Kant's Early Theory of Motion: Metaphysical Dynamics and Relativity

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Abstract

This paper examines the young Kant's claim that all motion is relative, and argues that it is the core of a metaphysical dynamics of impact inspired by Leibniz and Wolff. I start with some background to Kant's early dynamics, and show that he rejects Newton's absolute space as a foundation for it. Then I reconstruct the exact meaning of Kant's relativity, and the model of impact he wants it to support. I detail (in Section II and III) his polemic engagement with Wolffian predecessors, and how he grounds collisions in a priori dynamics. I conclude that, for the young Kant, the philosophical problematic of Newton's science takes a back seat to an agenda set by the Leibniz-Wolff tradition of rationalist dynamics. This results matters, because Kant's views on motion survive well into the 1780s. In addition, his doctrine attests to the richness of early modern views of the relativity of motion.

Introduction*

When Isaac Newton's *Principia* came out in 1687, natural philosophers on the Continent were quick to recognize the extent of its achievement. Some of them, though, found it hard to share all of Newton's assumptions. One was that space exists distinct from bodies. Another was his account of true—as opposed to merely apparent—motion: Newton analyses it as change of place in absolute space, hence as absolute motion.[1] Leibniz and Huygens wrote to each other in agreement that Newton was surely mistaken on this point. "I have reasons to believe that nothing breaks the general law of Equivalence [of hypotheses about which bodies move]," said Leibniz; previously, he had been blunter: "motion is not something absolute, but relative."[2] He shares Huygens' sentiment: "[That all motion is relative] I hold to be always true, and I am not impressed by the arguments and experiments of Newton in his *Principles of Philosophy*, whom I know to be wrong," had declared the Dutchman.[3] In calling all motion relative, Leibniz and Huygens echoed Descartes. Half a century before, he had asserted that each body has a true, or 'philosophical,' motion relative to bodies immediately surrounding it.[4]

It is then all the more puzzling to see Kant write in 1758 an essay titled *A New Doctrine of Motion and Rest* and claim, “I should never use [the terms ‘motion’ and ‘rest’] in an absolute sense, rather always relatively.”[5] The puzzle is that Kant seems to restate a view more than six decades old; yet he does so while proposing a *new* theory of motion. Is Kant guilty of false advertising? Was he simply ignorant of his predecessors’ claims? Or is there something true to his claim of novelty? In this paper, I intend to set the record straight. I reconstruct his early, pre-Critical account of true motion from the short but seminal paper *New Doctrine of Motion and Rest* (henceforth ‘*Neuer Lehrbegriff*’); I then use that reconstruction to answer three questions: What *is* young Kant’s theory of motion?; How does it relate to its predecessors in the classical age?; What really is new about his doctrine, and what were its motivations?[6]

The upshot of my exercise is twofold. First, it shows the young Kant deeply engaged with a foundational agenda for a dynamics of action-by-contact. He inherits both its priorities and the conceptual resources to handle them from Leibniz and Wolff. In addition, the young Kant rejects Newtonian absolute space as a metaphysical foundation for dynamics. These findings sow the seeds of considerable doubt about some influential current readings of Kant as a spokesman for Newtonian science. Secondly, I hope my hermeneutic exertions here will attest to how complex, nuanced and peculiar early modern ideas of the relativity of motion were. This should caution latter-day philosophers of physics who might approach early modern ‘relativists’ anachronistically, looking for nascent echoes of invariance, spacetime, transformation groups and similar post-classical vocabulary.

I begin with some extensive yet important background on the metaphysics of impact before Kant. Then I reconstruct Kant’s concept of motion and his views on relativity, and show that they aim to explain momentum exchanges in collisions, or the ‘communication of motion.’ I conclude with an evaluation of his account, and argue that it contributes to a broadly Leibnizian program in natural philosophy.

Background: Two Puzzles about Impact

Early modern physics scored a key success in 1668, when Wallis, Wren and Huygens independently discovered how to quantify the ‘communication of motion,’ or collisions between bodies in uniform translation. Their proposed solutions amounted to rules for predicting, from the initial masses and speeds of two non-rotating bodies, their velocities after impact. Crucially, these rules involved only kinematic terms: they were just formulas connecting initial and final 'motions.' No causal mechanism or dynamical model was proffered to explain the behavior of the two interacting bodies.

This explanatory gap drove Locke two decades later to complain that, although we are clearly aware that bodies have the "power of *communication of Motion by impulse*," nevertheless when we "enquire how this is done, we *are equally in the dark*." The process whereby, in impact, motion passes "out of one body into another" Locke finds "obscure" and "unconceivable."[7] Shortly after the laws of impact were announced, Leibniz filed a similar dual grievance. First, he charged that the rules of impact submitted to the Royal Society in 1669 fail to explain. They predict *that* the motions of interacting bodies will change according to the rules of impact; but they give no insight into *why* bodies behave as they do in a collision.[8] Leibniz gave voice to his criticism in letters to Henry Oldenburg, who relayed it to Huygens, hoping that he might be able to answer it:

> For [Leibniz] seems to think that neither you nor Mr. Wren have assigned the causes of these Phenomena that you examined in establishing your rules [of impact].[9]

Huygens would not be drawn into the investigation. Two years earlier, Oldenburg had asked him the same question, pressed by William Neile. At the time, Huygens had granted that reasoning on the causes of motion is "subtle," but objected that it is "very metaphysical."[10] And he thought that the metaphysical principles underlying the mechanism of impact, while probable [*verisimile*], cannot however be demonstrated with any certainty: "As to the reason that Mr. Neile demands—why a body gives motion to another body that it collides with—I do not think that it can be found by means of the better known principles."

Leibniz's dissatisfaction with kinematic rules of impact comes from their lack of explanatory import; in this sense, they leave the communication of motion obscure, as Locke had complained. But Leibniz had a second reason for discontent, echoing Locke's accusation that it is also incomprehensible. In all impact, it *appears* that 'motion' has been transmitted from one body to another.[11] The rules of impact confirm it quantitatively: in a collision, one body loses as much momentum as the other one gains. But surely, objected Leibniz, there can be no literal communication of motion: a body's motion – be it speed, velocity or momentum – is a property of the body itself, just like its shape and size. To affirm that, in impact, motion is literally transferred is to suppose that properties could migrate from one substance into another. That is absurd; there can be no genuine transfer of properties:

> I am not surprised if you find insurmountable difficulties where you seem to assume something as inconceivable as the passing of an accident from one subject to another… It is not true that a body loses as much motion as it gives to another; which seems to be conceived as if motion were a substantial thing and resembled salt dissolved in water.[12]

Locke complained about how obscure and inconceivable action by contact is, but Leibniz took action. Initially, he sought to ground the rules of collision in a 'theory of abstract motion,' a model of impact in which the bodies' inertia and mass was disregarded, and only their 'conatus' (instantaneous speed) plays an explanatory role for their motion after impact.[13] By 1680, Leibniz had come to reject attempts to explain impact without the help of dynamical factors like inertia and force. Ultimately, his new solution forced him to re-conceive the very notion of substance and to recast mechanics on wholly new foundations. A monograph may hope to do justice to his achievement; here, I can barely sketch it in the broadest of outlines. As an excuse, I offer that he only indirectly influenced the man at the center of my investigation; the young Kant takes issue with dynamical models and concepts inherited from Leibniz's successors rather than from the master himself. Very briefly, then: Leibniz sought to solve the two puzzles first by 'dynamicizing' the communication of motion—he turned collisions into encounters between internal 'moving forces,' and re-described impact as an occasion for each body to modify its own derivative force. "[E]very bodily mass … has already in it all the force that it can ever acquire, but the encounter with other bodies merely gives it its determination, or rather this determination only occurs at the time of the encounter."[14] Hence, no force is transferred in the Leibnizian picture of collision: writing to Christian Wolff, Leibniz admonishes that one "should know that forces do not migrate from one body into another [*vires non transire de corpore in corpus*]."[15] Secondly, he reformulated the rules of impact as conservation principles, or statements about dynamical magnitudes—momentum and *vis viva*—whose overall quantity across the system remains constant. The temptation to see any actual transference of motion in impact is thus greatly diminished, for Leibniz codified dynamical laws of force, not kinematic predictions about change in speed.[16] With these contributions, Leibniz started a research program into the foundations of impact mechanics that shaped German natural philosophy significantly; I will explain how Kant too engages with this program and attempts to correct it.[17] The program has two distinguishing features. (1) It demands a metaphysical picture of impact that does not affirm or entail that motion is literally transferred from one body to the other in a collision.

(2) It requires that kinematic regularities about changes of motion in impact be grounded in dynamical laws or claims about forces.

The post-Leibnizian consensus

In Germany, Christian Wolff strove to carry out Leibniz's mandate in metaphysical dynamics, though he was no slavish follower. In *Cosmologia Generalis* of 1731, he is eager to present impeccable mechanist credentials: "[i]n bodies, there can be no change unless by motion."[18] Predictably, Wolff also rejects action-at-a-distance: "A body cannot act upon another one unless it pressed against it [*in ipsum impingit*]." Impact and pressure are thus the fundamental forms of interaction between bodies; collisions retain the central role they had in Leibniz's mechanics, so they must be freed of problems.

Wolff makes good on Leibniz's first demand by depicting impact as an encounter between forces rather than motions. Material bodies are all endowed with three 'cosmological' attributes: extension, force of inertia, and active force. Ultimately, all corporeal change must be reduced to them and their laws.[19] When a body is in motion, its active force manifests itself as 'moving force' [*vis motrix*]; stationary bodies are passive, and by their force of inertia resist moving bodies that collide with them: "The principle of resistance to motion in bodies is called *Force of inertia*, or *passive Force*."[20] Together, these two forces suffice to explain what occurs in impact. When a moving body collides with another one at rest, the former exerts a striving [*nisus*] to change the latter from rest to motion. Thereby, it uses up a part of its moving force in order to overcome the latter's resistance: "In an action, each body expends that part of its force which suffices to overcome the [other body's] resistance."[21] With the moving force left over, the first body then pushes the second along, now devoid of resistance. If both are in motion but collide with unequal moving forces, the body with the greater force will defeat the lesser one; with the remaining force, it will then push it along, hence the two bodies will move together: "motion ensues in the direction of the stronger or greater [body]."[22] Finally, when two bodies collide with equal force, they come to rest relative to each other. This is because, their forces being equal, "there is no obvious reason" why one should begin to move along with the second rather than the other way around. Thus Wolff eschews the trouble of explaining how motion may be communicated, since no such transfer really occurs. Impact is first and foremost a dynamical interaction, not a purely kinematic process. It is an encounter between forces inherent in bodies, struggling to overcome one another. Motion only *appears* to have been transferred in impact; but that is just because one body expends as much moving force trying to move the other body as the latter uses up in resisting the former's efforts. The changes in the momenta of both are equal.

'Dynamicizing' collisions is just the first half of Wolff's foundational enterprise. He completes it with an effort to explain the kinematic change of motion in impact, to meet Leibniz's demand that the Wallis-Wren-Huygens regularities be explained. To that end, Wolff distinguishes between mere *rules* and proper *laws* of motion.[23] "The rules of motion we call those according to which the moving force [*vis motiva*] is modified in the collision of bodies."[24] In Wolff, modifications of motive force are changes in a body's 'celerity,' or speed. Hence, the 'rules of motion' are exactly the laws of impact discovered in the second half of the 17th century.[25] In contrast, "laws of motion are called the general principles of the rules of motion." These principles "lie in" [*inesse*] the rules of motion, which can be derived from them, he alleges. "These laws of motion Mathematicians assume, but do not prove; yet it behooves the Metaphysician to demonstrate them."[26] A law of motion, for Wolff, is grounded in the ontology of body and force, and must be derivable a priori from it; mere induction cannot yield proper laws of motion. In *General Cosmology*, he offers two such laws: a principle of inertia, and a claim that reaction is contrary to action.[27] He does 'deduce' both, from metaphysical features of bodies and the Principle of Sufficient Reason.[28] However, to claim that the 'rules' of motion are derivable from the 'laws' is misleading; Wolff derives the rules, elsewhere, but the laws do not come into the derivation.[29] His laws ground the rules only in the sense that they explain in loose, qualitative terms why the motion of bodies changes at all in impact, and why their changes in momentum are equal.

Transfusionism

The Wolffian model of impact rests on an anthropomorphic image in which two forces clash, with the stronger overcoming the weaker. In 18th-century German natural philosophy, there was also a minority view, the so-called transfusionist picture of action by contact. Presenting it, however, is a more difficult task, because we have to rely for its reconstruction on rather hostile sources. Kant attacks it outright in his Critical years,[30] and levels obscure, terse objections to it in *Neuer Lehrbegriff*. Still, it is not illegitimate to try and sketch the transfusionist view. Eric Watkins has already argued that supporters of physical influx, a doctrine about causation, could be expected to endorse a 'transfusionist' picture of body-on-body interactions.[31] Wolff's disciple Ludwig Philipp Thümmig, for instance, proffers a transfusionist explanation of impact. Briefly then, the view is this. When a moving body A collides with a body B at rest, it transfers, or pours into the latter, half of its 'moving force.' This is because, at the moment of impact, the two bodies A and B rest relative to each other—hence they become one body, since a body's parts are mutually at rest. Consequently, A's moving force will be uniformly distributed to all the parts of the new body AB.[32] Therefore, post impact both will move together with the same speed—just as the rules of inelastic collision predict. Yet transfusionists did not mean to say that there was any actual 'pouring over' of force in impact. Rather, they thought that the image of force flowing in from one body into another and homogeneously spreading itself throughout is a good *metaphor* of great use in understanding inelastic impact.[33] In any event, this much is clear: for transfusionists, there is indeed communication of motion in impact; but it must not be taken in a literal sense. Prima facie, at least, a transfusionist could claim to steer clear of Leibniz's warning that properties do not migrate across substances.

Some details in the Wolffians' effort to ground rules of impact in metaphysical laws may be fuzzy, but the idea itself is of vast importance. The young Kant is committed to the same project, as we will see presently. Moreover, Wolff's stature in the German-speaking world ensured that his metaphysics of impact congealed into a virtual consensus; Kant was among the very few to challenge it.[34] Of this shared view, three features are significant for my investigation of his theory of motion. It operates with a distinction between rules and proper laws of motion, from which the former "can and should be demonstrated."[35] Second, the Wolffians recognize only *two* such laws: the Principle of Inertia, and a law of action and reaction.[36] In addition, they insist that these laws are derivable a priori from ontological premises. Lastly, a metaphysical model of collision based on 'moving forces' must accompany these laws and rules. This, I believe, is the background needed to interpret Kant's *Neuer Lehrbegriff* correctly. He articulates a new doctrine of true motion in order to remove some weaknesses in the Wolffian consensus and so improve it. Conceptually speaking, his 1758 essay on motion has three parts. In the first, he refutes the 'traditional ideas' of true motion and rest, and replaces them with an account of true motion as irreducibly relational. From his concept of relative motion, he infers two a priori laws of motion. In part II, he attacks the Wolffians' view that impact is the encounter between an active 'force of motion' and a passive 'force of inertia' (when one body is at rest in the local frame). In the last part, he tries to show that his model explains the rules of inelastic collisions.

I The New Doctrine

Before we delve into it, I must make a crucial point; talk about whether motion is 'absolute' or 'relative' tends to obscure it. Neither Kant's new doctrine, nor the theories of motion he refutes, is an account of merely *apparent* motion (i.e. of how bodies appear to move when seen by various observers). Likewise, they are not accounts of what speakers mean by 'motion' in ordinary, everyday-life contexts. Rather, natural philosophers in the classical age were after a concept of *true* motion.[37] (I give a fuller discussion of this idea when I examine Kantian relativity.) The laws of early modern science, in particular the Principle of Inertia, presuppose it. Kant too is after the nature of true motion, although he is not always explicit.

He starts polemically, with a critique of the "common concepts" of motion and rest. The received view, he continues, explicates (true) motion as change of (true) place.[38] The trouble is that, left unanalyzed, the term 'place' is ambiguous. Kant sees two possible ways for the Wolffians to explicate it, and proceeds to reject both. 'True place' can mean position relative to a body (or system of bodies) truly at rest; or it may designate a part of an immovable, container-like whole: absolute space. Some terminology is needed. Let us call the first account 'global-frame relationism': it defines true motion as change of position relative to a *unique* frame of reference designated by one or more distinguished bodies.[39] These objects suffice to set up a frame with respect to which we could define and measure the motion and rest of *all* bodies; hence the attribute 'global.' The name 'relationism' is to indicate that it analyzes, or defines, true motion in terms of relations between bodies.[40] The second option is best called 'absolutism.' Like its alternative above, it *admits* that all bodies have a *unique quantity* of true motion; in contrast, it seeks to define it as velocity relative to space metaphysically distinct from matter, thus absolute. Kant thinks he can refute both positions, *ergo* the claim that true motion is change of true place.[41]

Kant and the work of Bradley

He charges that global-frame relationism is empirically empty: true motion must be a true quantity, yet a rightfully privileged frame of reference is beyond our empirical reach, he accuses. Take the Earth to mark that global frame; that won't do, for the Earth truly moves around the Sun. Then maybe the Sun could be the privileged frame relative to which bodies truly move? The Sun is not good enough, either—it is not certain that the Sun is truly at rest; rather it seems to be itself in motion. Earlier in the century, James Bradley had conjectured that the Solar System as a whole might be moving relative to the fixed stars. A global-relationist's last resort would be the frame of the fixed stars. But Bradley was unable to supply a definite measure of the Sun's velocity relative to the stars; moreover, he suspected that the fixed stars may not be so fixed, after all: long-term observations suggest there is relative motion among some of them. Kant invokes Bradley to refute global-frame relationism, but he is quiet about how the astronomer might support his argument. So, a look at Bradley would shed light on Kant's own reasoning.

Astronomer Royal between 1742 and 1763, the Reverend James Bradley had set out, late in 1725, to continue observations by Robert Hooke and Samuel Molyneux aimed at Gamma Draconis, the third and brightest star in that constellation, "…in order to try whether it had any sensible parallax."[42] Detecting stellar parallax in one star—an angular displacement relative to some neighboring celestial body—serves to compute the Sun's distance to that star. Unfortunately, Bradley's repeated attempts were unsuccessful, "there appearing therefore after all no sensible parallax in the fixed stars." Though parallax eluded him, in 1727 Bradley saw another series of "change[s] in the positions of the heavenly bodies, which, by reason of their smallness, had escaped the notice of [his] predecessors."[43] The new phenomena that puzzled Bradley were a change in the angular motions of some stars—a periodic change greater than the precession of the equinoxes would predict. To explain such collective changes of relative position, Bradley investigated several explanatory hypotheses; eventually, he closed in on nutation, a previously undetected, regular wobble of the Earth's axis.

Towards the end of his paper on nutation, Bradley allows himself to go into a few methodological reflections. He remarks that settling the issue of the true cause of the astronomical phenomena he has just discussed supposes the fixed stars "have no real motion in themselves, but are at rest in absolute space."[44] But that is far from certain, worries the Reverend: long-term, ever-more accurate observations had led 18th-century astronomers to conclude that "there appears to have been a real change in the position of some of the fixed stars with respect to each other." Such changes in relative position can be the result of various causes, explains Bradley. One *possible* cause is "if our own solar system be conceived to change its place with respect to absolute space." It must be stressed, however, that a motion of the Solar System as a whole through space was a *mere conjecture*, as far as Bradley was concerned. Astronomical data and instruments at the time were too uncertain to warrant any definitive conclusions. So, "it may require the observations of many ages to determine the laws of the apparent changes even of a single star: much more difficult therefore must it be to settle the laws relating to all the most remarkable stars." On the issue of the Earth's true motion in space, Bradley's prevailing mood in his two papers was of mild pessimism. Unable to measure any parallax, he could not even determine the Earth's distance to one star. And he had no definite answer to whether the Sun really moves relative to the stars, and how much.[45]

What did Kant make of these results? I submit that he took Bradley's doubts to spell the doom of global-frame relationism. He astutely objects that it is not enough to *explicate* true motion and rest as change of position relative to the fixed stars; for the doctrine to have empirical content, it must allow us to *measure* these distances, and so yield a determinate answer to the question: what is the Earth's *true velocity* relative to the stellar frame? And he reads Bradley's negative conclusions as undermining that possibility: neither stellar parallax nor a supposed solar translation in space is amenable to definite measurement, at least for the time being. Speaking of a sphere—about which Kant had been asking what its true motion relative to the stars might be—he concludes: "And now I get dizzy, and don't know any longer if [the sphere] rests or moves, in what direction and with what velocity."[46] Accordingly, *this* account of true motion—as change of position relative to a privileged, material frame of global extent—turns into a cul-de-sac: its concept of true motion is not empirically well-defined. Thus, it must be abandoned.

Absolutism

Next, Kant turns to the second competitor, the view that true motion is change of place in absolute space. He does not mention Newton by name, but the rejected position is clearly Newton's. Nor does he engage seriously with the Briton's arguments for absolute space, but gives it short shrift:

> And even if I wanted to imagine a mathematical space, empty of all creatures, as a container of bodies, this would not help me at all. For how am I to distinguish its parts and the different places that are not occupied by anything corporeal?[47]

The charge is terse, but not ineffectual; it hits a weak link in Newton's foundations of mechanics. Let me expand it on Kant's behalf. If true motion is change of absolute place, we need either markers to distinguish one absolute place from another or some physical way to measure how fast a body moves between them, i.e. its velocity in absolute space. But Newton admits openly that absolute places "cannot be seen and cannot be distinguished from one another by our senses.[48] And there is no mechanical experiment—certainly none involving collisions—that could help measure a body's absolute velocity. All evidence from observing bodies in impact is insufficient to determine whether a colliding body is at rest in absolute space or moves uniformly in it with some velocity *c*. Absolute space is useless if one asks, as Kant does, "in which direction and at what speed" a sphere lying on a table in a ship drifting downstream *really* moves. Collisions between such spheres are standard examples in the mechanical philosophy, yet there is no determinate way to measure the proper velocity of each body in impact. The Wolffians' task of grounding collisions in metaphysical laws of motion cannot succeed, for their concept of true, or proper motion, is defective:

> …there is something lacking in the expressions 'motion' and 'rest.' I should never use them in an absolute sense, rather always respectively [*respective*]. I should never say that a body rests, without adding with respect to which things it is at rest; and should never say that it moves without at the same time naming the objects with respect to which it changes its relation.[49]

As a prescription for elucidating the nature of true motion, this seems baffling. After all, Kant's critique began by noting, unoriginally, that if we think of motion as change of place relative to other bodies, then our judgments are unstable: ascribing motion or rest to an object varies as we vary the particular frame of reference. Surely now he is not about to urge a return to the same predicament! It turns out that he is not. When Kant says we should not predicate motion and rest absolutely, he means we must never use them as complete, monadic predicates: to say that a body X moves *simpliciter* is a mistake. Rather, we ought to attribute motion to *pairs*. True motion does not have a single subject: it is not the true motion of *a* body. It is an *irreducible* relation between a body and another one. Still, in the actual world any one body is a member of many such pairs. Which one is distinguished as the body's *true* motion? The privileged pair, Kant explains, is the set of two bodies about to collide; this is because only the bodies in this pair act upon one another, and effect mutual changes. For a body in impact, all other pairings are dynamically irrelevant: "You will grant me that, when talk about the effect which the two bodies have on each other through impact, the relation to other, external things has nothing to do with it."[50] As a pair, two colliding bodies are in a relation of approach [*Annäherung*]; Kant sees it as a "mutual relation," in which bodies have a "share" [*Anteil*]. To determine each body's share, hence how much "motion" [*Bewegung*] they have with respect to each other, he appeals to considerations of symmetry:

> …tell me if one can infer, from what happens between them, that one is at rest and only the second moves, and also which of them rests or moves. Must we not ascribe the motion to both, namely in equal measure? Their mutual

approach may be attributed to the one just as much as to the other.

Therefore, each body has an equal amount of respective motion. But there is a catch. Kant's talk about mutual approach really is about a *kinematic* factor: the spheres' speed relative to each other. Yet when he goes on to explain how to compute their respective (true) motions, he also takes masses into account, so as "to maintain a complete equality on both sides." Quietly, he switches to a *dynamical* perspective. This is for two reasons, about which he is equally silent. In general, when two unequal bodies collide, they have equal speeds only in a frame unsuitable for inertial mechanics: it is accelerated, so the rules of impact do not hold in it. Second, Kant wants an account of true motion as a dynamical magnitude, not just kinematic—because he will take it to express the true amount of 'moving *force'* with which the bodies interact. So, he takes the relative speed (before impact) and divides it inversely as the masses of the two bodies, then ascribes one to each body: the lower speed to the larger sphere, and vice versa. Of course, two bodies only have these particular speeds in a special frame, the center-of-mass of their impact. In that frame, he infers, they collide with equal true motion [*wahrhafte Bewegung*], which they have only with respect to each other [*respective*]. Kant has carried out a conceptual analysis of true motion to conclude that it is rightly understood only as a relation between a pair of bodies, which share in it equally. The share of each is equal to its momentum in a privileged frame, in which the pair's center of mass is at rest.

Kant on impact

What happens in that frame when they collide? Both objects are truly in motion, so each has a 'force' [*Kraft*] of motion. Kant cleverly privileges the frame in which the relevant objects always have equal momenta, thus their 'forces' are equal: "both move toward each other … the one with the same force as the other."[51] Because they meet with equal forces, these "cancel each other out" [*diese zwei gleichen Kräfte einander aufheben*]. As a result, the bodies come to mutual rest in the center-of-mass frame. It now becomes clear his topic has been inelastic collisions. They are, ultimately, encounters between two 'moving forces' that, in virtue of their equality, balance each other out. No motion is really communicated; if bodies stop after impact, it is because each one's own force has been annihilated by the other one's equal and opposite force. What is more, the Kantian picture of impact does not require two different kinds of dynamical causes, a 'force of inertia' and a 'moving force,' as it did for his German precursors. Neither body is ever truly stationary before impact, so there is no need to assume a 'force of inertia' in bodies at rest.

Now we see that his new doctrine of true motion was just a flanking maneuver to outwit the Wolffians and replace their metaphysical dynamics of impact with a leaner model. The real upshot was an improved account of collisions in terms of two forces of the *same* kind: two 'moving forces' present in both objects to the same extent. But Kant spends so much effort on the way to it, and is so terse about its details, that it is easy to miss.[52]

He pleads he cannot be thorough in "such rich material with such narrow limits on space." But also, he did not need to be explicit about all of his assumptions. In post-Leibnizian Germany, some were taken for granted. For instance, that "the active force of bodies is called *motive force*, because it accompanies local motion"; for that reason, "when we posit local motion, motive force [*vis motrix*] is also thereby posited."[53] Or that "if two bodies strive toward each other from opposite directions with equal force, neither will move." Likewise, that "if two inelastic bodies A and B collide frontally at speeds inversely as their weights, both come to rest." Above all, Kant and the Wolffians agree that impact is eminently understandable if presented as a conflict between 'moving forces.' Kant never makes that assumption explicit because he has no intention to challenge it. His new model of impact aims to correct a tradition, not replace it.

Kantian relativity

True motion, Kant insists, is always *respectiv*, or with regard to other bodies. Then he explains that it is a relation, hence relative. How does his view really differ from that of so many others before him who claimed motion to be relative? It does differ, rather drastically; but the language of relativity can obscure the gulf between Kant and others, and how radical his stance is.

Though not always explicitly, Descartes, Newton, Leibniz and Berkeley all assumed that, for each body taken *individually*, it is meaningful to ask whether it truly moves or is truly at rest.[54] This premise is threefold. **1** Almost to a man (Huygens was the great exception) these figures accepted that the phrase 'truly moves' is a complete predicate: a monadic property that is really distinguished from true rest. Hence, true motion has a *single* subject: it is properly predicated of individual objects. Of course, the *definitions* of the 1-place predicates 'true motion' and 'true rest' may employ relations; e.g., a relation to the body's immediate neighborhood or to the part of absolute space occupied by the body. But whatever relations the correct analysis of true motion may involve, the term itself is a predicate of a single argument.[55] Its reference is a *property* of a body, not a relation between bodies. **2** In addition, true motion and rest are real contraries: on pain of contradiction, one cannot affirm both of the same body at the same time. There are facts of the matter as to whether individual bodies move truly or are truly at rest. Some such facts are dynamical phenomena: e.g., true rotation is distinguished from rotational rest by the presence of centrifugal effects. Other facts are metaphysical; they may concern the force of bodies or their velocity in absolute space. **3** Lastly, these thinkers assumed, every body has a *unique* state of motion or rest. An object may move with respect to this or that body, system, or frame; but as far as its *true* motion is concerned, each body has exactly one.

The young Kant denies (1) and (3) explicitly. He first *refutes* the idea that motion is a complete predicate: "I should never use [the terms 'motion' and 'rest'] in an absolute sense…" True motion is a relation *irreducible* to monadic predicates, because it has *no* single subject. This is quite different from claiming that motion is relative in the sense that the monadic predicate 'truly moves' must be defined as motion relative to material bodies. Because for Kant (true) motion is fundamentally a relation, it is illegitimate to say of a single object that it moves. Moreover, a body does not have a unique true motion; it has as many as there are bodies dynamically related to it by an interaction. That is the gist of Kant's relativity. In the taxonomy proposed by Robert Rynasiewicz, he would count as a *strong relationist*.[56] Among Newton's opponents, some *shared* with him the conviction that true motion is a complete predicate, i.e. that motion has a single subject. They just objected to Newton's *definition* of it in terms of absolute places. Rather—they offered—true motion must be defined as a body's change of position relative to some privileged set of bodies. For that reason, we may call them 'weak' relationists: they *stop short* of denying the very idea that motion has a subject. Descartes, Leibniz and Berkeley all fit this profile.[57] Ever the sharp wit, Huygens saw clearly that weak relationists are closer to Newton than they suspect: "Mariotte distinguishes the relative celerity of two bodies from their 'proper velocities.' … Thus believe all of them; and Newton too."[58] In contrast, Huygens himself was a 'strong' relationist: he thought motion has *no* single subject but is only meaningful when said of pairs; moreover, he thinks in collisions there is *no* unique quantity to single out as the two bodies' true motion.[59]

Kant is already a radical in claiming, like Huygens, that motion has no single subject, but is irreducibly dyadic; as if that were not enough, he mixes it with the bold idea that bodies have true velocities in impact. It is an outlandish view, but Kant has convinced himself that only it can explain the communication of motion successfully. Let us see how he goes about that task.

Laws of motion

With the new doctrine in place, Kant infers two "corollaries." He claims they follow from it, though he never shows how to derive them. The first reads: "Every body, with respect to which another one moves, is also in motion relative to the latter; and it is impossible that a body should approach another one that is absolutely at rest."[60] Secondly, "[i]n impact, action [*Wirkung*] and reaction [*Gegenwirkung*] are always equal." The signs are overwhelming that, with these corollaries, Kant continues the Wolffian tradition of a priori laws of motion. He has only two, just as they did. His first law is meant to replace their first, the Principle of Inertia (which he demotes to "a law of some phenomenon known empirically" rather than derivable from a metaphysics of body, as Wolffians would have it). A force of resisting motion, inherent in bodies at rest, is needless if the aim is to explain impact, reasons Kant.[61] He has just 'proved' that in *all* collisions the two bodies move equally; neither is ever truly at rest. Hence a 'force of resistance' is superfluous, and so is the metaphysical principle that codifies it: Wolff's Law of Inertia. (More on this presently.) Further, Kant's second corollary is a principle of action and reaction, just like the Wolffians' second law of motion; he merely gives 'reaction' a somewhat different meaning. Also, he seeks a priori laws of motion in order to ground collisions—by explaining the rules of impact. The same animus drives both him and the Wolffians.

His law of action and reaction deserves a gloss. Wolff and his followers had one, too, but left no room for Newton's Second Law in their foundations.[62] As a result, it is unclear what action and reaction are or how to quantify them, in post-Leibnizian dynamics. They hint that action is measured by the amount of 'moving force' an active body must expend to overcome the passive body's resistance in impact.[63] Less transparent is their measure of reaction, especially if the second body is stationary, hence endowed only with a 'force of resisting' grounded in inertia. Often, the Wolffians offer obfuscation and verbiage, patently unsure about what to say; Kant steers clear of their troubles, but not by importing from Newton the concept of impressed force. Rather, he argues that no body is truly at rest in impact, so both equally move, hence they have the same amount of moving force. Ergo, the measure of both 'action' and 'reaction' is the *momentum* of each body in the center of mass frame of their collision.[64] They are equal because the two momenta are so. Wolff's difficulties vanish, but at a heavy price: Kant's laws of motion only hold in a special frame; the equivalence of all Galilean frames is not on his mind.

Finally, note how far all this is from Newton. The *Principia* rests on three laws of motion; the young Kant has only two. Impressed force is absent from his dynamics, so his Principle of Action and Reaction differs in both meaning and scope from the Third Law. At this stage, at least, Kant does *not* define true motion in terms of his proposed laws of motion; quite the opposite, in fact—the laws appear to be inferred from a previously articulated doctrine of motion.[65] He has designed both doctrine and dynamical laws to handle impact only, not orbital motion, as Newton did. Further, Kant dissents from Newton in spirit too, not just in letter: he aims to obtain laws of motion 'from mere concepts,' as he will put it later; the Briton was adamant that his laws could only be had by induction.[66] Add to this Kant's drastic views about the relativity of motion: not only is he not tempted by Newton's absolute space, but he rejects the very idea that true motion is change of true place, or a complete predicate. On this point, the two thinkers could not be further apart. Believers in a supposed justification of 'Newtonian' science by Kant should see all this as a challenge; but on the interpretation I have proposed, a link to Newton is predictably absent: in 1758, Kant is only after solutions to some problems in the Leibniz-Wolff agenda for metaphysical dynamics.

II Polemic Engagement

In the middle part of his essay, Kant refutes two claims: the notion that the *vis inertiae* is a true force of bodies, and Leibniz's law of continuity. It appears to be an unrelated insertion between two connected parts of his main topic: the nature and laws of true motion (I), and how the new doctrine grounds the rules of impact (III). Yet there is a way to read the middle part such that it fits coherently with the other two sections. I propose that the targets of section II are: first, Wolff's understanding of collision as the encounter between active 'moving force' and passive inertia; second, the transfusionist model of impact.

Against *vis inertiae*

First, Kant rejects the notion that inertia is an inherent force in bodies at rest, conceived as a passive power to resist changes in their state of motion. Recall that a *passive* force of resisting—named 'force of inertia'—was part of Wolff's model of causality in impact. Kant accuses it twice: (a) it is superfluous: one does not need it to explain impact; (b) the concept of a force of inertia is incoherent. Charge (a) assumes Kant's own picture of collision and his first law of motion: in impact, neither body is truly at rest. Accept that, and the Wolffians' force of inertia becomes a needless addition; impact is an encounter of bodies in equal true motion, hence equally *active*:

> Now that I have shown how that which one had falsely regarded as rest with respect to the impacting body is in fact a motion relative to it [*beziehungsweise*], it becomes self-evident that this force of inertia was devised without any real need. Moreover, in each collision, a motion of one body is met with that of another body, which opposes the former with the same degree of motion. This explains quite easily and clearly the equality of action and reaction, and there is no need to think up a special type of natural force [*Naturkraft*].[67]

The argument for charge (b) is that a 'force of inertia' is inconsistent with the model of impact it is supposed to explain. If a body A is at rest, Kant argues, it is in a state of equilibrium of forces, for that is a consequence of rest. Then how is A supposed to displace itself instantaneously into an imbalance of force so that, at the moment of impact, it has a force of resistance directed at the incoming body B? There has to be one, he reasons, since B slows down post collision; so, a force in A must have countered it:

> But how is it that, as soon as the impacting body touches the one at rest, the latter is supposed to suddenly change itself into a state of motion or striving directed at the approaching body, in order to destroy in the latter a part of its force?[68]

The notion that stationary bodies possess a force of resistance has fatal flaws, he thinks. The attack shores up his own model of impact as a clash between two equal and opposite moving forces that cancel each other.

Continuity and impact

The reasoning in the second half of Section II is more obscure. Explicitly, he argues against a version of Leibniz's Law of Continuity. This is a rich and fascinating topic, on which I cannot digress here at any length.[69] Suffice it to say that, in his essay on motion, Kant distinguishes between a logical and a physical law of continuity. He wants to retain continuity as a "logical rule"—this move is the seed of a later development: continuity as a regulative idea, in the First Critique. The "physical law" of continuity reads, according to Kant:

> a body does not transmit its force to another body all at once, but only so that the former transfers its force to the latter by means of all the infinitely small degrees from rest to a determinate velocity.[70]

This physical principle Kant wants to reject. The argument he deploys is, again, twofold: (1) the law is arbitrary; although many scientists assume it tacitly in their proofs, it can never be conclusively confirmed, but it is always easy to disprove; (2) if one assumes that a body's force is transmitted gradually through all intermediate degrees, a sort of Zeno's paradox ensues: body A would never get to transmit any force to B, since it would have to give it an infinity of intermediate degrees. Kant believes these two objections are enough to make the 'physical' Law of Continuity untenable, so he proposes to abandon it.

I submit that he wants to reject it so as to undermine the transfusionists' model of impact. Kant takes it to be an essential premise in transfusionism that in collisions one body 'transfers' some of its motion to the other *by degrees*, hence *continuously*, as the Law of Continuity prescribes. Presumably, he can topple transfusionism if he disproves this premise. His reasoning on this point is extremely obscure, but I think we can reformulate the argument thus: if there really is communication of motion in impact, then the following should happen. Take two *perfectly hard* (i.e. rigid) bodies A and B of equal mass. Let A be at rest with respect to us, the observer, and let B approach it with a speed = $2v$. If communication of motion really takes place, then we should see B come to rest, and A begin to move with speed $2v$. That is because B has to transmit its 'moving force' all at once to A. But we know that this doesn't happen; what occurs instead is that both A and B move together with speed v. What is the explanation of that fact? A transfusionist, Kant implies, tacitly assumes that force is transmitted by continuous degrees, until both A and B reach the same degree of 'moving force' and then move together. But this account relies on an assumption—the physical law of continuity—that is a mere hypothesis, and can never be confirmed.[71] Dismiss the law, and transfusionism becomes untenable. Both the Wolffians and the minority opposition have thus been refuted, Kant believes. To show that his model is better, now he must put it to work.

III The Rules of Impact Explained

At last, Kant moves to fulfill the original promise: show that his theory of motion grounds the rules of impact. He thus continues the program of Leibniz and Wolff, but more soberly. The Wolffians averred that rules of impact can and should be deduced from metaphysical laws of motion; Kant only hopes to explain them by his doctrine. That is just as well, because a strict deduction requires extra premises that only induction could justify. So, he opts for a "key to the explication [*Erläuterung*] of the laws of impact." The key has two components: (1) a *dynamical* model of impact in the center-of-mass frame (call it **M**), where two bodies truly and equally move with respect to each other—in Kant's sense of true motion. The model yields "the rules of the relation that colliding bodies enter into, with respect to each other."[72] (2)

He also has a *kinematic* report of how their motion *appears* to an observer at rest in another frame **R** in uniform translation relative to **M**. It describes "the change of [the colliding bodies'] outer state with regard to the space in which they exist." But this change of motions relative to an observer is "only the outer phenomenon of what occurred immediately between them; and it is the latter that one needs to know," he insists. To connect the two hinges, he has an unstated principle, the composition of (uniform) motions. A body's motion relative to one frame can be compounded from (1) its motion in another frame and (2) the motion of the two frames relative to each other.[73] One component can be dynamically affected—e.g. by external forces—while leaving the other unchanged (obviously, the *total* composite motion changes if a component has changed).[74] He has already spelled out and defended his dynamics of impact, so he now summarizes:

> What takes place, in a collision, between the two bodies mutually acting upon each other, is already clear, according to our doctrine expounded above. It is simply this: the reciprocal action and reaction of both bodies is equal, and both bodies, subsequent to the impact, come to rest relative to each other—that is to say, when they collide frontally, and if we abstract from any elastic force [*Federkraft*]."[75]

That holds only in the center-of-mass frame **M**, whereas the rules of impact predict motions as observed in other frames. Consider a sphere B of 3 pounds at rest in **R** and another sphere A of 2 pounds approaching it with a speed of 5 'degrees.'[76] The relevant rule of impact says that, post collision, both A and B will move together at a speed of 2 (relative to **R**). Kant explains the prediction thus. A's speed of 5 in **R** was composed of two motions: a speed of 3 relative to **M** and a speed of 2 that **M**, together with bodies in it, had relative to **R**. A's speed of 3 in **M** is 'cancelled out' [*aufgehoben*] by the impact with B. But their collision leaves intact **M**'s speed relative to **R**; the two spheres interact with each other, not with their 'space.' Body A shared in the motion of that frame, so it still has it; that is why it moves with 2 'degrees' of speed after impact. In turn, B only appears stationary in **R**. Its apparent rest is the result of two equal and opposite motion: a speed of 2 relative to **M**, and **M**'s own contrary speed of -2 degrees relative to **R**; the sphere B shares in it. Its collision with A only cancels out the former speed, but the sphere retains the latter. Hence, someone looking on in **R** will see B move after impact with speed 2, just as the rules of impact lead us to expect.

Thus Kant fulfills the Leibniz-Wolff mandate. In his model, there is no transfer of motion in impact, only an equality of forces; and the way they balance each other explains the subsequent kinematics of colliding bodies.

A Qualified Success

Kant's early theory is not without its difficulties. I have already complained that he operates with an ambiguous concept of motion. To prove that two bodies collide with equal 'respective' motions, he equivocates on the meaning of 'motion': he starts with premises about their relative speed, and infers that they have equal momenta. If only getting dynamics from mere kinematics were so easy!

Also, it is unclear what he would say when a *third* body enters the picture. Take a collision between A, B, and C. What is body A's true motion? Does it have one respective to B and another one respective to C? Does it move respective to B and C together? Or would he say that each has a true motion with respect to the center-of-mass frame of the system? Some commentators do see Kant move precisely in this direction.[77] And they think that this answer, only implicit in *Neuer Lehrbegriff*, is spelled out and strengthened through Kant's adoption of absolute space in the 1780s. But danger lurks in this reading. The peril is that it collapses into precisely the position that Kant sets out to refute in the 1758 paper: global-frame relationism, or the view that true motion is change of place (i.e. velocity) relative to a privileged global frame. For, if in a multi-body system the Kantian true motion of each is velocity relative to the center-of-mass frame, the following ensues. A mechanical system is part of a larger, more encompassing system. The latter, in turn, is also contained in an even vaster one, and so on up to the ultimate mechanical ensemble, the "systematic constitution on the great scale" of Kant's cosmology.[78] This ultimate system, including all material objects in it, will have its own center-of-mass frame, with the origin at "a mass which may serve as its fulcrum." Kant would have to say that, ultimately, each body has a unique quantity of true motion, defined as velocity relative to the center-of-mass of the world. But that is just a version of the very theory of motion Kant sets out to criticize: the view that motion is *change of true place* relative to a distinguished frame of global extent. If we construe Kant's reading that way, we would have to write him off as hopelessly confused.

A third shortcoming is that he makes no attempt to analyze rotation. Circular motion remained a conundrum for relationists.[79] One source of difficulty is rigid rotation: two bodies can be in true motion around each other, yet there is no change of relative distance between them. Such are the two globes connected by a chord, in Newton's Scholium to the Laws of Motion; such are the parts of a rigid body rotating in an empty universe. It is a mystery how Kant would analyze such situations or in what sense he would say they move respective each other. In 1758, he seems oblivious to their challenge. In his defense, the early concept of motion was designed for a different type of events—collisions, or interactions that *always* involve actual changes of relative distance. But that is a dodge of limited effect, because his early doctrine grounds only a point-mass treatment of impact. Yet that is not the only possible one, nor is it the most informative. Colliding objects can also be studied as extended bodies, rigid or deformable, not just as point masses. In fact, at the time Euler was doing just that.[80] And the rub is that, in this enriched treatment of impact, the motion of each body is a combination of translations and *rotations*. Circular motion, therefore, remains inevitable. Kant will ponder it, with mixed results, much later, in *Metaphysical Foundations of Natural Science*.

Lastly, his claim to novelty is more nuanced than it appears. Before him, others had resorted to the center-of-mass frame as the key to taming impact. In 1668, Wren began by declaring that, when two bodies collide, their "Velocities proper and most Natural are inversely proportional to the Bodies," i.e. their masses.[81] Around 1677, Leibniz also toyed with the idea that "if two bodies collide, the speed must be understood to be distributed between them in such a way that each runs into the other with the same force [*eadem vi*]."[82] Even the general tenor of Kant's project – to start with a conceptual analysis of (true) motion so as to arrive at laws of impact – has a Leibnizian precedent. Late in 1669, at work on his theory of abstract motion, Leibniz notes that, since our senses can be deceiving as to the nature of motion, we must investigate it by reason [*ratione*]. The "reasons of motion," he follows, must be demonstrated "not from matters of fact or the testimony of senses, but from the definitions of terms."[83] Of course, Kant did not know about the young Leibniz's explorations in a priori mechanics. In any event, undeniably novel is the peculiar brand of Kantian relativity offered in *Neuer Lehrbegriff*.

Conclusions

Kant's pre-Critical theory of motion is best understood against the background of his rationalist predecessors in Germany, notably Leibniz and Wolff. It is a strong relationist doctrine meant to support a metaphysical dynamics of impact and laws of motion a priori. His engagement with these topics attests the influence, well into the eighteenth century, of a research agenda and conceptual resources that go back to Leibniz. Further, his case makes plain how rich and complex early modern views about the relativity of motion were, and challenges us to approach them on their own terms rather than try to read modern relativity principles into their proponents. Not unimportantly, I have shown that there is no temptation to cast him as a competitor to Newton's account of true motion. Not only is the young Kant's encounter with Newtonian absolutism cursory, but he also has different ideas about what a theory of motion should do. So, to expect him to provide, in later years, an alternative to Newton's absolute space and motion would require a watershed event in his natural philosophy after 1758. If such a dramatic change of mind cannot be documented, it ought to give some pause to advocates who think that the mature Kant labored to set *Newtonian* physics upon secure foundations.

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Notes

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[1] Newton's explication of true motion – as translation in absolute space – is in his Scholium to the Laws of Motion, in Isaac Newton, *The Principia: Mathematical Principles of Natural Philosophy*, trans. I. Bernard Cohen and Anne Whitman (Berkeley: University of California Press, 1999), 408-415.

[2] G.W. Leibniz to Christiaan Huygens, in *Oeuvres Complètes de Christiaan Huygens* (The Hague: 1888-1950), vol. X, 645 (henceforth abbreviated as *OCCH*); second quotation: Leibniz, "Motion is Not Something Absolute," in *The Labyrinth of the Continuum*, trans. and ed. Richard T.W. Arthur (Yale University Press, 2001), 333. Unless otherwise noted, all translations are my own.

[3] Huygens to Leibniz, 29 May 1994, in *Oeuvres Complètes*, vol. X, 614.

[4] Cf. R. Descartes, *Principia Philosophiae*, II.25, in *Oeuvres de Descartes*, ed. Ch. Adam and P. Tannery, vol. VIII-1 (Vrin, 1964), 53.

[5] I. Kant, *Neuer Lehrbegriff der Bewegung und Ruhe, und der damit verknüpften*

Folgerungen in den ersten Gründen der Naturwissenschaft (A New Doctrine of Motion and Rest, and of its Consequences for the First Principles of Natural Science), in vol. II of the *Akademie-Ausgabe* of the works of Kant (Berlin: Georg Reimer, 1912), 13-26. I use the standard practice of quoting from Kant's work by indicating the volume and the page number in the German Academy edition.

[6]Kant's account of motion in his mature period, subsequent to the publication of the First Critique, has received a certain amount of attention. See, among others, K. Pollok, "Kant's Critical Concepts of Motion," *Journal of the History of Philosophy* 44(2006), 559-575; Michael Friedman, "Metaphysical foundations of Newtonian science", in his *Kant and the Exact Sciences* (Harvard University Press, 1992), 136-165; Martin Carrier, "Kant's relational theory of absolute space", in *Kant-Studien*, 83(4), 1992, 399-416. In contrast, Kant's early views on motion have been far less thoroughly investigated. See Fr. Kaulbach, *Der philosophische Begriff der Bewegung. Studien zu Aristoteles, Leibniz und Kant* (Köln, 1967), 80ff; Th. S. Hoffmann, "Der Begriff der Bewegung bei Kant," in *Zeitschrift für philosophische Forschung*, 45(1991), 38-59.

[7]J. Locke, *An Essay concerning Human Understanding*, Book II, Ch. xiii, § 28, ed. P.H. Nidditch (Oxford University Press, 1975), 311.

[8]A few examples might help us understand Leibniz's charge. "The Velocities of Bodies proper and most natural are inversely proportional as the Bodies. Hence the bodies R, S having proper Velocities, will retain them even after impact."—C. Wren, "Lex Naturae de Collisione Corporum," *Philosophical Transactions*, 43(Nov. 1668), 868. "If a hard Body at Rest is struck by another equal Body, after their contact the latter will come to Rest, whereas the former will acquire all the Celerity that was in the Impacting Body."—Chr. Huygens, "Regulae de Motu Corporum ex mutuo Impulsu," *ibidem*, 46(Apr. 1669), 925.

[9]H. Oldenburg to Chr. Huygens, 7 April 1671, in *OCCH*, vol. VII, 56. Leibniz was not alone in his quest for an explanation of the laws of impact. Shortly before him, William Neile, a young natural philosopher in Britain, objected to both Wallis and Huygens for not having "solidly reasoned upon the physical principles of Motion," as Oldenburg writes to Huygens on 16 September 1669. "[Neile] would like, however, that some skillful man take it upon himself to discover the true nature and the true principles of motion, *from which those rules may be inferred* which you, Mr. Wren and Dr. Wallis have so learnedly proposed; and he adds that all that has been achieved so far only teaches us that Nature observes such and such laws and proportions in the motion of bodies—but that nothing worthwhile has

yet been proposed regarding *the physical cause by means of which a body moves and changes its place*," Oldenburg adds on October 17 (my emphasis). Cf. *OCCH*, VI, 494, 504.

[10]Huygens to Oldenburg, 10 August 1669, in *OCCH*, VI, 481. Next quotation: Huygens to Oldenburg, 30 October 1669—*ibidem*, 514.

[11]An example is needed; for ease of grasp, I shall assume two bodies of equal mass. Let sphere A be at rest in the 'lab frame' of an observer, say on a billiard table in a room, and sphere B approach it with a velocity of 4 m/s. If the two bodies are perfectly elastic, after they collide A will begin to move at 4 m/s, while B comes to rest. If they are perfectly inelastic, both will move with an equal velocity of 2 m/s. A gains as much velocity as B loses. In both cases, it strongly appears that a body's motion passes over to the other one.

[12]Leibniz, *New Essays on Human Understanding*, Book II, Ch. xxiii, § 28, trans. and ed. by P. Remnant and J. Bennett (Cambridge University Press, 1981), 218.

[13]Conatus is just a kinematic quantity, thus insufficient by itself to explain *changes* in motion. To fix this, Leibniz appears to give it some causal-dynamical import: he postulates that conatus can propagate itself fully to infinite distances through all obstacles [*obstantia*]. As a result, when bodies collide "there can be simultaneously in one and the same body several contrary conatuses." To compute the resulting, total conatus after impact, he uses the parallelogram of velocities. Cf. "Theoria Motus Abstracti" (1670-1), in W. Kabitz, H. Schepers (hrsg.) *G.W. Leibniz: Philosophische Schriften*, Series VI, Bd. 2 in the *Akademie Ausgabe* of the works of Leibniz (Berlin, 1966), 176-185 and 258-275 (henceforth abbreviated as A6.2, followed by page number). On *Theoria Motus Abstracti*, see A. Hannequin's doctoral dissertation *Quae fuerit prior Leibnitii philosophia* (Paris: G. Masson, 1895), 54-86; Ph. Beeley, *Kontinuität und Mechanismus*, (Stuttgart, 1996), Ch. 10, 13; R.S. Woolhouse, "Leibniz's Collision Rules for Inertialess Bodies Indifferent to Motion," *History of Philosophy Quarterly* 17(2000), 143-157.

[14]Cf. *The Leibniz-Arnauld Correspondence*, trans. H. T. Mason (Manchester, 1967), 148.

[15]In his response to a letter from Christian Wolff dated 31 December 1710—cf. C. I. Gerhardt (hrsg.), *Briefwechsel zwischen Leibniz und Christian Wolff* (Halle, 1860), 131.

[16]In this respect, Leibniz took the exact opposite of the path Huygens traversed in his quest for the laws of collision. Huygens' early researches into the matter are marked by attempts to measure the 'force of percussion' – a dynamical magnitude

– exerted in impact. They appear to have been unfruitful, until he resolved to approach the problem in purely kinematic terms, which quantify only changes in velocity resulting from (elastic) collisions. Cf. Richard S. Westfall, *Force in Newton's Physics* (American Elsevier, 1971), 148ff; G. Mormino, *Penetralia motus: La fondazione relativistica della meccanica in Christiaan Huygens* (Nuova Italia Editrice, 1993).

[17]I chose to speak of a research program primarily because of Leibniz's demonstrable influence upon 18th century philosophy in Germany. However, pondering the mechanism whereby motion is communicated in impact also preoccupied some in Britain, for instance, where an inclination for Leibnizian dynamics was not widely shared (though a few were aware of his foundational work). See, for instance, Henry Home, "Of the Laws of Motion" and John Stewart, "Some Remarks on the Laws of Motion," both in *Essays and Observations, Physical and Literary* (Edinburgh: Hamilton & Balfour, 1754), 1-69 and 70-140. In contrast, Hutton's dictionary takes a quietist approach to the problem: "Communication of motion, or how a body in motion communicates the same to a body at rest, by coming into contact with it, is also a subject which has been as much controverted by philosophers as [the Law of Inertia], and after all, is as little understood as the continuation of motion, the cause of gravity, and other speculative inquiries of a similar nature." Cf. art. 'Motion,' in Charles Hutton, *A Philosophical and Mathematical Dictionary*, 2nd ed. (London, 1815), vol. II, 72.

[18]Christian Wolff, *Cosmologia Generalis*, ed. Jean Ecole, vol. II.4 of Wolff's *Gesammelte Werke* (Hildesheim: Olms, 1964), § 128. Next two quotations: *ibidem*, § 320.

[19]Wolff, *Cosmologia*, § 138. Next two quotations: *ibidem*, § 130 (emphasis in the original).

[20]Wolff's distinctions between active and passive bodies, moving force and force of resistance crucially depend on the possibility of drawing a well-defined distinction between true rest and true motion. Yet he never ventures to say what true motion amounts to. He does define motion as change of place; by implication, true motion must be change of true place. But there is no account of true place in Wolff's natural philosophy. A second difficulty gravely compounds his predicament. Wolff bases his typology of forces and his laws of motion on impact; however, collisions can never supply any evidence sufficient to distinguish between the true rest and true motion of bodies involved in them. That is one aspect of Galilean relativity in classical mechanics.

[21]Wolff, *Cosmologia*, § 346. Next two quotations: *ibidem*, § 342 and § 341.
[22]Obviously, this outcome only holds for perfectly inelastic collisions. Elastic impact comes later in Wolff's account, because explaining it requires, besides the two 'cosmological' forces above, the introduction of a third element: 'elastic force.'
[23]The distinction appears to pre-date him. Descartes introduces laws [*leges*] and rules [*regulas*] to handle impact, in *Principles* II.37-46; but he does not explain what relationship, if any, is between the two groups of statements. Also, Wolff sometimes resented being described as a Leibnizian, and we know that he owed some of his philosophical education to Cartesians, e.g. Pohl, Neumann, and Tschirnhaus: Ch. Corr, "Christian Wolff and Leibniz," *Journal of the History of Ideas* 36(1975), 243ff. This presses the issue of whether talk of a post-Leibnizian consensus may be just *post hoc, propter ergo hoc*. The very brief answer is, no; whatever influence Cartesianism may have exerted on Wolff, it did not extend to the analysis of impact. Wolff's demand that the rules be derived from the laws comes from Leibniz, not Descartes. Secondly, he embeds his laws of motion in a metaphysical dynamics of forces—active and passive, moving and resisting—that is recognizably Leibnizian. See also J. Ecole, "Cosmologie wolfienne et dynamique leibnizienne," *Les études philosophiques* 19(1963), 3-9.
[24]Wolff, *Cosmologia*, § 302.
[25]Here's an example. "If in the impact of two inelastic bodies the larger body is slower whereas the smaller is faster, then the motion of both will always be retarded if the inverse ratio of their masses is smaller than the ratio of their speeds." See Wolff, *Cosmologia*, § 407. Cf. also § 393.
[26]*Ibidem*, § 303.
[27]*Ibidem*, § 309 and § 318. I hesitate to say that Wolff's second law of motion is Newton's Principle of Action and Reaction, because it is unclear what the measure of 'action' and 'reaction' is, in Wolff's mechanics. Not so in Newton: both terms denote impressed forces, measured by Newton's *Lex Secunda*. But Wolff lacks both the notion of impressed force and a measure for it.
[28]For instance, the Law of Inertia is deduced in three steps, in §§ 304, 305 and 309 of *Cosmology*. Among the premises he summons are that "All bodies resist motion" and that "the force with which a body is impelled is the sufficient reason for the actuality of motion of the movable." Cf. also § 129.
[29]He deduces the rules of impact in "Elementa Mechanicae," volume II of his *Elementa Matheseos Universae* (2nd edition, 1748), Ch. XII, 'De Motu Corporum ex Percussione,' §§ 532-40. But all of his derivations rest on two premises: the

conservation of linear momentum, and a version of the Principle of Sufficient Reason. Wolff’s two metaphysical laws of motion are deductively idle.

[30]See Kant, *Metaphysische Anfangsgründe der Naturwissenschaft,* ed. K. Pollok (Felix Meiner: Hamburg, 1997), 106f.; in the *Akademie* edition, 4: 549-50.

[31]Eric Watkins, “Kant’s Justification of the Laws of Mechanics,” in E. Watkins (ed.), *Kant and the Sciences* (Oxford University Press, 2003), 147f.

[32]Cf. L. Ph. Thümmig, *Institutiones Philosophiae Wolffianae,* editio nova, vol. I (Leipzig: Renger, 1740), §§ 63-4.

[33]Thus says Gottsched, *Erste Gründe der gesamten Weltweisheit*, 6th edition (Leipzig: Breitkopf, 1756), § 1067. See also Eric Watkins, “The Development of Physical Influx in Early Eighteenth-Century Germany: Gottsched, Knutzen, and Crusius,” *Review of Metaphysics* 49(1995), 304.

[34]See, for instance, G.E. Hamberger, *Elementa Physices*, 3rd edition (Jena: Meyer and Sons, 1741), §§ 22-37; J.H. Winckler, *Institutiones Philosophiae Wolfianae* (Fritsch, 1735), §§ 669-723; M. Chr. Hanov, *Philosophia Naturalis sive Physica Dogmatica*, vol. I (Halle, 1762), §§ 1-10, 41-2; N. Burkhaeuser, S.J., *Institutiones Metaphysicae*, vol. I (Würzburg, 1771), §§ 607, 619-29; Fr. Chr. Baumeister, *Institutiones Metaphysicae*, 2nd edition (Wittenberg, 1774), §§ 433-50; D. Beck, *Institutiones Metaphysicae*, 2nd edition (Salzburg, 1780), §§54-69, pp. 203-217.

[35]Thümmig, *Institutiones Wolfianae*, § 43.

[36]We must credit Eric Watkins with first discovering this important fact. Cf. Eric Watkins, “The Laws of Motion from Newton to Kant,” *Perspectives on Science* 5(1997), 311-48.

[37]Robert Rynasiewicz had the first clear recognition of this aspect of early modern natural philosophy and of the conceptual options available to protagonists before Kant. See, primarily, his two-part article “By Their Properties, Causes and Effects: Newton’s Scholium on Time, Space, Place and Motion,” in *Studies in History and Philosophy of Science*, **26**(1995), 133-153 and 295-321, and also his “On the Distinction between Absolute and Relative Motion,” *Philosophy of Science*, Vol. 67, No. 1, (2000), 70-93. In recent years, two other philosophers of physics have taken up the absolute-vs-relational controversy in the classical age from the vantage point of true motion and its problematic. See Nick Huggett, *True Motion* (in preparation), Chs. 1-3, and Oliver Pooley, *The Reality of Spacetime* (in preparation), Chs. 1 and 2.

[38]By the ‘received view,’ he means most likely the Wolffians’ concept of motion. This is, for instance, how Wolff defines it: “Motion is a continuous change of place

[*Motus vero est continua loci mutatio*]"—Wolff, "Elementa Mechanicae, *op. cit.*, § 2. Further evidence is Kant's opening his essay with a short *pro domo* to express hope that "gentlemen who use to dismiss as chaff … all ideas" that have not been ground at the "mill of Wolff's celebrated system" will look kindly upon his attempt. As I mentioned, Wolff does not spend much time explaining what place – or rather, true place – may be. Hamberger is among the few at the time to mention that place can be either relative or absolute. See Hamberger, *Elementa Physices*, §§ XIIf.

[39]To specify a frame of reference completely, we have two ways. One requires the existence of an extended, rigid body. Any four non-coplanar points in this body will be enough to determine an origin and three (rigid) axes. Another one is by finding four non-coplanar particles or mass points. Note that this is not yet an *inertial* frame: either setup could be rotating or otherwise accelerated, which would make it unsuitable as a basis for the laws of classical mechanics.

[40]Some philosophers of science refer to relationism also as 'relationalism.' Relation(al)ism is really a name for two families of views about true motion.

[41]There is a third option here, which Kant never considers. It is Descartes' position: each body does indeed have a true, or 'philosophical,' motion, whose proper analysis is 'transference from its vicinity as defined by surrounding bodies (taken to be at rest).' This is indirect evidence that Kant engages with the Wolffians rather than 17th century thinkers like Descartes, Leibniz, Newton and Huygens. The dual typology of true motion – as either translation relative to a global frame or to absolute space – is in Wolff, Hanov, and Hamberger.

[42]Cf. James Bradley, "Letter to Dr. Edmund Halley giving an account of a new-discovered Motion of the Fixed Stars," published in *Philosophical Transactions* no. 406 (1728), 637ff.—in Stephen P. Rigaud, *Miscellaneous Works and Correspondence of James Bradley* (New York: Johnson Reprint Corporation, 1972), 4. Next quotation: *ibidem*, 15.

[43]J. Bradley, "A Letter to the Rt. Hon. George Earl of Macclesfield, concerning an apparent Motion observed in some of the Fixed Stars," in Rigaud, *Works of Bradley*, 17.

[44]*Ibidem*, 39. Next three quotations: *ibidem*.

[45]In fact, it took another century for these two questions to receive an answer: reliable evidence that the Solar System moves relative to the fixed stars was not available until William Herschel, early in the 19th century; and only in 1838 did Friedrich Bessel succeed in finding the first measurable parallax. On Herschel, see his papers "On the Direction and Velocity of the Motion of the Sun and Solar System" (1805),

and "On the Quantity and Velocity of the Solar Motion" (1806), in J.L.E. Dreyer (ed.), *The Scientific Papers of Sir William Herschel*, vol. II (London: The Royal Society, 1912), 317ff. and 338ff. On Bessel, cf. his "Bestimmung der Entfernung des 61sten Stern des Schwans," in *Astronomische Nachrichten* 16(1838), 66-95.

[46]Kant, *Neuer Lehrbegriff*, 2: $17_{15\text{-}17}$.

[47]*Ibidem*, 2: $17_{23\text{-}27}$.

[48]Newton, *Principia*, 410.

[49]Kant, *Lehrbegriff*, 2: $17_{17\text{-}22}$.

[50]*Ibidem*, 2: $18_{5\text{-}10}$. Next two quotations: *ibidem*. Obviously, Kant's paradigm example is a collision in which all external forces are balanced; e.g., two billiard balls on a frictionless flat table.

[51]Kant, *Lehrbegriff*, 2: $19_{15\text{-}18.}$

[52]Some scholars did, in fact, miss this crucial aspect. Kant repackages his early dynamics of impact, with very little change, in the later *Metaphysical Foundations of Natural Science* (1786). Trying to grapple with it there, without attention to its early origins and Wolffian background, has led some to claim that "Kant's analysis [of communication of motion] … is purely kinematical, involving no causal mechanisms at all, let alone dynamist mechanisms, in the explanation." Cf. Howard Duncan, "Inertia, the Communication of Motion, and Kant's Third Law of Mechanics," *Philosophy of Science* **51**(1984), 93.

[53]Wolff, *Cosmologia*, §§ 137, 148. Next two quotations: *ibidem*, § 341; "Elementa Mechanicae," § 538.

[54]For Descartes, a body's true motion was the speed of transference from its immediate vicinity; for Newton, velocity relative to absolute space; for Berkeley, velocity relative to the fixed stars. Leibniz is harder to pin down, but he appears to support my contention. In 1677, he proclaimed a "remarkable fact: motion is something relative, and one cannot distinguish exactly which of the bodies is moving." About two years later, he glosses it as the view "that it is not possible to determine which subject [motion] is in."—*Labyrinth of Continuum*, 229 and 257. In 1694, Huygens asks Leibniz if he really thinks it "absurd to say that there is no real motion" [*absonum esse nullum dari motum realem*]. He confirms: "if motion, or rather the moving force of bodies, is something real (as it seems we ought to admit), it must well have a subject."—*OCCH*, vol. X, 614 and 645. If Leibniz thought the continuum is a labyrinth, his views on true motion are no less of a maze. Instructive attempts to map it are: Earman, *World Enough and Space-Time*, 15ff., 71ff., 116-22, 130-4; J.T. Roberts, "Leibniz on Force and Absolute Motion," *Philosophy of Science*

70(2003), 553-573; P. Lodge, "Leibniz on Relativity and the Motion of Bodies," *Philosophical Topics* 31(2003), 277-308; E. Slowik, "The 'dynamics' of Leibnizian relationism: Reference frames and force in Leibniz's continuum," *Studies in History and Philosophy of Modern Physics* 37(2006), 617-634; N. Huggett, *True Motion*, Chapter 3 (in preparation); A. Jauernig, "Leibniz on Motion and the Equivalence of Hypotheses," *The Leibniz Review* 18(2008), 1-40.

[55]I owe this insight to Robert Rynasiewicz and Nick Huggett.

[56]Following Sklar's lead, some have dubbed 'relationism' all early modern views that rejected Newton's absolute space—L. Sklar, *Space, Time, and Spacetime* (University of California Press, 1974), 167ff. Subsequently, Rynasiewicz argued cogently that we need more care in describing these stances, and distinguished 'weak' from 'strong' relationism. See Rynasiewicz, "By Their Properties…", 'Part II: The Context,' 295f., and also his "Absolute vs. Relational Theories of Space and Time: A Review of John Earman's *World Enough and Space-Time*" in *Philosophy and Phenomenological Research* 55(1995), 675-87.

[57]The most plausible construal of Berkeley's theory of true motion is that he thought of it as velocity relative to the stars. See W.A. Suchting, "Berkeley's Criticism of Newton on Space and Motion," *Isis* 58(1967), 186-197, and Kenneth P. Winkler, "Berkeley, Newton and the Stars," *Studies in History and Philosophy of Science Part A* 17(1986), 23-42.

[58]Chr. Huygens, "Fragment 10," translated from Mormino, *Penetralia Motus*, 246.

[59]As proof, I adduce some telling remarks from *Codex Huygens 7A*, a projected preface to a treatise on the laws of impact. Because of Huygens' premature death in 1695, the preface remained an inchoate collection of fragments in Latin and French. I translate from a splendid critical edition of the *Codex* by Gianfranco Mormino; Arabic numerals refer to fragments, letters denote sections within a fragment. "In vain does one ask what this true motion is—for what good is it? … For there is no difference between the relative rest and motion of free bodies." (6.I) "One cannot conceive in any way what the true and simple motion of a single entire body is—nor does it differ from the state of rest of the same body." (9.A) "I contend that there is no proper celerity. Instead of saying, 'no matter what their proper velocities may be,' [Mariotte] ought to say, 'no matter what their velocities relative to some other body.'" (10.A) Huygens is not oblivious to how unusual his position is: "I saw that all authors who had written on this matter commonly express a different opinion than mine. Their position is that the true and proper motion of a body is one thing,

and that a body's motion relative to some other body is quite another thing." But "if we look carefully into the nature of motion, we shall find that this true motion and rest, as nearly all of them understand it, not only cannot be known, but does not exist at all in the universe. I do not doubt that such a view will strike very many as a strange paradox and far from the truth." (7.A).

[60]Kant, *Neuer Lehrbegriff,* 2: $19_{27\text{-}29}$.

[61]Wolff, *Cosmologia*: "When a body B moves another body A at rest or changes the latter's motion, B acts, whereas body A suffers [*patitur*]" (§ 133). But "the action of a patient body upon the active one [*actio patientis in agens*] is called reaction" (§ 313). And "a body reacts through its force of inertia" (§ 316). "By its force of inertia, a body resists all changes" (§ 132). That is why "the principle of resistance to motion in bodies is named *Force of inertia*, or *passive Force*" (§130).

[62]In Newton's mechanics, both action and reaction denote impressed forces, measured by his *Lex Secunda*. For two bodies A and B in impact, the measure of A's action is the change of momentum it induces *in B*. Eric Watkins confirms that the post-Leibnizians who dabbled in natural philosophy generally ignored the Second Law.—E. Watkins, "Kant's Justification of the Laws of Mechanics," *Studies in History and Philosophy of Science* 29(1998), 542ff.

[63]Wolff, *Cosmologia*, § 316: "in the action of forces, it is not expended more than the [other] body is capable of reacting." See also Leibniz's disciple Jakob Hermann: "…the part of the total force of a body which must be expended in overcoming the resistance of the patient body, is that force from which action properly springs [*manat*]."—J. Hermann, *Phoronomia* (Amsterdam, 1716), 378.

[64]Wolff lets slip that, when two spheres A and B collide, the direction of A's action is the direction of its momentum vector. Cf. his *Cosmologia*, §§ 317-8. The implication is that a body's action is the quantity of momentum it expends in a collision. Kant gives no indication that he rejects this part of the Wolffian model.

[65]I must signal this fact because some prominent interpreters of Kant's *Critical* doctrine of motion take him to start with dynamical laws and define true motion in terms of them. He supposedly "conceives the laws of motion rather as conditions under which alone the concept of true motion has meaning: that is, the true motions are just those that *satisfy* the laws of motion."—M. Friedman, "Metaphysical foundations of Newtonian science," 143, emphasis in the text. It is clear that, at this time of the pre-Critical phase, Kant's procedure is the exact reverse of Friedman's construal.

[66]"These [laws of motion] are deduced from phenomena and made general by

induction: which is the highest evidence that a proposition can have in this philosophy."—Newton to Roger Cotes, 28 March 1713, in Isaac Newton, *Philosophical Writings*, ed. A. Janiak (Cambridge University Press, 2004), 118. And in an unpublished draft of Query 31 in his own *Optics*: "And the first thing to be done in Philosophy is to find out all the general laws of motion… And in this search metaphysical arguments are very slippery. A man must argue from phenomena."—quoted in J.E. McGuire, 'Force, Active Principles, and Newton's Invisible Realm,' *Ambix* 15(1968), 170f.

[67]Kant, *Neuer Lehrbegriff,* 2: $20_{5\text{-}13.}$

[68]*Ibidem*, 2: $20_{34\text{-}37.}$ Kant's phrase 'striving directed at the approaching body' is a dead giveaway that he has Wolff's account in mind. Cf. Wolff, *Cosmologia*, § 319: "The force of inertia is a striving [*nisus*] exerting itself against the striving of the active body."

[69]Besides Arthur's masterful introduction to the *Labyrinth of the Continuum*, the reader may consult P. Beeley, *Kontinuität und Mechanismus*, Ch. 3, 4, 7, 8; D. Anapolitanos, *Leibniz: Representation, Continuity and the Spatiotemporal* (Kluwer, 1999), 55ff.; M. Otte, "Das Prinzip der Kontinuität," *Mathematische Semesterberichte* 39(1992), 107ff.

[70]*Ibidem*, 2: $22_{3\text{-}6}$.

[71]*Ibidem*, 2: $21_{28\text{-}30}$.

[72]Kant, *Neuer Lehrbegriff*, 2: 23_{33f}. Next two quotations: *ibidem*.

[73]When the two component motions make an angle, the parallelogram rule gives the magnitude and direction of the composite motion: make the two component motions into the adjacent sides of a parallelogram; the diagonal represents the resultant motion. For collinear motions, the components are added or subtracted, depending on their direction. I am reluctant to call it, anachronistically, the vector addition of velocities or the coordinate transformation rule for Galilean frames. In the period from Galileo to Lagrange, 'composite motion' had at least four different meanings, and it spanned both kinematics and dynamics. Kant explores one sense of this term in "Metaphysical Foundations of Phoronomy," the first chapter in his 1786 tract on natural philosophy. I untangle some of these historical threads in my forthcoming paper "Phoronomy: the Structure and Limits of Kant's Kinematics."

[74]That Kant tacitly assumes it becomes apparent when he argues that "the impact cancels [the colliding bodies' motions], but not the motion of the surrounding space, in which nothing is effected [*gewirkt*]."—Kant, *Neuer Lehrbegriff*, 2: 24.

[75]Kant, *Neuer Lehrbegriff,* 2: $23_{28\text{-}33}$.

[76]This is the first of Kant's two examples, but he botches its presentation inexplicably. He starts with A = 3 lbs and B = 2 lbs, then, in the middle of the account, makes a mysterious switch to a reverse situation in which A weighs 2 lbs and B is 3 lbs. An explanation for this mistake eludes me.

[77]For instance, Michael Friedman, "Metaphysical foundations," 151ff.

[78]I. Kant, "Universal Natural History and Theory of the Heavens," in W. Hastie (trans.), *Kant's Cosmogony* (New York: Greenwood: 1968), 47. Next quotation: *ibidem*.

[79]Cf. John Earman, *World Enough and Space-Time* (Cambridge, Mass: MIT press, 1989), 61.

[80]See his papers "De communicatione motus in collisione corporum sese non directe percutientium" (1744), in *Commentationes Mechanicae ad Theoriam Corporum Rigidorum Pertinentes*, vol. 8.1 in Euler's *Opera Omnia* (Zurich, 1954), 7-26; "De collisione corporum gyrantium" (1772), *ibidem*, 369-402. Cf. also István Szabó, *Geschichte der mechanischen Prinzipien* (Basel, 1987), 453ff.

[81]C. Wren, "Lex Naturae de Collisione Corporum," *Philosophical Transactions* (1668), 867f.

[82]Leibniz, "Space and Motion are Really Relations," in Arthur, *Labyrinth of Continuum*, 225.

[83]Leibniz, "De Rationibus Motus," A6.2, 160.