper, I wish to respond to Arthur by attempting to clarify the issues involved in the apparent dispute between us. Ironically, in light of the fact that Arthur billed his response as a defense of both Leibniz *and* Carlin, I will also attempt to make clear that there is an absolutely fundamental point of disagreement between the position that Arthur defends and the position that was originally defended by Carlin—a point of disagreement that Arthur ignores in his own paper.

Arthur's substantive remarks begin with an attempt to establish that "Leibniz had good reasons for upholding the part-whole axiom as constitutive of quantity, and for rejecting the assumption that an infinite aggregate is a whole." He seems to suggest that these "good reasons" are grounded in the supposed fact that in the case of "the continuous magnitudes of geometry . . . the part-whole axiom unambiguously applies" (Arthur, 1999: p. 107). He illustrates his point by describing a paradox presented by Leibniz in two papers written in the 1670s,[1] a paradox that Arthur dubs "Leibniz's Diagonal Paradox":

> Suppose lines to be infinite aggregates of indivisible points. Now the points on the diagonal of a rectangle can be put into 1-1 correspondence with the points on one of the sides by drawing lines parallel to the bottom line of the

> rectangle, so there are as many points in one as the other; yet *the magnitude of the side is clearly less than the magnitude of the diagonal* [my emphasis], i.e. is equal to a part of it. Thus the part is equal to the whole, contrary to the part-whole axiom assumed as a premise. Since we began with well-defined wholes, the contradiction must result from supposing them to consist in infinitely many indivisible points. It therefore follows that the lines cannot be composed of points or indivisibles. [ibid.: p. 108]

In the part of this passage that I have emphasized, Arthur has Leibniz assuming that the magnitude of the side of a rectangle is obviously "less than" the magnitude of the side of the diagonal of the rectangle. He then has Leibniz concluding that the side of the rectangle "is equal to a part of [the diagonal]," and thence that "the part is equal to the whole, contrary to the part-whole axiom assumed as a premise." But the sense in which the side is said to be "less than," and hence equal to a "part" of, the whole (i.e., the diagonal) makes appeal to a *congruence criterion of "less than,"* whereas the sense in which the side is said to be equal to, and not less than, the whole (the diagonal) makes appeal to the criterion of equality with which the passage began, namely, that in terms of a one-to-one correspondence of component points. Thus no inconsistency with the part-whole axiom will be forthcoming unless it is assumed that the criterion according to which the side of the rectangle fails to be less than the diagonal is the same criterion for "less than" that is involved in the part-whole axiom; and there seems little reason to think that these criteria need be the same.[2] Thus if the part-whole axiom is interpreted as invoking the *congruence* criterion of equality, rather than the criterion of one-to-one correspondence of component points, then the side and diagonal of a rectangle will indeed satisfy the part-whole axiom, consistently with the assumption that "lines [are] infinite aggregates of indivisible points." It seems, then, that Arthur is wrong to assume that the part-whole axiom "unambiguously applies" in the case of continuous geometrical magnitudes; for it applies if "less than" is read as invoking a congruence criterion of equality, but fails to apply if "less than" is read as invoking a criterion of equality in terms of one-to-one correspondence of component parts. Thus I will not grant that Leibniz had "good reasons" for *generally* "upholding the part-whole axiom as constitutive of quantity," although he may have had good reasons for upholding the part-whole axiom for certain purposes and in certain contexts—for example, in the context of comparing finite quantities, like the *lengths* of finite lines, but *not* in the context of comparing *infinite* quantities, like *the number of points* in lines of finite length. There is no need, then, to assume, as Arthur does, that the option I should have proposed to resolve Galileo's paradox is "to discard the part-whole axiom and define equality in terms of 1-1 correspondence" (Arthur, 1999: p. 107), at least if that is to be understood as meaning that I should have proposed discarding the part-whole axiom across the board, in every context and for every purpose. For the part-whole axiom may be retained as long as it is understood that "equality" can be defined in different ways for different purposes and that not all legitimate construals of "equality" need satisfy the part-whole axiom. All that is required to escape paradoxes like Leibniz's "Diagonal" is the exercise of care sufficient to recognize when different senses of "equality" are in play within a given argumentative context. Thus the "Diagonal Paradox" gives no good reason for thinking that lines cannot be composed of an infinite number of points. In my previous paper I also argued that Leibniz's various attempts to prove that the notion of infinite number or infinite whole implies a contradiction are unsound, since, as Cantor argued was true for all such arguments (see Brown, 1998: p. 122), they appeal to premises which generally hold for finite numbers or aggregates but not for infinite numbers or infinite aggregates. Thus while Leibniz may have *thought* he had good reasons for dismissing infinite number and infinite wholes on the grounds that they generally imply a contradiction, the fact remains that he did not.

I hasten to add that there may well be circumstances under which one might have good reasons for adopting the position that certain multiplicities cannot be wholes. For all of his enthusiasm for lots of infinite numbers and wholes, even Cantor himself adopted the strategy of holding that some multiplicities should not be counted as wholes, as in the following famous passage from a letter he wrote to Dedekind on 3 August 1889:

> A collection [*Vielheit*] can be so constituted that the assumption of a "unification" of *all* its elements into a whole leads to a contradiction, so that it is impossible to conceive of the collection as a unity, as a "completed object." Such collections I call *absolute infinite* or *inconsistent* collections. [Cantor, 1932: p. 443; as quoted in Dauben, 1979: p. 245]

After inconsistencies in notions like *the set of all ordinals* and the *set of all sets* came to light, Cantor eliminated the difficulties by restricting his theory to the realm of consistent sets. But Leibniz thought that he had already found a contradiction in the very notion of an infinite number or whole, and he consequently adopted the strategy of refusing to allow *any* infinite collection to count as a "completed object" or whole, much as Cantor was later to adopt the strategy of refusing to allow a *set of all sets*, or a *set of all ordinals*, to count as a "completed object." But it is one thing to adopt a strategy for dealing with a purported difficulty; it is quite another to be justified in the use of that strategy. And therein lies the difference between Leibniz and Cantor. For Leibniz did not, in fact, have a sound argument for establishing that the very notion of an infinite number or of an infinite whole is inconsistent, and hence he had no sound argument for holding that there could not be *any* infinite numbers or wholes. To say the very least, Leibniz's claim that *no* infinity of things can be a whole was "premature" (Levey, 1999: p. 160, note 11). And had he not jumped the gun in rejecting the possibility of infinite number and infinite wholes, Leibniz, having already surmounted the prejudice against actual infinities, would have been well placed to anticipate the discoveries of Cantor and Frege by at least two hundred years.

Arthur next turns to address the question, "How can a body be infinitely divided and yet not have an infinite number of parts?" After discussing the nature of convergent infinite series, Arthur asks: "But doesn't the fact that converging series have a sum show that they are wholes, and have an infinite number of terms?" He then quotes Leibniz's response from "Infinite Numbers" (1676), which clearly suggests that Leibniz now interprets the sum of an infinite series in terms of a sequence of partial sums whose terms approach a limit:

> Whenever it is said that a certain infinite series of numbers has a sum, I think that nothing other is meant than that any finite series with the same rule has a sum and that the error always decreases as the series increases, so that it becomes as small as we would like. For numbers themselves absolutely *per se* do not go to infinity, since then there would be a greatest number. [A.VI.iii.503]

Arthur supposes that Leibniz's method for dealing with convergent infinite series also supplies him with a way to avoid talk of infinite number in the case of infinitely divided bodies:

> In the same vein, Leibniz is easily able to talk of infinite multiplicities, for instance, of the divisions within a body, without committing himself to infinite number: in such a case there are more divisions than any assignable number. However, there is no last division, just as "there is no last number of an [infinite] series, since it is unbounded," so we must conclude "that an infinity of things is not one whole, i.e. that there is no aggregate of them." [Arthur, 1999: pp. 109-110]

The point about there being no "last number" in an infinite series and no "last division" in a body is one that Arthur repeats a page later (Arthur, 1999: pp. 111-112); but this focus on the question of a "last number" or a "last division" leads Arthur to ignore a different problem that arises here. For even if there is no last number of an infinite series, it is nonetheless natural to think that in such a series an infinity of terms is given and hence that the *whole* of it can be assigned an infinite *cardinality*. Thus, for example, while there is no greatest natural number, i.e., no last term in the ordered sequence of natural numbers, 1, 2, 3, 4, 5, 6, etc., the natural numbers nevertheless have an infinite cardinality, namely, $\aleph_0$. Similarly, because the terms in the infinite series, 1/2, 1/4, 1/8, 1/16 + etc., for example, can obviously be placed in one-to-one correspondence with the natural numbers—indeed with a proper subset of the natural numbers (say, by pairing each term of the series with the natural number found in its denominator)—the total number of terms would again be what we now designate as $\aleph_0$. Leibniz himself even seems to have recognized that the terms in an infinite series could have what we now call a "cardinality" that transcends the finite numbers. Thus in a passage from "Infinite Numbers" that occurs shortly after the one quoted by Arthur, Leibniz writes:

> Thus if you say that in an unbounded series there exists no last finite number that can be written in, although there can exist an infinite one: I reply, not even this can exist, if there is no last number. To this reasoning I have nothing other to respond than that the number of terms is not always the last number of the series. That is, it is clear that even if finite numbers are increased to infinity, they never . . . reach infinity. This consideration is quite subtle. [A.VI.iii.504]

Samuel Levey has offered the following comment:

> In seeing his way clear to the fact that the number of terms in the series of natural numbers — the cardinality of the naturals — is not itself in the series, but rather lies outside it, Leibniz places himself well ahead of the majority of his peers and predecessors on the topic. Further, taken at face value, his claim that "the number of terms is not always the last number of the series" touches quite directly on the concept of cardinality, and conceives of a series's cardinality as a number. In the crucial case of the infinite series, the number of terms is the cardinal number "infinity" (waiving Cantor's distinctions between higher and lower transfinite cardinals), despite the fact that there is no corresponding infinitieth element in the series. Were it stable, this insight would place Leibniz closer still to the nineteenth-century work on the foundations of mathematics. [Levey, 1998: p. 84]

But Levey goes on to argue that the insight in Leibniz is not stable: Leibniz is blocked from ultimately accepting the conclusion that there are an infinite *number* of terms in an infinite series because he supposes that the very notion of infinite number involves a contradiction. And what holds for infinite series holds for the infinite divisions in bodies: Leibniz is blocked from accepting that there are an infinite *number* of parts in a body because he supposes that the notion of infinite number is generally contradictory. So Leibniz denies that an infinity of things can make a whole in order to escape commitment to infinite number.

It is worth considering a little more closely Leibniz's approach to infinite series, for it is reflective of a more general stance that he sometimes takes in his philosophy of mathematics. In his early period, Leibniz sometimes assumes a decidedly *constructivist* attitude toward numerical infinities—that is, as Levey puts it, "he sometimes tends to think of numbers as forming an *indefinitely extensible* and *essentially incomplete* multitude rather than an actual and determinate infinity of terms" (Levey, 1999: p. 153). From a constructivist perspective, then, the natural numbers, for example, are *not* regarded as a given, completed whole. They are instead regarded as presenting the mind with only a *potential* field for the construction of ever larger numbers, without end, but in accordance with a rule: given any natural number n, "add 1" to construct its successor, $n + 1$. Thus in *Pacidius Philalethi*, Leibniz declared:

> I believe it to be the nature of certain notions that they do not admit of perfection, and not even of completion, nor likewise of a greatest of their kind. Number is such a thing. [A.VI.iii.551]

The constructivist stance can clearly be detected in the passage quoted earlier from "Infinite Numbers," in which Leibniz adopts what we may call, following Benardete (see Benardete, 1964: p. 20), an "operational approach" to infinite series — similar to the one developed by Cauchy in the nineteenth century. Since it eliminates all reference to any actual infinite, this way of approaching infinite series enables Leibniz to eliminate any reference to a last term in an infinite series—something to which he previously seemed to be committed by his standard technique for finding sums of infinite series (see Levey, 1999: pp. 70-74). His interest in eliminating talk of a last term in an infinite series is obvious given his belief that the notion of infinite number is contradictory. For if there were a last term in an infinite series, there would have to exist an infinite *ordinal* number that would designate its position in the series. But we have seen that a question can still be raised about the number of terms in an infinite series—the *cardinality* of the set of terms that enter into the series—even if we do not suppose that there is a last term in the series. The operational approach supplies an answer. For the sum of an infinite series, on this approach, is to be explained in terms of a *potentially* infinite sequence of partial sums, where each partial sum is *constructible* from its predecessor in accordance with a rule. Each partial sum contains only a *finite* number of terms, but each such sum approaches closer to a given limit than its predecessors. Still, none of the partial sums, however far along the sequence of partial sums they may be situated, ever actually equals the limit, since no *last* sum of the sequence of partial sums can be constructed. So on the operational approach, it is clear not only that there is to be no talk of a last term of an infinite series, but also that there is to be no talk of an *actual* infinity of terms in an infinite series. An infinite series is to be conceived as containing only a *potential* infinity of terms. So an *actual* infinity of terms is never supposed to be given in the series, and, again, the limit is never precisely reached by any of its partial sums. Thus we may distinguish, again following Benardete (Benardete, 1964: p. 55), two kinds of infinite series: *actually infinite series*, conceived as containing an actual infinity of terms and hence as actually reaching their limits, and *potentially infinite series*, conceived as containing only a potential infinity of terms and hence as not actually reaching their limits. This distinction between actually infinite series and potentially infinite series is of the essence in our consideration of Leibniz. For Leibniz seems to have understood the distinction and, at least in "Infinite Numbers" (1676), opted unambiguously in favor of potentially infinite series, which fail to reach their limits (see Levey, 1998: pp. 79-80).

On the other hand, it is not at all clear that Leibniz can extrapolate, as Arthur suggests that he does, from the mathematical case to the case of actual bodies. Recall that after claiming that "for Leibniz the denial of infinite number, in the sense of a completed collection or whole . . . is equivalent to denying the existence of a last term, even an infinitieth, in an infinite series, which is consequently conceived as approaching its sum as a kind of ideal limit," Arthur immediately adds that "similarly, there is no bound to an infinite division, such as occurs in any body, so that the infinite parts of a body do not constitute it as an infinite collection or true whole." But the fact that there is "no bound," or least part, of an infinitely divided body has no bearing on the question of whether such a body is a completed collection, containing an infinity of parts, implying the existence of an infinite cardinal number. What does have such a bearing, however, is that *all* the divisions in such a body are conceived by Leibniz to be *actually given*. And it is in light of this realization that the move from Leibniz's treatment of mathematical series to actual bodies is seen to be especially problematic. Arthur treats the move as straightforward, but it is far from that. Given Leibniz's operational treatment of infinite series, it is natural to suppose that the series is not *actually* completed, that it is not, to use Arthur's expression, "a completed whole." But the same cannot be said for actual bodies. For Leibniz is quite unambiguous, throughout the whole of his philosophical career, in stating that the divisions in actual bodies, as opposed to those imagined in what he considered to be *ideal* mathematical continua, are *determinate*, and thus *actually*, and not just *potentially*, infinite. For example, in "On Motion and Matter," a work composed during the first half of April 1676, Leibniz denies that there can "be as many things [as] numbers" because even though there is an actual infinity of things, the "multitude of things is something determinate [*certa*], while that of numbers is not" (A.VI.iii.495). During his mature period Leibniz often repeated this point about the parts of actual bodies being determinate, unlike the parts of mathematical continua, which are only potential and hence indeterminate.[3] At this point it is also worth recalling the passage from Leibniz's letter of January 1692 to Foucher that I quoted in my original paper:

> I am so in favor of the actual infinite that instead of admitting that Nature abhors it, as is commonly said, I hold that Nature makes frequent use of it everywhere, in order to show more effectively the perfections of its Author. Thus I believe that there is no part of matter which is not—I do not say divisible—but actually divided; and consequently the least particle ought to be considered as a world full of an infinity of different creatures. [G.I.416]

So unlike infinite series operationally conceived, an actual body is not to be conceived as involving only a potential infinite. Given that the body is *actually* divided to infinity, as Leibniz contends, *all* of the parts, or divisions, of the body are *actually given*, and in *that* sense it *is* a "completed whole." The number of its parts is thus reasonably said to be an *infinite* cardinal number. Again, we know why Leibniz would have resisted this consequence: he thought that he had established that the very notion of infinite number was contradictory. But in that he was wrong, and Cantor and Frege were able to establish that $\aleph_0$, for example, is no more contradictory than the number 5. So if Leibniz did reject the assumption that infinitely divided bodies are wholes because he thought that infinite number and infinite wholes are generally contradictory, he did so for a patently bad reason.

Arthur tells us that

> Leibniz adopts the subtle position that there is an actual infinity of things, if infinity is understood syncategorematically, so that there are more things than any assignable number, but there is no infinite collection of things. This allows him, as he says in the passage quoted by Carlin (23, n. 23), to enunciate things about the infinite, provided he does so in "distributive mode," and not collectively. "So it can be said that every even number has a corresponding odd number, and vice versa; but it cannot on that account accurately be said that the multiplicities of odd and even number are equal." There is nothing inconsistent about this position. [Arthur, 1999: p. 110]

There is nothing inconsistent about this position, of course, at least if one rejects the possibility that one-to-one correspondence of elements might yield a perfectly reasonable and consistent sense of "equality." But Leibniz had no good argument against such a possibility. So even if Leibniz's position to this point may be regarded as consistent, and even if it enabled him to avoid the paradoxes that he thought were generated by the notions of infinite number and infinite whole, that position cannot yet be regarded as well motivated. As before, it is one thing to argue that a certain strategy will resolve a purported problem; it is quite another *to be justified in the use of that strategy*. For to be justified in the use of a strategy to resolve a purported problem—in this case with the notions of infinite number and infinite whole—the purported problem must *actually be* a problem, and not just a pseudo problem that has arisen as an artifact of a defective consideration. And while it is true, as Arthur points out, that in his book on Leibniz, Russell remarked that "the principle that infinite aggregates have no number 'is perhaps one of the best ways of escaping from the antinomies of infinite number'" (ibid.), it is also true that in his later book, *Introduction to Mathematical Philosophy*, Russell argued much in the spirit of the position I have been defending here:

> This property [that the number of natural numbers is the same as the number of even natural numbers] was used by Leibniz (and many others) as a proof that infinite numbers are impossible; it was thought self-contradictory that "the part should be equal to the whole." But this is one of those phrases that depend for their plausibility upon an unperceived vagueness: the word "equal" has many meanings, but if it is taken to mean what we have called "similar" [i.e., standing in one-to-one correspondence], there is no contradiction, since an infinite collection can perfectly well have parts similar to itself. Those who regard this as impossible have, unconsciously as a rule, attributed to numbers in general properties which can only be proved by mathematical induction, and which only their familiarity makes us regard, mistakenly, as true beyond the region of the finite. [Russell, 1918: pp. 80-81]

And it should also be noted that even in his book on Leibniz, Russell expresses no approval for Leibniz's suggestion that an infinite aggregate or multitude is not a whole, arguing that "the assertion of a whole is involved even in calling it a multitude" (Russell, 1937: p. 117). Russell's point seems to be that any attempt to refer to a multiplicity of things will commit one to the existence of a whole consisting of the set of those things. But this does seem too strong, and there may be sound reasons for rejecting Russell's contention under certain circumstances. Still, it is worth considering at this point Leibniz's standard take on the meaning of "part" and "whole," as, for example, in the following passage from *Definitiones notionum metaphysicarum atque logicarum* (1685):

> If many things are posited, then by that very fact it is understood that some single thing is immediately posited; the former are said to be the parts; the latter, the whole. And in truth it is not necessary that they exist at the same time or in the same place; it is sufficient that they be considered at the same time. Thus, from all the Roman emperors together we construct a single aggregate. [S.481]

On the basis of this account it does not seem entirely unreasonable to think that when all the parts of the universe are posited, say, then they are immediately understood to form a whole; or that when the parts of a body are posited—even supposing that they are infinitely many—they are also immediately understood to form a whole. We know that Leibniz resisted these inferences because he held that infinite number and infinite wholes are generally contradictory. But, again, he was mistaken about that premise and without it his strategy of refusing to allow an infinity of things to count as a whole loses its rationale. It should be noted, however, that to the passage quoted above Leibniz immediately adds that "in truth . . . no entity that is truly one [e*ns vere unum*] is composed out of parts" (ibid.). But then this suggests that *a whole* is something other than a *genuine* or *substantial unity*.

As we shall see, a failure to keep the notions of *a whole* and of *a genuine unity* clearly distinguished has introduced some confusion into Arthur's discussion of the exchange between Carlin and myself. I admit that I may have been the source of some of this confusion by suggesting in my original paper that various of Leibniz's doctrines seem to require that "the world have more than the merely 'verbal unity' that [Leibniz] attributes to it in the letter to Des Bosses of 11 March 1706" (Brown, 1998: p. 118). I now think that many of the arguments I presented for this are more appropriately treated as arguments for thinking that the world has a soul, and hence is a genuine unity, than for thinking that the world is a whole,[4] and I am grateful to Arthur for helping me to clarify my thoughts on this matter. So perhaps Arthur is right when he contends that "Leibniz's much beloved doctrine of the 'connection of all things' [for example] is easily interpreted [in the distributive mode]: if each thing is connected with those with which it is in contact, and there is no vacuum, then 'all things are connected' is perfectly intelligible without the supposition of an infinite collection" (Arthur, 1999: p. 110). But then the point has nothing particularly to do with the fact that the collection in question would be infinite. Leaving aside for the moment Leibniz's metaphysics of infinitely divided body, if we suppose there were just five bodies, say, and that they were all in contact, then the doctrine of the connection of all things could again be interpreted as Arthur has suggested, and "all things are connected" would be perfectly intelligible without the supposition of a finite collection of five things. But the real question is: "What would bar us from considering the five bodies together as a whole?" And if we suppose that there are infinitely many bodies, as Leibniz does, what would bar us from considering this infinity of bodies to be a whole, with an infinite number of parts? Insofar as the answer to this question should rest upon the assumption that infinite number and infinite wholes are generally contradictory, as it seems Leibniz's answer would, then the answer would not be cogent; for infinite number and infinite wholes are *not* generally contradictory.

Moreover, it can at least be doubted whether *all* of Leibniz's talk about the *whole* world can reasonably be understood in the distributive mode. This is especially true in the case of expressions that are definite descriptions of the actual world, serving to pick it out from all other possible worlds. Thus, for example, Leibniz refers to this world as *the most harmonious world* or, indeed, as *the best of all possible worlds*. Such expressions as these seem to presuppose a comparison of this system of substances, *as a whole*, to other such systems, also considered *as wholes*. If God is to judge which world is the best or most harmonious, it seems it must determine truths about the whole collectively and not just truths that hold distributively of the world. As Leibniz says at *Theodicy* § 225 concerning God's choice among possible worlds: "The divine Wisdom distributes all the possibles it had already contemplated separately, into so many universal systems which it further compares the one with the other" (G.VI.252 = H.267). However that may be, it seems perfectly reasonable to say that the world is a whole in the sense, at least, that all of its parts are *actually given*. So absent a sound argument to the effect that infinite number is generally contradictory, we may reasonably say that if the world contains an *actual* infinity of creatures, as Leibniz does, then the cardinality of the set of creatures is an infinite number.

Arthur, however, maintains that "Leibniz can speak of matter as an infinite aggregate of substances without it committing him to an infinite collection":

> For the argument for this infinitude is that every portion of matter, however small, contains a substance or substances within it. This argument involves only the syncategorematic infinite, and the idea of monads distributed everywhere in matter, but not matter as a collection of monads. [Arthur, 1999: p. 110]

If we grant that for Leibniz bodies are not collections of monads because monads are not strictly *parts* of matter, it is nonetheless true for Leibniz that bodies are *actually* divided to infinity. In particular, the organic bodies of corporeal substances (considered apart from their souls) are divided into an infinity of other organic bodies as parts. Thus although strictly speaking an organic body is not composed of *corporeal substances* as parts (since the parts of an organic body are, strictly speaking, only the *organic bodies* of the corporeal substances that enter into it), an organic body, like any other bit of matter, seems to be, as Leibniz says in the *Theodicy*, an "infinite accumulation of substances" (G.VI.232 = H.249). And this brings us to the nub of the issue in the discussion between Carlin and myself. For the problem, as it was originally conceived by Carlin, is this:

> Why . . . should we admit that infinite aggregates, like the world, cannot admit of a soul? After all, organic bodies, according to Leibniz, just are an accumulation of infinitely many substances, yet he clearly thought they had souls. [Carlin, 1997: p. 7]

I will return to the problem posed by Carlin shortly, but I want first to consider what Arthur says in concluding his historical remarks on Leibniz:

> I hope [they are] enough to establish the material equivalence for him of the following: No infinite number ≡ no infinitely large magnitude ≡ no infinitely small actual ≡ no last term in an infinite series ≡ no totality of parts in an infinite division. [Arthur, 1999: p. 111]

But exactly what Arthur hopes he has established is not altogether clear from what he says here—whether it is that Leibniz *merely accepted* the equivalences in question, or that Leibniz *thought he had good reasons for accepting* the equivalences in question, or, finally, that Leibniz *in fact established* the equivalences in question. But my interest has only been in whether infinite number and infinite wholes are, in fact, generally contradictory, and whether Leibniz *actually* established that infinite number and infinite wholes are generally contradictory. So, too, I am only interested in whether the equivalences in question actually hold and whether Leibniz actually established them. For reasons I have already given, I do not think that infinite number and infinite wholes are generally contradictory, so I certainly do not think that Leibniz established that they are. As regards the purported equivalences, I do not think that "no infinite number" is equivalent to "no infinitely large magnitude," if "infinitely large magnitude" means "infinitely large extensive magnitude," as the universe is supposed to be for Leibniz; and thus, too, I do not think that Leibniz established such an equivalence. For infinite number, as I have already argued, could reasonably be thought to be implied by the existence of bodies actually divided to infinity, whether or not there was any extensively infinite magnitude. On the other hand, if "infinitely large magnitude" is intended, as I suppose, to denote *both* extensively *and* mereologically infinite magnitudes—that is, *both* infinitely extended magnitudes *and* finitely extended bodies that are nonetheless *actually* divided into an infinity of parts—then I do not think that either "no infinitely large magnitude" or "no infinite number" is equivalent to "no infinitely small magnitude." For as I shall argue shortly, a body could be *actually* divided to infinity without that implying the existence of an "infinitely small magnitude." And as I have already argued, a body *actually* divided to infinity can reasonably be said to imply the existence of an infinite cardinal number that is the number of its parts. Thus, again, I certainly do not believe that Leibniz established the equivalences in question, regardless of whether he accepted them or thought that he had good reason for accepting them. Moreover, and again for reasons already stated, I certainly do not think that "no infinite number" or "no infinitely large [mereological] magnitude" is equivalent to "no last term in an infinite series," and consequently, again, I certainly do not think that Leibniz established that it is. And if Levey's interpretation of the passage considered earlier from "Infinite Numbers" is correct—that Leibniz recognized that the terms of an infinite series could have an infinite cardinality quite apart from the question of whether there is a last term in the series—then Leibniz himself did not even accept the equivalence in question, let alone establish it. Finally, I do not think, again for reasons already given, that "no last term in an infinite series" is equivalent to "no totality of parts in an infinite division," so even if Leibniz *accepted* the equivalence in question, or *thought that he had a good reason for accepting* it, I certainly do not think that he *actually had a good reason for accepting* it.

Arthur enumerates a number of consequences that he supposes would have followed for Leibniz had he embraced infinite number and wholes. We are told that "there would be an infinite number of infinitesimals in a finite quantity: therefore infinitesimals would be actual parts, and Leibniz's philosophy of mathematics would have been completely different" (Arthur, 1999: p. 111). If, as it appears, the intent here is to provide reasons why Leibniz did well not to embrace infinite number and infinite wholes, then Arthur adopts a very curious argumentative strategy. That his acceptance of infinite number and infinite wholes would have had the consequence of making "Leibniz's philosophy of mathematics . . . completely different" is certainly—even trivially—true; but *that* can scarcely count as a *good reason* for Leibniz's decision *not* to embrace the notions in question: it is one thing simply to accept or reject a certain notion, and it is quite another to do so for good reason. I have argued that the reason Leibniz offered for rejecting the notions in question—namely, that they are generally contradictory—was based on arguments that are unsound

As to the specific claim that on the assumption of infinite number and infinite wholes, "there would be an infinite number of infinitesimals in a finite quantity: therefore infinitesimals would be actual parts," it must be said that on Leibniz's view the consequent must in any case be true. For that the divisions within matter must finally resolve themselves into either infinitesimals or minima is something that seems to be guaranteed by Leibniz's assumption that *every part* of matter is actually divided to infinity. This has been shown in detail by Samuel Levey (see Levey, 1999: pp. 149-152); but Levey also argues that Leibniz's attempt to explain how finite bodies might be divided to infinity by analogy with convergent infinite series did not force him to adopt a model in which *every part* of matter is conceived to be infinitely divided. All that is required is that there be no smallest bit of matter—just as there is no least term in a convergent infinite series. The alternative model suggested by Levey is what he calls the "diminishing pennies" model (see ibid.: p. 143). We are to imagine a stack of pennies, where the first in order is assumed to be half an inch thick. Each successive coin is supposed to be half as thick as its predecessor, so that in terms of their thickness, the pennies form an infinite series of magnitudes: 1/2 inch + 1/4 inch + 1/8 inch + etc. The series will have an infinity of terms, but no *last* term and hence no *least* term. There are two important points to note about Levey's "diminishing pennies" model: first, none of the pennies is itself conceived to be further divided; secondly, the sequence of pennies is "open" on one end, that is, there is no penny of least thickness in the sequence. These conditions ensure that while there are an infinity of pennies, every penny will be of finite size: there will result no infinitesimal or minimal pennies. If we momentarily leave out of account Leibniz's commitment to the doctrine that *every part* of matter is actually divided to infinity, then Levey's "diminishing pennies" model seems to fit nicely with what Leibniz says in reply to Johann Bernoulli's suggestion that it is inconsistent to suppose that a body could be actually divided to infinity and yet not possess any infinitely small parts:

> Although . . . I hold as certain that a part of matter is actually subdivided as far you please, I do not on that account think it follows that there exists an infinitely small portion of matter; even less do I admit that it follows that there is any absolutely minimum portion of matter. . . . Let us suppose that in a line [the subdivisions] 1/2, 1/4, 1/8/ 1/16, 1/32, etc. are actually given and that all the terms of this series actually exist. You infer from this that there is also given an infinitieth term. But I think that nothing follows from this except

> that there is actually given an assignable finite fraction as small as you please. [GM.III.536]

However, we need to remind ourselves that the model of infinitely divided matter that Leibniz did adopt, as in the following passage from *Pacidius Philalethi*, requires that *every part* of matter be infinitely divided:

> The division of the continuum is not to be considered as being like the division of sand into grains, but as being like the division of a paper or a tunic into folds. . . . It is as if we should suppose a tunic with folds multiplied to infinity in such a way that there is no fold so small that it is not subdivided by a new fold. . . . And the tunic cannot be said to be resolved all the way down into points. Rather, though some folds are smaller than others to infinity, bodies are always extended and points never become parts, but always remain only extrema. [A.VI.iii.555]

Now as I have said, Leibniz's model of how matter is infinitely divided should entail that the divisions resolve ultimately into either infinitesimals or minima, that is, indivisible points. But by the time he was writing the *Pacidius*, Leibniz had already adopted the position that infinitesimals are *fictions*, useful in mathematics, but with no real existence. And it was in a part of the *Pacidius* that precedes the present passage that Leibniz had formulated what Arthur calls the "Diagonal Paradox," which eventually moved him to adopt the position that indivisible points, or minima, cannot be parts of anything, for they are to be understood only as extrema, which are modes and not things. This view is reflected in what Leibniz says at the end of the passage just quoted, and it is reinforced a few pages later, where he writes:

> I believe that there is no portion of matter which is not actually divided into more parts, so that no body is so small that there is not in it a world of an infinity of creatures. . . . Nevertheless, it is not to be allowed on that account that a body . . . is divided into points . . . because indivisibles are not parts, but the extrema of parts. For that reason, even though each thing is subdivided, it is nevertheless not resolved all the way down into minima. [A.VI.iii.565-566]

But now, if it is true that Leibniz's doctrine that *every part* of matter is actually divided to infinity requires that the divisions in matter resolve ultimately into infinitesimals or into minima, why did Leibniz think, as in the passages we have just been considering, that his model of the division of matter—which, following Levey, I will refer to as the "folds model"—did not result in the division of matter into either infinitesimals or minima? To understand the answer that Levey proposes for this, we need to recall that Leibniz's constructivism and his operational treatment of infinite series, which fits perfectly within a more general constructivist stance toward mathematical infinities, enabled him to eliminate talk of both a last term of an infinite series and of an actual infinity of terms in an infinite series. This is because, on the operational account, an infinite series is regarded as only a *potentially* infinite series, not an *actually* infinite series. Thus on the operational account of infinite series, an infinity of terms is never assumed to be *actually given*. It is for this reason that I earlier argued that Arthur's move from Leibniz's analysis of infinite series to his analysis of infinitely divided bodies cannot be regarded as unproblematic. For unlike the terms of an infinite series operationally construed, the parts of actual bodies, according to Leibniz, are *actually given*: bodies are *actually*, not just *potentially*, divided to infinity. So Leibniz had two quite different accounts of the infinite in play. His constructivist stance in mathematics led him to treat the mathematical infinite as merely *potentially* infinite, whereas his metaphysics of divided matter led him to treat the divisions in bodies as *actually* infinite.

This brings us, then, to Levey's speculation about why Leibniz apparently failed to realize that his doctrine that *every part* of matter is actually divided to infinity must imply that the divisions of matter ultimately resolve into either minima or infinitesimals:

> With such a dualistic view of the infinite, it is critical that Leibniz keep his thinking about mathematical infinity well apart from this thinking about real infinity. I suspect, however, that he does not and that his constructivism spills over disastrously into his philosophy of matter. [Levey, 1999: p. 155]

To put the matter as succinctly as possible, Levey argues that the reason Leibniz does not see that his commitment to the doctrine that *every part* of matter is infinitely divided must lead to matter's resolving into either minima or infinitesimals is because his constructivism hides from his view the fact that in an *actual, completed* division, as opposed to a *potential* division, *all* the divisions must *actually be given at once*. They are given in what Levey terms "the actual state of the division":

> It is the limit state, and as it falls outside the series of finite levels but encompasses them all, we might call it the "omega level." Only at the omega level will it appear that any given part of matter occupying some finite level contains an infinity of actual parts. . . . And only at the omega level, with all the cuts actually in place, will the outer surfaces of the finite parts of matter be seen to belong to limit parts [i.e., either infinitesimals or minima], for only

> when everything has actually been subdivided are the surfaces seen to be actually separated from any finite part. [Levey, 1999: p. 156]

Now Arthur seems to think that infinitesimals can only be avoided by denying that the infinity of parts in a body constitute a completed whole, but that is *not* the only way that infinitesimals might be avoided. For if a body is conceived to be infinitely divided after the manner of Levey's diminishing pennies model, then the parts may be conceived as constituting a completed whole, in the sense that an infinity parts are *actually given*, without thereby supposing that there are any infinitesimal parts. The assumption that leads to the conclusion that a body actually divided to infinity must resolve ultimately into either infinitesimals or minima is, as we have seen, the assumption that *every part* of matter is itself actually divided to infinity. But perhaps Arthur realizes this and means to suggest that given the assumption that *every part* of matter is divided to infinity, the only way to avoid infinitesimals is by supposing that the parts of bodies do not form a completed whole. He may thus see what Levey argues Leibniz may not have seen, namely, that if a body is regarded as a completed whole, in the sense that *every part* of it is *actually* divided to infinity, then the divisions must ultimately yield infinitesimals, which Leibniz dismissed as fictions. So I suppose that Arthur may also think that if there are no infinitesimals, as Leibniz maintained, then a body cannot be regarded as a completed whole in the sense that *every part* of it is *actually* divided to infinity. But then what are we to make of Leibniz's unambiguous commitment to an *actual* infinity of divisions within every part of matter? Here Arthur may be committing with eyes wide open the mistake that Levey supposes Leibniz made more blindly: perhaps he is allowing "his constructivism [to spill] over disastrously into his philosophy of matter," or rather into his interpretation of Leibniz's philosophy of matter. In line with the constructivist perspective, talk of an *actual* infinity of divisions within matter gives way to talk of a merely *potential* infinity of divisions within matter, just as talk of an *actual* infinity of terms in an infinite series gives way to talk of a merely *potential* infinity of partial sums, each of which is itself actually finite. So then the divisions within the parts of a body are never understood as *actually* given to infinity, and so the divisions are not 'complete,' and so, finally, the parts of a body do not form, to use Arthur's expression, "a completed collection or whole" (Arthur, 1999: pp. 110-111).

On the other hand, it is not altogether clear that *Leibniz*, at any rate, actually realized that his commitment to the doctrine that every part of matter is actually divided to infinity entails that bodies are not wholes, despite the fact that such would seem to follow rather directly from the former doctrine in conjunction with the doctrine that an infinity of things cannot make a whole. This problem about how bodies can be wholes for Leibniz is what Levey has called the "unity problem":

> The *unity problem* is this: if each and every part of matter is further divided into parts, and those parts into further parts ad infinitum, then any part of matter you specify will contain an infinity of parts. But this is impossible, on pain of Galileo's paradox. As we saw earlier, in Leibniz's view nothing can contain an infinity of parts without violating the axiom that the part is less than the whole; or, nothing that is truly *one* or *whole* can have an infinity of parts. Leibniz's escape from the paradox . . . is to allow that there can be an infinity of "parts" as long as they are not actually parts of any one thing. In the present case, however, since *any* part of matter we specify would be subject to the precisely same infinite division into parts, it follows that *no* part of matter can truly be one or a whole. But to say that something is not truly one is to say that it does not truly *exist*. Thus in the folds model of matter's infinite division, since no part of matter can truly be one, *there can't be any matter*.[5]
>
> Although this problem with Leibniz's model of matter is not difficult to see, I find no evidence that Leibniz himself ever sees it. Across his writings he readily endorses both the part-whole axiom (with its attendant claim that there can be no infinite wholes) and the model of matter as actually divided into parts that are actually subdivided into parts ad infinitum. [Levey, 1999: p. 146]

Levey has suggested that Leibniz's apparent failure to perceive the unity problem might again have been due to his allowing his constructivism to spill over into his analysis of actuals. As we have seen, Levey supposes that the reason Leibniz may have failed to see that his doctrine that *every part* of matter is actually divided to infinity must lead to the conclusion that matter resolves ultimately into either infinitesimals or into minima is because his constructivism concealed from him the fact that in an *actual* division, as opposed to a *potential* division, *all* the divisions must *actually be given at once*, even if there is no *last division*. The problem of matter's resolving into either infinitesimals or into minima would not be visible from a constructivist point of view since the problem only becomes visible at what Levey calls the "omega level"—a level which simply does not exist for the constructivist. But Levey suggests (see Levey, 1999: pp. 156-157) that the unity problem may also have remained hidden from Leibniz's view since it, too, only becomes visible at the omega level. If Levey is right about all of this, then Leibniz may well have thought that bodies constituted wholes even though he officially held that they are actually divided to infinity and that an infinity of parts cannot make a whole.

In connection with the points made in the last few paragraphs, consider the following passage from *New Essays* II.xiii.21:

> Yet M. Descartes and his followers, in making the world out to be indefinite so that we cannot conceive of any end to it, have said that matter has no limits. They have some reason for replacing the term 'infinite' by 'indefinite,' for there is never an infinite whole in the world, though there are always wholes greater than others *ad infinitum*. As I have shown elsewhere, the universe itself cannot be considered to be a whole. [NE.151]

It is worthy of note that Leibniz argues that although "there is never an infinite whole in the world," "there are always wholes greater than others *ad infinitum*." This tends to support rather strongly the suggestion that Leibniz did hold, contrary to what Arthur maintains, that bodies of finite extensive magnitude are wholes, even though he officially regarded them as actually divided to infinity. But this passage is otherwise difficult to interpret. On the one hand, it is tempting to think that Leibniz's guarded endorsement of the Cartesian view that the world is "indefinite," rather than infinite, should be interpreted as indicating his endorsement of the view that the world is only *potentially* infinite rather than actually infinite. This reading is supported by the fact, already discussed, that Leibniz adopted an operational approach to infinite series as part of a more general commitment to mathematical constructivism: just as the partial sums of an infinite *divergent* series are seen, on the operational approach to infinite series, to grow larger and larger without there ever being given a partial sum that is *actually* infinite, so too, in the passage from the *New Essays*, Leibniz might be read as treating the universe as a *potentially* infinite sequence of *finite partial* sums of the parts of the material universe, in which there is never given an *actually infinite* sum of *all* of the parts of the material universe. In this way we could see Leibniz's treatment of the extensive infinite as running parallel to his treatment of the mereological infinite; in each case Leibniz could be seen as allowing his mathematical constructivism to spill over into the analysis of actual bodies. And once again the actual infinite would have been exchanged for the potential infinite in order to avoid commitment to infinite number.

On the other hand, Arthur does not shrink from granting that because of his acceptance of the principle of plenitude Leibniz did hold that "the universe is actually infinite in magnitude" (Arthur, 1999: p. 112). To support this interpretation, Arthur quotes the following passage from *Catena mirabilium demonstrationum* *de Summa rerum* (1676): "since there is no reason determining or limiting [the] size [of space], it will be the greatest it can be, i.e., absolutely infinite" (A.VI.iii.585; cf. §§ 21-22 of Leibniz's fourth letter to Clarke). But Arthur adds the following comment in a footnote:

> Leibniz's use of the term 'absolutely' here is not very happy. Note, however, that he does not contradict himself in saying that the world is infinite in magnitude. As he says [in a letter to Des Bosses of 11 March 1706], this means "that it extends beyond any magnitude that can be assigned" (G.ii.304): the world is syncategorematically infinite, but not categorematically so. [ibid., p. 116, note 22]

Arthur does not see Leibniz as rejecting the actual infinite—and would thus not interpret the passage we have been considering from the *New Essays* as doing so. Rather, Arthur interprets Leibniz as holding that while the world is actually infinite in extension, it cannot be regarded as *an infinite whole*. Why not? Well because, as Leibniz tells us in the *New Essays* passage, "I have shown elsewhere [that] the universe itself cannot be considered to be a whole"—and that would appear to go back to his supposed 'proof' that infinite number and infinite wholes are generally contradictory, a 'proof' that was part and parcel of the motivation behind his adoption of the operational approach to infinite series in the first place. To this I can only reply by repeating my earlier point: the argument that the world cannot be a whole because infinite wholes and infinite numbers are generally contradictory is not sound, and hence Leibniz's rationale for denying the world to be a whole is without merit.

Returning to Arthur's criticisms, we are told that if infinite numbers and wholes are assumed,

> bodies would be real wholes, since there would be a determinate, though infinite, number of parts into which they were divided. Matter would therefore be real, and would not need immaterial principles to complete it. The monadology would be unneeded. By extension, human bodies would form real unities without the need of immaterial souls . . . [ibid., p. 111]

I begin by again simply noting the obvious fact that even if his acceptance of infinite number and infinite wholes should have entailed that Leibniz's philosophical system would have been very different from what it turned out to be can scarcely count as a good reason for thinking that Leibniz was well advised *not* to accept infinite number and infinite wholes or that he actually had *good reason* for not accepting them. But that Leibniz is committed to there being an actual infinity of parts and divisions within *every part* of matter—even if he did not always keep clearly in mind all the consequences of this—is something I take as given. Arthur seems to be objecting to the idea that there would be "a determinate . . . number" of such parts if infinite number and infinite wholes are admitted. But there would not be a "determinate" number n, say, of parts in the sense, to use Jonathan Bennett's phrase, of "'just so many and not a single one more or less,' this being understood to require a value of n such that $(n + 1) > n$" (Bennett, 1974: p. 128). Such a sense of "determinate" holds only for finite numbers. But furthermore, even if Leibniz did reject the claim that there was a *determinate number* of parts in matter, he did, as I argued earlier, certainly accept the view that the parts of matter are *determinate* and *actually infinite*; and because he had no sound argument to the effect that infinite number and infinite wholes are generally contradictory, I have argued that he had no good reason to deny that there is an infinite *number* of parts in matter.

But leaving that point aside, we need to consider why, on Arthur's view, it should follow from the supposition that bodies are wholes that "matter would therefore be real." So consider the following passage from *Definitiones notionum metaphysicarum atque logicarum* that I quoted in part earlier:

> If many things are posited, then by that very fact it is understood that some single thing [*unum aliquod*] is immediately posited; the former are said to be the parts; the latter, the whole [*totum*]. And in truth it is not necessary that they exist at the same time or in the same place; it is sufficient that they be considered at the same time. Thus, from all the Roman emperors together we construct a single aggregate [*unum aggregatum*]. In truth, however, no entity that is truly one [*ens vere unum*] is composed of parts. Every substance is indivisible and whatever has parts is not an entity [*entia*], but only a phenomenon. From these considerations the ancient philosophers correctly attributed substantial forms, such as minds, souls or primary entelechies, to those things that they said made up an *unum per se*. And they denied that matter by itself is a single entity [*unum ens*]. Certainly those things that lack these [substantial forms] are no more a single entity [*unum ens*] than a pile of sticks; indeed, they are no more real entities [*entia realia*] than rainbows or mock suns. [S.481]

As I suggested earlier, this passage appears to draw a very sharp distinction between a *whole*, on the one hand, and an "entity that is truly one," or an "*unum per se*," that is, a *substantial unity*, on the other. And the distinction that Leibniz appears to draw here is drawn in many other passages as well. In fact, in his original paper Carlin offered any number of good reasons (see Carlin, 1997: pp. 8-9), with textual basis, for thinking that *whole* and *substantial unity* are *not* equivalent notions for Leibniz—not the least being the fact that, as in the present passage, Leibniz's definitions of a *whole* seem to require that a whole have parts, whereas he insists that substantial unities do not have parts. But then it does not seem to follow, as Arthur suggests, that if bodies are considered to be wholes, then "matter would . . . be real." For as Leibniz suggests here, and in many other places as well, aggregates can make wholes without that implying that they are "real entities," for "they are no more real entities than rainbows or mock suns." Thus the only way that I can understand why Arthur should think that for Leibniz the assumption that bodies are wholes would imply that they are substantial unities is by supposing that he interprets Leibniz as holding that *a whole* and *a substantial unity* amount to the same thing. This would explain why he thinks that it would follow from the assumption that bodies are wholes that "human bodies would form real unities without the need of immaterial souls." But such an understanding of what Leibniz means by *a whole* seems to fly rather squarely in the face of what Leibniz says in the passage we are now considering, as well as in the face of what he says in the other similar passages that Carlin reviewed in his original paper.

As to whether matter "would need no immaterial principles to complete it," I again do not see why this should follow from the assumption that bodies are wholes in the sense suggested by Leibniz's definitions of "whole." In fact I agree with Arthur when he writes that for Leibniz a "Cartesian body, regarded as pure extension, is not something complete, and cannot be a substance" (Arthur, 1999: p. 111). But there are many reasons that Leibniz presents for this, dynamical considerations, for example: pure extension cannot act, and hence cannot be a substance, and hence must be completed by immaterial principles, genuine substances that *can* act and thus give rise to the forces that are found in nature.[6]

But furthermore, even a *Leibnizian* body cannot be a substance by Leibniz's lights, and that brings me to Arthur's claim that if Leibniz accepted bodies as wholes, "the monadology would be unneeded." The passage from *Definitiones notionum metaphysicarum atque logicarum* suggests otherwise. For that passage suggests that something could be a whole, or a "single aggregate," without thereby being an *unum per se*, that is, a genuine substance. So even if bodies were wholes, according to Leibniz, it seems that they would not, *ipso facto*, be *una per se*, or genuine substances. To gain genuine substances, we are told, as "the ancient philosophers perceived," there is need to add "substantial forms, such as minds, souls, or primary entelechies," which constitute the active forces in what Leibniz called "monads." Without these there could presumably be no corporeal substances, even if bodies were assumed to be wholes.

So I am not at all persuaded that Arthur is right to suppose that had Leibniz embraced infinite number and infinite wholes "he would have produced a system unrecognizable as the one we know as Leibnizian" (Arthur, 1999: p. 111). And I will repeat here for a final time what I have already said twice before, namely, that even if his acceptance of infinite number and infinite wholes should have entailed that Leibniz's philosophical system would have been very different from what it turned out to be can scarcely count as a good reason for thinking that Leibniz was well advised *not* to accept infinite number and infinite wholes or that he actually had *good reason* for not accepting them.

Arthur may believe that in my original paper I myself was construing Leibnizian wholes as substantial unities—perhaps due to unclarities in my presentation there. But I do not, in fact, construe them so. But this misunderstanding of how I understood the Leibnizian notion of a whole in my original paper leads Arthur to misstate my position rather badly, as in the following passage:

> Of course, establishing the failure of Brown's suggested remedy[7] does not automatically resolve the difficulty both Carlin and Brown were trying to address, . . . so let me turn to that now. . . .
>
> I believe Brown is correct to suggest that Leibniz's denial of infinite number (infinity understood collectively) precludes a body's being a true whole just as surely as it precludes the world's being a true whole. But Brown appears to assume, falsely, that Leibniz wants finite bodies to be true wholes, which is why he insists Leibniz would have done better to have accepted infinite numbers. [Arthur, 1999: p. 112]

Arthur appears to think that I was contemplating the possibility that finite bodies, as such, might have souls and that I wished to maintain that Leibniz "wants bodies to be true wholes," where by "true wholes" Arthur seems to mean "substantial unities." But neither Carlin nor myself understand Leibniz as holding that *a whole* is the same as *a substantial unity*, and I was certainly not attempting to maintain that Leibniz thought that finite bodies as such, as opposed to corporeal substances, were substantial unities. For again, as it was originally formulated by Carlin the problem that he and I were addressing was this:

> Why . . . should we admit that infinite aggregates, like the world, cannot admit of a soul? After all, organic bodies, according to Leibniz, just are an accumulation of infinitely many substances, yet he clearly thought they had souls. [Carlin, 1997: p. 7]

So the point I was trying to make is this: *if* Leibniz maintains that the universe is not capable of being the body of a corporeal substance because it is an infinite extensive magnitude and hence not a whole (where "whole," of course, is *not* here to be understood as equivalent to "substantial unity"), then by the same argument it should follow, contrary to the supposition that there are corporeal substances with organic bodies of finite extensive magnitude, that *no* body, not even those of finite extensive magnitude, is capable of being the body of a corporeal substance. For the argument we have been considering from Leibniz seems to conclude that the universe, which is an infinite extensive magnitude, cannot be a whole because otherwise there would have to be infinite number—something that Leibniz thought he had established as contradictory. But given that the organic body of a corporeal substance, considered apart from its soul, is also an accumulation of infinitely many substances—which also seems to imply the existence of infinite number—it would seem that it cannot be a whole either, and consequently that it cannot, contrary to hypothesis, be the body of a corporeal substance. And so conversely: *if* Leibniz allows that a body of finite extensive magnitude *can* be a whole, and hence be capable of possessing a soul, as Carlin argued in his original paper, then it would seem that the universe could also be a whole and hence be capable of possessing a soul. For *if* an infinitely divided body of finite extensive magnitude can be a whole, despite the fact that that would imply the existence of infinite number, then one cannot reasonably deny that the universe is a whole simply because that would imply the existence of infinite number.

But Arthur disagrees with my argument because he thinks that, unlike the universe, which is an infinite extensive magnitude, a body of finite extensive magnitude, as understood by Leibniz, does not entail the existence of infinite number:

> A finite body can comprise an arithmetical unity, though not a true one, because the parts within parts, each of which contains either a substance or an aggregate of substances, are progressively smaller. Consequently they can "sum" to a finite quantity in the same way that a converging infinite series can, without there being an infinite number, or, equivalently, a last part or last term of the series. But if every corporeal substance is contained within a larger body, the analogous series for the whole world is divergent. Consequently, such a world would not possess even the arithmetical unity requisite for its semi-reality as a body or well-founded phenomenon. Thus Leibniz's philosophy of the infinite does allow him to conclude that the world cannot be a body: if it were infinite, there would be infinite magnitude, and therefore infinite number, which he has rejected. [Arthur, 1999: p. 112]

This is a very confusing passage. What complicates the discussion of our disagreement is that unlike Carlin and myself, Arthur again seems to assume that by "a whole" Leibniz means "a substantial unity," which is apparently why Arthur says that a "finite body [is] not a true [unity]," even though the discussion is ostensibly about whether a finite body is a *whole*. Moreover, in his original paper Carlin argued that it is precisely because a body of finite extensive magnitude is an "arithmetical unity" that it can be called a "whole" (see Carlin, 1997: p. 12), while in the present passage Arthur seems to be denying precisely that. For he seems to be denying that a body of finite extensive magnitude is a whole, even while conceding that it is an arithmetical unity. So it is hard to see why, or in exactly what sense, Arthur takes himself to be in agreement with Carlin (see Arthur, 1999: p. 112). Arthur's argument seems to be that a body of finite extensive magnitude can be an *arithmetical unity*, although *not* a *whole*, while the universe cannot even be an *arithmetical unity*. But Carlin's argument was quite different, namely, that a body of finite extensive magnitude *is* a *whole* precisely because it *is* an *arithmetical unity*, while the universe *cannot be* a *whole* precisely because it is *not* an *arithmetical unity*. Thus Carlin and Arthur agree that for Leibniz a body of finite extensive magnitude can possess arithmetical unity, while the universe cannot, but beyond that their arguments are really at cross purposes, and it is misleading to suggest otherwise. In any event, my argument has been that whether or not a body of finite extensive magnitude is an arithmetical unity in the sense that Arthur seems to understand it, it is nonetheless, according to Leibniz's official doctrine, *actually* divided to infinity and hence an actual infinity of parts is given in it. Thus, and in that sense, the parts can reasonably be regarded as forming a completed whole, whose cardinality should be an infinite number.

There is some confusion in Arthur's claim that, unlike the parts of a universe of infinite extensive magnitude, the parts of a body of finite extensive magnitude "can 'sum' to a finite quantity in the same way that a converging series can, without there being an infinite number, or, equivalently, a last term of the series." It is true that, unlike a divergent series, a convergent series has a finite sum, but that has nothing to do with the fact that such a series has "no last term." A divergent infinite series also has no last term. The point, then, would seem to be simply that a convergent infinite series has a finite sum, whereas a divergent infinite series does not have such a sum. But, as was discussed before, whether infinite series—convergent or not—commit one to the existence of infinite number is *not* simply a matter of whether they have a finite sum; it is also a matter of whether one takes them to be *actually* infinite series, that is, series in which *all* the terms are assumed to be given, even though *no last term* is assumed to be given. So it is a mistake for Arthur to persist in insisting that "there being an infinite number" is equivalent to there being "a last term of the series." Earlier I argued that Leibniz's operational approach to infinite series suggests that he wanted to treat such series as only *potentially* infinite, in which case infinite series need not be understood as implying infinite number. But this holds equally for convergent *and* divergent infinite series: on the operational approach all of the partial sums of an infinite divergent series are as finite as those of an infinite convergent series. Thus whether a particular infinite series is convergent or divergent has nothing at all to do, on the operational approach, with whether it commits one to the existence of infinite number; for on the operational approach, neither a convergent nor a divergent series implies infinite number.

But if we turn from the mathematical case of infinite series to the physical case of bodies, it may be possible to see why his constructivism might have led Leibniz to think that bodies of finite extensive magnitude could be regarded as wholes while at the same time leading him to hold that bodies of infinite extensive magnitude could not. Consider first that at any level in the division of a body of finite extensive magnitude—including even that level that Levey terms the "omega level"—the magnitude of the sum of the parts given at that level must equal the magnitude of the body in question, which by hypothesis is finite. Moreover, insofar as Leibniz's constructivism made him oblivious to the possibility of an omega level of division, as Levey argues that it did, Leibniz would not have seen himself as committed to there being any level of division at which an infinity of parts in a body of finite extensive magnitude is actually given—although this could be managed only by exchanging his official doctrine of matter *actually* divided to infinity for the ersatz doctrine of matter only *potentially* divided to infinity. Thus even if the infinite *division* of a body can never be complete from the constructivist perspective, the parts at any specified level of division are conceived to be actually given, and with them *the body* must itself be conceived to be *completely given as a whole*. So once the constructivist assumptions have done their work and safely eliminated the specter of encountering a level of division at which an *actual* infinity of parts might be given, Leibniz can freely accept the view that bodies of finite extensive magnitude are wholes. On the other hand, a universe of infinite extensive magnitude cannot be conceived to be given as a whole according to Leibniz, since that would imply the existence of an infinite number as its measure. Thus in the passage from the *New Essays* Leibniz does not conceive the parts of the universe to be given through the division of a pre-existing whole, as he does when considering the parts of a body of finite extensive magnitude, but rather the whole is conceived to be given, if at all, by construction, *through the addition of parts*. But it is clear that at no point in the outward progress of the addition of material parts will we be met with a sum that totally exhausts the magnitude of the universe, and hence at no point can the universe actually be conceived to be completely given as a whole. In the end, then, I believe it likely that Leibniz did allow his constructivism to spill over into his analysis of the extensive infinity of the universe: it, too, has become a merely potential infinity. In the *New Essays* passage the universe is not conceived to be given all at once, at some omega level—as we must suppose any *actual* infinite to be[8]—but rather sequentially through the addition of its parts. And thus at no point in the attempt to conceive the universe through its parts—in contrast with the attempt to conceive a body of finite extensive magnitude through its parts—can the universe be conceived to be given as a whole, with a magnitude that is exhausted by the sum of the parts given at that point.

As before, if we take seriously the implications of Leibniz's doctrine of infinitely divided matter and refuse to allow constructivism to spill over into the analysis of actuals, then it is not at all clear that the analogical move from the case of infinite series to the case of actual bodies is sound. For unlike infinite series interpreted operationally, bodies—according at least to Leibniz's official doctrine—are *actually*, and not just *potentially*, divided to infinity. Thus an *actual* infinity of divisions and parts are supposed to be given in them. Consequently, even if there is no last division in a body, or no least part of a body, it does not follow that the number of parts in a body *actually* divided to infinity cannot reasonably be said to be infinite. And to insist that such a body cannot be a whole on the grounds that otherwise a contradictory notion, namely, infinite number, will be the result is simply to persist in a mistake; for the notion of infinite number is not generally contradictory. Furthermore, if the argument we are considering against the universe's being a whole turns upon a supposed contradiction implied by infinite number—as Arthur seems to assume and as I believe Leibniz intends—and if Leibniz held that bodies of finite extensive magnitude could be wholes, as Carlin argued in his original paper, then there seems to be as much of a problem for finite bodies actually divided to infinity as there is for the universe, which is supposed to be of infinite extensive magnitude. So if Arthur believes, as he says, that I was "correct to suggest that Leibniz's denial of infinite number (infinity understood collectively) precludes a body's being a true whole just as surely as it precludes the world's being a true whole," then his quarrel is as much with Carlin as it is with myself. For in his original paper, Carlin took the position that for Leibniz bodies of finite extensive magnitude, even though infinitely divided, were *wholes*—although decidedly *not* substantial unities—while the world, because it is an infinite extensive magnitude, was not a whole. The argument against this in my original paper was entirely hypothetical and need not be repeated here. But in light of the considerations raised by Levey, it is somewhat clearer why it might never have occurred to Leibniz that a body of finite extensive magnitude should fail to be a whole—even while officially asserting that it is actually divided to infinity. For by allowing his constructivism to spill over into his analysis of actual bodies, Leibniz may have been prevented from considering what Levey calls the "omega level" — the level at which *every part* of matter is supposed to be *actually* divided to infinity, and thus the level at which the conflict between the assumption that bodies are wholes that are actually divided to infinity and the assumption that an infinity of things cannot make a whole would have finally come clearly into view. In this paper I have recently added some further considerations that might make it somewhat clearer why Leibniz may have thought that he could consistently hold that a body of finite extensive magnitude is a whole while at the same time denying that a body of infinite extensive magnitude, as he held the universe to be, could similarly be a whole. In any event, I still maintain that Leibniz's argument to the effect that infinite number and infinite wholes are generally contradictory is not a sound argument, and hence his denial of infinite number and infinite wholes, insofar as it is predicated upon that assumption, is not well motivated. This was one of the main points I was trying to establish in my original paper.

Before concluding this discussion, I should like to mention a final point to be borne in mind when considering the issue of the relationship between *wholes* and *substantial unities* in Leibniz's thought. If we should entertain the thought that the universe has a soul, then it can no longer be conceived to be a mere accumulation of an infinity of substances, or as consisting of an infinity of parts. For if we suppose it to possess a soul, it will be a corporeal substance, and a corporeal substance, for Leibniz, is not a mere accumulation of an infinity of substances, nor does it consist of an infinite number of parts. Rather a corporeal substance is a *substantial unity*, which, as we have seen, Leibniz often seems to distinguish rather sharply from a whole consisting of parts. But as we have also seen, Arthur seems to suppose, contrary to what Carlin and I were assuming in our original discussion, that by "a whole" Leibniz means "a substantial unity." If that claim could be clearly made out, both Carlin and I would have to modify our positions accordingly. But my main point would remain unchanged, namely, that to the extent that Leibniz's denial of infinite wholes, including infinite numbers, turns on his claim that they are generally contradictory, to that extent is his denial without sound foundation.[9]

Received 11 October 2000

Gregory Brown
Department of Philosophy
University of Houston
Houston, TX 77204-3785
gbrown@uh.edu

Notes

[1] One of these is *Pacidius Philalethi* (1676), and Leibniz's statement of the paradox in that work may be found at A.VI.iii.549-550.
[2] This point has recently been made by Samuel Levey (see Levey, 1998: p. 62; cf. also Benardete, 1964: pp. 47-48).
[3] See, for example, G.II.268, 278, 282, from the correspondence with De Volder.
[4] Thus, for example, in my original paper I mentioned that in section 61 of the *Monadology* Leibniz approvingly quotes Hippocrates to the effect that "all things conspire," but then I went on to note that "as Leibniz was well aware, it was the supposed universal sympathy of all things that led 'the ancients,' to which he so frequently refers, to postulate the existence of a world soul" (Brown, 1998: p. 120).
[5] Levey seems to treat the fact that the "folds model" leads to the conclusion that "there can't be any matter" as indicating that the folds model must be defective from the standpoint of Leibniz's wider philosophical position. But of course those who interpret Leibniz as an idealist on the question of the reality of matter would not take the fact that the folds model leads to the conclusion that there can't be any matter as necessarily indicating a defect in that model, since on their view Leibniz did in fact assert that the only things that are real in the strict metaphysical sense are non-extended monads. Levey later argues, however, that it is not the unity problem, but what he calls the "dependency problem" (see Levey, 1999: pp. 147-149)—namely, the problem that arises from Leibniz's assumption that every part of matter is ontologically dependent on other parts of matter, *ad infinitum* — that eventually drives Leibniz to the monadology: "the theory of matter then loses its status as a description of ultimate reality and yields to a metaphysics of monads" (ibid., p. 158).
[6] Then, too, there is Levey's point, mentioned in the previous note, that what drove Leibniz to the monadology was not the unity problem but his view that

every part of matter is ontologically dependent on other parts of matter, *ad infinitum*.

[7] It should be noted that when I said that Leibniz ought to have embraced infinite number, I was not intending to suggest a "remedy" for the problem raised by Carlin. I was simply offering a criticism of Leibniz to the effect that once he had embraced the actual infinite, he should have embraced infinite number as well—as Cantor also seems to have thought. For again, although Leibniz may have thought that he had a sound argument to show that infinite number is generally contradictory, he in fact did not.

[8] Thus Bertrand Russell observed that "the notion of infinity. . . is primarily a property of *classes*, and only derivatively applicable to series; classes which are infinite are given all at once by the defining property of their members, so that there is no question of 'completion' or of 'successive synthesis'" (Russell, 1914: p. 160).

[9] I should like to thank Laurence Carlin and Mark Kulstad for their helpful comments on an earlier draft of this paper.

Abbreviations

H = *Theodicy*. Edited by Austin Farrer. Translated by E. M. Huggard. La Salle: Open Court, 1985.

S = *Gottfried Wilhelm Leibniz: Fragmente zur Logik*. Edited and translated by Franz Schmidt. Berlin: Akademie Verlag, 1960.

References

Arthur, Richard. 1999. "Infinite Number and the World Soul: in Defence of Carlin and Leibniz." *The Leibniz Review* 9, pp. 105-116.

Benardete, José A. 1964. *Infinity: An Essay in Metaphysics*. Oxford: Oxford University Press.

Bennett, Jonathan. 1974. *Kant's Dialectic*. Cambridge: Cambridge University Press.

Brown, Gregory. 1998. "Who's Afraid of Infinite Numbers? Leibniz and the World Soul." *Leibniz Society Review* 8, pp. 113-125.

Cantor, Georg. 1932. *Gesammelte Abhandlungen mathematischen und philosophischen Inhalts*, edited by E. Zermelo. Berlin: Springer Verlag.

Carlin, Laurence. 1997. "Infinite Accumulations and Pantheistic Implications: Leibniz and the 'Anima Mundi.'" *Leibniz Society Review* 7, pp. 1-24.

Dauben, Joseph. 1979. *Georg Cantor: His Mathematics and Philosophy of the Infinite*. Cambridge: Harvard University Press.

Levey, Samuel. 1998. "Leibniz on Mathematics and the Actually Infinite Division of Matter." *Philosophical Review* 107:1, pp. 49-96.

_____. 1999. "Leibniz's Constructivism and Infinitely Folded Matter." In *New Essays on the Rationalists*, edited by Rocco Gennaro and Charles Huenemann, pp. 134-162. New York: Oxford University Press.

Russell, Bertrand. 1914. *Our Knowledge of the External World*. London: Allen and Unwin.

_____. 1918. *Introduction to Mathematical Philosophy*. London: Allen and Unwin.

_____. 1937. *A Critical Exposition of the Philosophy of Leibniz*. London: Allen and Unwin.