Solving the Lucky and Guaranteed Proof Problems*

Stephen Steward, Syracuse University

Abstract

Leibniz's infinite-analysis theory of contingency says a truth is contingent if and only if it cannot be proved via analysis in finitely many steps. Some have argued that this theory faces the Problem of Lucky Proof—we might, by luck, complete our proof early in the analysis, and thus have a finite proof of a contingent truth—and the related Problem of Guaranteed Proof—even if we do not complete our proof early in the analysis, we are guaranteed to complete it in finitely many steps. I aim to solve both problems. For Leibniz, analysis is constrained by three rules: an analysis begins with the conclusion; subsequent steps replace a term by (part of) its real definition; and the analysis is finished only when an identity is reached. Furthermore, real definitions of complete concepts are infinitely complex, and Leibniz thinks infinities lack parts. From these observations, a solution to our problems follows: an analysis of a truth containing a complete concept cannot be completed in a finite number of steps—indeed, the first step of the analysis cannot be completed. I conclude by defusing some alleged counterexamples to my account.

Leibniz famously offered a proof-theoretic account of modality called the *infinite-analysis theory of contingency*. I am going to defend that theory from two related objections: the *Problem of Lucky Proof* and the *Problem of Guaranteed Proof*. They are widely thought to be serious worries for Leibniz's infinite-analysis theory, but I will argue that Leibniz's views on analysis, definition, and infinity directly imply solutions to these problems. In §1, I state the Guaranteed Proof Problem and two versions of the Lucky Proof Problem. Then I argue for a straightforward solution to each problem in §§2-4. Finally, in §5, I address some alleged counterexamples to my proposal.

1. The Problem of Lucky Proof

Leibniz's infinite-analysis theory of contingency says that a truth is contingent if and only if it cannot be proved by analysis in finitely many steps.[1] I will say more about analysis in the next section, but a few initial remarks are in order. Analysis is a particular method of proof. An analysis begins with the conclusion to be proved, and each subsequent step replaces a term by its definition (or part of its definition).

* *This paper won the 2014 Leibniz Society of North America Essay Competition.*

The analysis is finished—and the conclusion proved—when, and only when, an axiom is reached through the process of substituting *definiens* for *definiendum*.[2] For Leibniz, the axioms are *identities*, such as 'A is A', 'A is not not-A', and 'everything is as it is'.[3] So, according to Leibniz's infinite-analysis theory, a truth is contingent if and only if it cannot be converted into an identity by replacing terms with their definitions in finitely many steps.

The *Problem of Lucky Proof* was first raised as an objection to that theory by Robert Adams (1994), and prominently discussed by John Hawthorne and Jan Cover (2000), and Gonzalo Rodriguez-Pereyra and Paul Lodge (2011). The problem arises regarding truths that Leibniz thinks satisfy his criterion for contingency, such as "Peter denies Jesus." Leibniz explains why that truth is contingent: "The concept of Peter is complete, and so involves infinite things; therefore one never arrives at a perfect demonstration" (*General Inquiries* 74, cited in Adams, 34).[4] Leibniz doesn't spell out all the details, but the idea seems to be this. Since Peter is a *substance*, his concept is complete—that is, it contains all the predicates true of Peter (AG 41). Since there are infinitely many predicates true of Peter, his concept is infinitely complex. And since Peter's concept is infinitely complex, we cannot convert truths about Peter into identities in finitely many steps.

Adams challenges Leibniz on that last step. He writes,

> Even if infinitely many properties and events are contained in the complete concept of Peter, at least one of them will be proved in the first step of any analysis. Why couldn't it be Peter's denial? Why couldn't we begin to analyze Peter's concept by saying, 'Peter is a denier of Jesus and…'? Presumably such a Lucky Proof must be ruled out by some sort of restriction on what counts as a step in an analysis of an individual concept, but so far as I know, Leibniz does not explain how this is to be done. (Adams, 34)

Hawthorne and Cover put the problem as follows: "even if complete concepts decompose into infinitely many simple concepts, this does nothing to show that [whenever s is contingently F] proving the containment of F in the concept of s requires an infinite analysis. The analysis might be luckily short" (HC, 153).

Rodriguez-Pereyra and Lodge claim that, if the Lucky Proof is a genuine problem, there is a further problem, independent of luck, which they call the *Problem of Guaranteed Proof*. They write,

> For even if we are unlucky and it takes a long time to uncover a particular predicate in the definition of the subject, it will always be uncovered in some finite number of steps. The point can be seen more clearly if we associate each one of the infinitely many concepts constituting Peter's concept with a natural number and we imagine that our analysis uncovers those constituent concepts according to the order of natural numbers. Then no matter what number the

> concept 'denier of Christ' is associated with, it will take only a finite—but probably very large—number of steps to reach this concept from the beginning of our analysis. In this case, although the full decomposition of the infinitely complex concept 'Peter' will not be completable in a finite number of steps, every concept composing 'Peter' can be found in 'Peter' after a finite number of steps. Thus, there will be a proof of every [true] proposition of the form 'Peter is F'. (RPL, 223)

I have quoted at such length partly to make it clear that my opponent is no mere straw man—it is a group of several respected Leibniz scholars, all of whom think the Lucky Proof is a serious problem in need of a solution.[5] My main goal, however, is to distinguish the different issues. Adams offers a particular proof that he claims *might* demonstrate a contingent truth in a finite number of steps.[6] Rodriguez-Pereyra and Lodge claim that if Adams' proof might demonstrate a contingent truth, then *any* true proposition can be proved in finitely many steps. Hawthorne and Cover offer a slightly different challenge: we have not been *shown* that it takes an infinite analysis to uncover a concept in every complete concept. I will address each of these concerns in order.

2. Adams

Let us begin with Adams' presentation of the problem. He thinks that even if Peter's concept contains infinitely many properties, one of them will be proved in the first step of the analysis. Furthermore, it could turn out that the first step in the analysis is "Peter is a denier of Jesus and…."[7] If that were the first step, Adams thinks we would have proved, in a finite number of steps (namely *one*), a truth that Leibniz counts as contingent. Adams presumes that some restriction on what counts as a step in an analysis must rule out such a Lucky Proof, but he cannot tell what that restriction could be.

There are three flaws in Adams' version of the problem. First, the first step in the relevant analysis *could not* be "Peter is a denier of Jesus and…." As Adams himself notes, "*Analysis*, for Leibniz and the seventeenth century, was a method of proof beginning with the conclusion to be proved and working back to the axioms from which it follows" (Adams, 27, emphasis in original). So the analysis begins with the conclusion to be proved—"Peter denies Jesus"—not "Peter is a denier of Jesus *and*…."

Second, Leibniz is clear about what counts as a step in an analysis. He writes, "demonstrating is nothing but displaying a certain equality or coincidence of the predicate with the subject… by resolving the terms of a proposition and substituting a definition or part of one for that which is defined" (AG 96).[8] The analysis starts with "Peter denies Jesus", and any subsequent step should replace a term by its definition (or part of one). "Peter is a denier of Jesus and…" merely adds some elided conjuncts to the initial claim ("Peter denies Jesus"). It does not replace a term by (part of) its definition, and thus is not the right sort of step to appear in a Leibnizian analysis. The right sort of step would replace "Peter" (or another term) by its definition.

Third, even if we could begin the analysis by saying, 'Peter is a denier of Jesus and…', we would not thereby have proved that Peter denies Jesus. As Leibniz says, "in necessary propositions, when the analysis is continued indefinitely, it arrives at an equation that is an identity; this is what it is to demonstrate a truth with geometrical rigor" (AG 28). To demonstrate a truth, we must arrive at an identity, and before we reach an identity, we haven't demonstrated anything. (Of course, "in contingent propositions one continues the analysis to infinity through reasons for reasons, so that one never has a complete demonstration" (ibid.).)

The point here is that analysis is a distinctive method of proof. We start with an initial claim, and derive some other claims, using only certain rules (we may only replace terms by definitions and their parts). Eventually we reach an identity. At the end of this process, we have proved our original claim, but we have not proved it *before* the end of the process.

An analogy may help. In a *reductio ad absurdum* proof, we start with an initial claim, derive some other claims, and eventually reach a contradiction. At the end of this process, we have proved the *negation* of our original claim. It would be an elementary misunderstanding of *reductio* proofs to think that anything has been proved before a contradiction is reached. Adams' thought that something "will be proved in the first step of any analysis", before an identity is reached, involves a similar misunderstanding of the nature of analysis.

So far I have tried to clarify three points about the distinctive kind of proof Leibniz calls analysis: (1) Every analysis *begins* with the claim to be proved; (2) every *step* in an analysis substitutes (part of) *definiens* for *definiendum*; (3) no analysis *proves anything* until an identity is reached. Given those three points, it is impossible for "Peter denies Jesus" to be proved in the way Adams envisions—just by saying "Peter is a denier of Jesus and…." The sort of Lucky Proof Adams suggests does not start with the conclusion to be proved; it makes a step other than substituting (part of) a definition for a term; and it does not end with an identity.

3. Rodriguez-Pereyra and Lodge

Rodriguez-Pereyra and Lodge claim that *if* the Lucky Proof is a genuine problem, so is the Problem of Guaranteed Proof. There are compelling objections to their conditional claim, but those are not my main concern.[9] I will argue that the same insights about analysis that dissolved Adams's Lucky Proof Problem can be combined with Leibniz's views on definitions and infinity to solve the Guaranteed Proof Problem.

Rodriguez-Pereyra and Lodge suggest that to see the Problem of Guaranteed Proof, we should associate each constituent of Peter's concept with a natural number and imagine our analysis uncovering those constituent concepts in the order of the natural numbers. *If* analysis really works the way they are imagining, then they are right: each concept—including 'denier of Jesus'—will be uncovered in some finite number of steps.

But it's not true that each step in an analysis uncovers a single constituent concept. Rather, each step in an analysis replaces a term either by its definition or by part of that definition. As I'll now argue, the appropriate kind of definition uncovers all the constituent concepts at once, rather than one at a time. Furthermore, when the defined concept is complete, it has infinitely many constituents—and a concept with infinitely many constituents cannot be replaced by its definition or even by part of it.

Leibniz distinguishes between *nominal* and *real* definitions. Nominal definitions "contain only marks of a thing to be distinguished from other things," and thus uniquely identify their *definienda*, but they do *not* show "whether the notion defined is possible" (AG 26, 57). A real definition, on the other hand, "establishes that a thing is possible", and "makes known the possibility of a thing" (ibid.). Leibniz writes, "we cannot safely use definitions for drawing conclusions unless we know first that they are real definitions, that is, that they include no contradictions, because we can draw contradictory conclusions from notions that include contradictions, which is absurd" (AG 25; see also AG 57). So for Leibniz, we must know that a definition is real before we can use it in a demonstration.

Rodriguez-Pereyra and Lodge use this insight in their solution to the Lucky Proof Problem. They claim that "for Leibniz one has not proved that 'Peter denies Christ' unless one has also proved that 'Peter' is a consistent concept [i.e., it has a real definition], a task which requires the full decomposition of the concept 'Peter'. So even if the concept 'denier of Christ' is found in 'Peter' at the early or not so early stages of one's analysis, there is never a point at which one has completed the proof of 'Peter denies Christ'" (RPL, 223). They also say that Peter's concept is infinitely complex, and thus the full decomposition of Peter's concept cannot be completed in a finite number of steps (RPL, 228; see also HC, 152-5).

I agree with them that a proof of 'Peter denies Christ' must show that 'Peter' is a consistent concept. I also agree that that task requires the full decomposition of the concept of Peter—otherwise, Peter's concept might contain some hidden contradiction (RPL, 228-9)—and that complete concepts such as Peter's are infinitely complex. But Rodriguez-Pereyra and Lodge say their solution "does not constitute a restriction on what counts as a step in an analysis of an individual concept" (228). My solution differs from theirs on this point.

Each step in an analysis replaces a term by its real definition *or part of it*. So, the first step in an analysis of "Peter denies Jesus" can only replace "Peter" by its definition or by part of its definition.[10] The step cannot replace "Peter" by its entire definition, because that definition is infinitely long and thus not finitely stateable. But at first glance, it seems like a step in an analysis could replace "Peter" by a part of its definition, and a finite analysis containing that step could be completed as follows:

> Peter denies Jesus
> A denier of Jesus denies Jesus

Nothing I've said so far tells against this analysis: it starts with the conclusion to be proved; by all appearances, its only step replaces a term ('Peter') by part of its definition ('denier of Jesus'); and it ends with an identity.[11] Despite the appearances, Leibniz would deny that "denier of Jesus" is part of the definition of "Peter," because of his views on infinity. In several places, Leibniz denies that infinities are *wholes* and that they have *parts*.[12] This implies that definitions containing infinitely many terms do not have single terms as parts. Although the concept 'Peter' *contains* the concept 'denier of Jesus', and the real definition of 'Peter' *includes* 'denier of Jesus', the constituent concept 'denier of Jesus' is nonetheless not technically a *part* of the infinitely complex concept 'Peter'. Thus we cannot substitute the one for the other in an analysis, even though we may replace a term by part of its definition. The point generalizes to all infinitely complex concepts: their constituent concepts are not parts, and thus we may not substitute constituent concepts for infinitely complex concepts in an analysis.[13]

So, regardless of whether we try to replace "Peter" by its entire definition or a part of it, we will fail. There is no Lucky or Guaranteed Proof because *we cannot state the first step*, rather than, as Rodriguez-Pereyra and Lodge would have it, because no matter how many steps we take, we cannot show that Peter's concept is consistent.[14] A side-by-side comparison will highlight the differences between Rodriguez-Pereyra and Lodge's conception of analysis and mine.

Rodriguez-Pereyra & Lodge's conception	*Steward's conception*
Peter is a denier of Christ.	Peter denies Christ.
Peter is an apostle.	He who denies Christ and . . . denies Christ.
Peter is also called Simon.	
. . .	

For them, each step in the analysis lists a single predicate of Peter, and 'Peter denies Christ' cannot be proved because no finite number of steps suffices to list all the predicates of Peter, and thus Peter's concept cannot be proved consistent in finitely many steps. We could carry out the analysis to a million steps, but that would not be enough to complete it. For me, the first step in the analysis must replace 'Peter' with its real definition—not by part of that definition, since infinitely complex definitions have no parts. Since the definition is infinitely long, we can never complete the first step in the analysis.

Here is a reason to prefer my conception of analysis to Rodriguez-Pereyra and Lodge's. They acknowledge that analysis "is conducted by substituting definitions for the *analysanda*" (RPL, 222), and that the definition of 'Peter' is infinitely complex (RPL, 228). But those things entail that the first step in the analysis of 'Peter denies Christ' will replace 'Peter' by its infinitely long definition. My conception of analysis also entails this result, while theirs is inconsistent with it.

I have a way of interpreting their view so that it is true. I can agree that one predicate is uncovered in each step of the analysis, if "step" means "conjunct of a line in a proof." But if, as I think they intended, "step" means "line in a proof," I do not think each step uncovers a single predicate.

Rodriguez-Pereyra suggested in conversation that their account accommodates certain of Leibniz's claims in a more satisfying way than mine. For instance, Leibniz writes, "in contingent propositions one continues the analysis to infinity *through reasons for reasons* [*per rationes rationum*]" (AG 28, my emphasis). The picture suggested by the text is this: we begin with a contingent proposition, we give a reason for that, then we give a second reason for the first reason, a third for the second, and so on to infinity. But my account is different: we start with a contingent proposition, begin giving a reason for that (which involves replacing a term by its infinitely long definition), and never finish. We only list conjuncts of an infinitely long definition, none of which are reasons for the others.

I grant that my account does not mesh well with this text, but I do not see any other account in a better position. Rodriguez-Peryera and Lodge's account might seem better-suited to this text because on their account, an analysis typically has many lines. But notice that none of the lines in the proof are *reasons* for other lines (except, perhaps, each line is a partial reason for the contingent truth under analysis). Their account does not feature "reasons for reasons" either.

Other extant accounts of analysis face a similar problem. Giovanni Merlo (2012) offers an account on which a proof of a contingent truth must show that some world in which it is true is better than any world in which it is false. On his view, the reasons for a contingent truth are comparative facts, like "this world is better than that world." The world-comparisons together constitute a sufficient reason for a contingent truth, but none of the world-comparisons are reasons for any others. So, Merlo's account also fails to accommodate "reasons for reasons."

The one view that may accommodate "reasons for reasons" is Hawthorne and Cover's. They give an account on which the proof that something is contingently F must show that the infinitely many inclinations (i.e. appetitions) that thing has toward being F outweigh its inclinations against being F (HC,158ff.). On their view, the reasons for the thing being F are the inclinations, and these reasons are weighed against one another. Now suppose something is contingently inclined to be F; this is a reason for the thing's being F. Their account implies that a proof of this contingent inclination must show that the thing's inclinations to be inclined to be F outweigh its inclinations to be not inclined to be F. These second-order inclinations may be considered reasons for the first-order inclination, which is itself a reason that the thing is F. So, Hawthorne and Cover's view does seem to accommodate "reasons for reasons." However, Rodriguez-Pereyra and Lodge have raised powerful objections to their account: it is not supported by the texts, and it implies that any initial inclinations are had necessarily (RBL, 224-26). So, I will set Hawthorne and Cover aside.

The only other way I see to accommodate this text is plainly inconsistent with other things Leibniz says. The sort of reason given by an analysis involves replacing a term by its definition (e.g. the reason it is a triangle is that it has three angles). For an analysis to involve "reasons for reasons," it would have to involved *nested definitions*, such as $A =_{df} B + C$, where B has some further definition. If we could use a nested definition in an analysis, that analysis would involve reasons for reasons (for instance, satisfying the definition of B is the reason it is B, and being B is (partly) the reason it is A). However, we cannot use nested definitions in analyses. Leibniz says that we cannot use a definition in a proof unless we know first that it does not contain any contradictions. But we cannot know that a definition like the one given of A does not contain any contradictions. Since B has a further definition, it might contain contradictions. So, we cannot use nested definitions in analyses. But nested definitions are the only way I see (besides Hawthorne and Cover's) to include reasons for reasons in an analysis.

So my account is not alone in struggling to accommodate the "reasons for reasons" text. It may remain a recalcitrant text, and if so, failure to accommodate it shouldn't count against any one theory. Or perhaps some future interpretation will be able to accommodate it. I only claim that no extant theory accommodates this text *better* than my account.

The fact that nested definitions cannot appear in analyses raises a difficult issue. Consider this claim: "He who denies Christ and does everything else Peter does denies Christ."[15] It seems to have two interesting features: (1) it is equivalent to the claim that Peter denies Christ, which Leibniz thinks is contingent; and (2) it is an identity, so Leibniz would think it is necessary. And it's very plausible that (3) whatever is equivalent to a contingent truth it itself contingent. But (1), (2), and (3) commit Leibniz to the view that some truth is both necessary and contingent.

One diagnosis would point to an equivocation on 'equivalent.' There does seem to be a sense in which, whenever x is F, 'x' is equivalent to 'the thing which is F and has all the other features x has.' But the sense of 'equivalent' in which a Leibnizian should accept (3) is the sense in which two claims are equivalent if and only if one can be converted into the other by replacing terms by their real definitions. These two senses of 'equivalent' are not equivalent in any sense. A Leibnizian can (and should) deny that (1) is true in the 'conversion-by-definition' sense, because the definition, "Peter $=_{df}$ he who denies Christ and does everything else Peter does" is a nested definition and thus not a real definition for Leibniz. For the same reason, she should deny that (3) is true in the other sense of 'equivalent.' On this diagnosis, all three claims are true, but they are consistent because of the equivocation on 'equivalent.'

Another diagnosis would reject (2). Rodriguez-Pereyra and Lodge argue that identities can only contain primitive terms (RPL 231ff.). Since "He who denies Christ and does everything else Peter does denies Christ" contains terms with real definitions (e.g. "Christ," "Peter"), it does not only contain primitive terms. So, on their view, it is not an identity.

I do not have decisive reasons for preferring one diagnosis to the other. It will matter only once more, when I respond to an objection in section 5. Nothing else in the paper hangs on this. So, let's move on to the remaining version of the Lucky Proof Problem.

4. Hawthorne and Cover

Hawthorne and Cover put the Problem of Lucky Proof as follows: "even if complete concepts decompose into infinitely many simple concepts, this does nothing to show that… proving the containment of F in the concept of *s* requires an infinite analysis" (HC, 153). Given the conception of analysis I have been arguing for, we can easily show that no finite analysis can prove that *s* is F.

Suppose that the concept of *s* decomposes into infinitely many simple concepts.[16] Then the real definition of '*s*' must list all those simple concepts. Otherwise the concept of *s* might contain some hidden contradiction, and thus it won't be a real definition for Leibniz. So the analysis will look like so:

s is F.
The thing that is F and G and … is F.

Infinitely many predicates are elided from the second step. That step cannot be stated in full, and so the analysis cannot be completed. So, *contra* Hawthorne and Cover, I have shown that whenever the concept of *s* decomposes into infinitely many simple concepts, proving the containment of F in the concept of *s* requires an infinite analysis—in the sense that the analysis contains an infinitely long step, rather than containing infinitely many steps.

Now I turn to some counterexamples from Giovanni Merlo (2012).

5. *Merlo*

Merlo (2012) presents several counterexamples to a view called Complexity.

Complexity: A truth is contingent iff it contains infinitely complex concepts.

Two facts about Complexity deserve comment. First, the term 'truth' here only refers to *derivative* truths; in this sense, Leibnizian *identities* are not truths (Merlo 2012, 10). Second, I am not committed to Complexity, though I am committed to its right-to-left direction. If a truth contains infinitely complex concepts, then there cannot be a finite proof of that truth—since the infinitely complex concepts cannot be replaced by (parts of) definitions. But for all I have said, a truth could fail to have a finite proof for other reasons than complexity of concepts. Perhaps a truth could contain *infinitely many* finitely complex concepts and be unprovable for that reason. Perhaps there are other ways for a truth to fail to have a finite proof. All I claim is that if a (derivative) truth has infinitely complex concepts, that truth lacks a finite proof by analysis, and thus is contingent according to Leibniz's infinite-analysis theory.

Since I do not endorse the left-to-right direction of Complexity, Merlo's counterexamples to it are no threat to my view (Merlo 2012, 17-18). I take no stand on whether there are contingent truths that lack infinitely complex concepts. My view is consistent with either option.

Merlo also offers counterexamples the right-to-left direction of Complexity, which I endorse, so I owe a response to those examples. Two of his examples are "I shall be what I shall be" and "I have written what I have written" (Merlo 2012, 17). Merlo acknowledges that both of these claims are identities (17; Rodriguez-Peryera and Lodge (231ff.) disagree). But if they are identities, then they are not counterexamples to Complexity or to my view. Complexity is a claim about *derivative truths only*, and my view only implies that *derivative* truths with infinitely complex concepts are contingent. An identity containing infinitely complex concepts *can* be resolved into an identity, and thus proved by analysis, in a finite number of steps—zero. So these two attempted counterexamples are unsuccessful.

Most troubling for me is Leibniz's frequent insistence that God exists necessarily (AG 19, 218, 235-40). Modulo the infinite-analysis theory, it implies there must be a finite proof of "God exists." But my view implies that there cannot be a finite proof of a derivative truth that involves an infinitely complex concept. So, my view is in trouble if it implies that the concept of God is infinitely complex—and Merlo gives two arguments that it has this implication (Merlo 2012, 16).

According to the first argument, God has an infinitely complex concept because he is a substance (AG 218), and "each and every individual substance contains the whole series of things in its complete notion… and to that extent contains something of the infinite" (AG 100, cited in Merlo 2012, 16).

According to the second argument, there cannot be *any* infinitely complex concepts unless the concept of God is infinitely complex; and since there are infinitely complex concepts, God's concept is infinitely complex. As Merlo puts it, "a concept is infinitely complex if and only if it resolves into infinitely many simple or primitive positive concepts. But we know that the simple or primitive concepts that enter into any complex concepts are all and only the absolute attributes of God. So, to the extent that any concept is infinitely complex, the absolute attributes of God must be infinitely many and… the concept of God… must be infinitely complex, too" (Merlo 2012, 16).

Each argument fails to refute my position. The second argument relies on the premise, "a concept is infinitely complex if and only if it resolves into infinitely many simple or primitive positive concepts." But this is false. An infinitely complex concept may be built out of finitely many primitive concepts, as long as the primitive concepts are *combined* in infinitely many ways. Suppose that the concept 'natural number' resolves exclusively into the concepts 'one' and 'successor of' in the following way: something is a natural number if and only if it is one, or the successor of one, or the successor of the successor of one, or…. The infinitely complex concept 'natural number' resolves into finitely many primitive concepts. Analogous examples can be generated with other iterative concepts, such as 'ancestor of.'

Someone might object that 'natural number' isn't really an infinitely complex concept.[17] Even if there is an infinitely long definition of 'natural number' along the lines I suggested above, there is also a *finite* definition of 'natural number,' and this shows that the concept is only finitely complex. The finite definition might go a number of different ways. Here is one:

> The number 1 is a natural number, and any successor of a natural number is also a natural number.

Someone might reasonably classify the preceding statement as a definition because it can help someone come to understand the concept. However, it is not the right kind of definition to feature in an analysis. In an analysis, a term must be replaced by its definition. But there is no part of the above 'definition' (other than 'natural number') that could replace 'natural number' in an arbitrary sentence without changing its truth conditions. For the purposes of analysis, the right kind of definition should have this form: x is F $=_{df}$ x is ________. Then a step in an analysis could replace 'F' with whatever fills the blank.

Here is a second attempted definition:

> x is a natural number $=_{df}$ x is 1 or the successor of a natural number.

This at least has the right form to be used in an analysis, but it is not a Leibnizian *real* definition. 'Natural number' appears in the *definiens*, so it is a circular definition. This is a particular kind of nested definition: the *definiens* contains a term with a further definition. Because of this, we cannot be certain that the defined notion is possible. Thus, we cannot use definitions like this in analyses. Neither attempted definition is appropriate for use in analysis, so we have not seen a persuasive reply to my argument that an infinitely complex concept can be resolved into finitely many primitive concepts. And if that argument succeeds, it undermines Merlo's second argument that God's concept is infinitely complex.[18]

Unlike the second argument, the first (that God's concept is infinitely complex because he is a substance) successfully establishes that God has *an* infinitely complex concept. However, it does not show that *the* concept of God is infinitely complex, because it has not been shown that God has only one concept. In addition to a complete concept of God that is infinitely complex, Leibniz invokes an *incomplete* concept of God-as-perfect-being when he says that God exists necessarily.

In brief, my argument for the incomplete concept is that Leibniz characterizes a concept of God in such a way that it does not meet a constraint on being a complete concept. The constraint is that a complete concept must contain *all* the predicates of a thing. This implies that whatever is true of a God is contained in God's complete concept. So God's complete concept includes constituent concepts such as 'created

Adam,' 'always chooses the best,' and 'did not create a sub-optimal world.' In fact, God's complete concept includes *infinitely many* predicates: all the predicates true of him.

Yet in many texts Leibniz uses a concept of God that is 'without limits, without negation' (AG 218). Since this concept is *without negation* it cannot contain the predicate 'did not create a sub-optimal world.' But a complete concept of God must contain that predicate, since it is true of God. So, the concept of God that is 'without negation' must not be a complete concept.

If Leibniz sometimes uses an incomplete concept of God, it would be good to know *when* he is using that concept rather than the complete concept, and *what* the incomplete concept's constituents are. The second question is relatively easy: the incomplete concept of God contains each of the perfections (AG 56). I think there are finitely many of those, and I rejected Merlo's argument that there are infinitely many. The first question is harder, because Leibniz is less than explicit about when he uses the incomplete concept. I submit that Leibniz is using the incomplete concept of God whenever he defines God as a perfect being. The concept of a perfect being contains only the perfections—not the *negative* predicates—and a concept that doesn't contain all of a thing's predicates is an incomplete concept.

As an illustrative example, consider Monadology 45 (AG 218):

> *God alone (or the necessary being) has this privilege, that he must exist if he is possible. And since nothing can prevent the possibility of what is without limits, without negation, and consequently without contradiction, this by itself is sufficient for us to know the existence of God* a priori.[19]

Since Leibniz presupposes here that God is without negation, he is using the incomplete concept of God. If God's incomplete concept is finitely complex, we can make sense of the above passage: God's concept can be replaced by its definition, and thus a derivative truth such as 'God exists' can be converted into an identity, such as 'the being that is omniscient, omnipotent, … and exists, exists.' Other passages where he argues for God's necessary existence reflect this same focus on God-as-perfect-being (AG 56, 235-40).

I have argued that in the *incomplete* sense of 'God', God exists necessarily. It is still true, though, that in the *complete* sense of 'God', my view implies that God exists contingently. A position roughly equivalent to mine can be stated in contemporary *de re*/*de dicto* terms. The *de dicto* claim that *some* absolutely perfect being exists may be necessary, but the *de re* claim that *this* being (who is absolutely perfect) exists is contingent. And this seems exactly right—since it is possible that God created a different world, it is possible that there is a God different than the actual one.

6. Conclusion

Leibniz' infinite-analysis theory of contingency says that a truth is contingent if and only if it cannot be proved by analysis in finitely many steps. In this paper, I tried to answer two related objections to the infinite-analysis theory: the Problems of Lucky and Guaranteed Proof. I argued for a particular conception of analysis: an analysis begins with the conclusion to be proved, each subsequent step replaces a term by its definition or part of its definition, and the analysis is complete when an identity is reached. I also invoked Leibniz's claim that only real definitions can be used in demonstration, and his view that infinities are not wholes. Together those claims entail that there can be no Lucky Proof or Guaranteed Proof, because an analysis of any truth involving a complete concept must contain an infinitely long step that cannot be completed. Leibniz's conception of analysis and his views on infinity straightforwardly imply a solution to the Lucky and Guaranteed Proof Problems.

I also answered three kinds of counterexamples from Merlo (2012). One only refutes the converse of my view. Another is an unproblematic identity. The third is Leibniz's claim that God exists necessarily. I considered and rejected two arguments that my account cannot accommodate the necessity of God's existence. On my account, Leibniz's claim is true since Leibniz is using a concept of God that is incomplete and finitely complex.[20]

Received 6 December 2014

Stephen Steward
Syracuse University
Department of Philosophy
541 Hall of Languages
Syracuse, NY 13244
steve.steward@gmail.com

References

Adams, Robert (1994). *Leibniz: Determinist, Theist, Idealist*. Oxford: Oxford University Press.

Hawthorne, John, and Jan Cover (2000). "Infinite Analysis and The Problem of the Lucky Proof." *Studia Leibnitiana* 32: 151-165.

Merlo, Giovanni (2012). "Complexity, Existence and Infinite Analysis." *Leibniz Review* 22: 9-36.

Rodriguez-Pereyra, Gonzalo, and Paul Lodge (2011). "Infinite Analysis, Lucky

Proof, and Guaranteed Proof in Leibniz." *Archiv für Geschichte der Philosophie* 93: 222-236.

Cook, Roy T. (2000). "Monads and Mathematics: The Logic of Leibniz's Mereology." *Studia Leibnitiana* 32: 1-20.

Notes

[1] See, for example, *On Contingency* (A VI, 4, 1649–1652/Gr 302-6/AG 28–30), *On Freedom* (FC 178-85/AG 94-98), and *The Source of Contingent Truths* (C 1-3/Gr 325-26/AG 98-101). The formulation in the main text has a minor problem: it is circular, since the modal term 'can' appears in the analysans. We can avoid that problem with a more precise but more awkward formulation than I provide in the main text: a truth is contingent if and only if there is no finite proof of it by analysis.

[2] See Adams, 27, Hawthorne & Cover (hereafter abbreviated as "HC"), 152, n. 6 and references therein, and Rodriguez-Pereyra & Lodge (hereafter "RPL"), 222.

[3] See AG 30-31, and AG 28, 96, 99. I discuss a controversy about identities in n. 11.

[4] In some contexts it is worth carefully distinguishing *terms* and their *definitions* from *concepts* and the *constituents*. But that distinction is irrelevant to the issues I wish to discuss, so I will slide freely between claims about terms and concepts.

[5] Adams raises the problem without offering a solution; Hawthorne and Cover and Rodriguez-Pereyra and Lodge offer competing solutions.

[6] Except for the one occasion where I'll explicitly discuss another proof method, every instance of 'proof' or its cognates should be understood to mean 'proof by analysis.'

[7] Presumably the ellipsis elides the rest of Peter's predicates, though Adams doesn't make this entirely clear.

[8] The quotation continues: "or in other cases at least displaying the inclusion so that what lies hidden in the proposition and was contained in it virtually is made evident and explicit through demonstration." It's unclear what exactly Leibniz means here, and what this adds to the sentence. It seems like the way to "display the inclusion" and make it "explicit" is to replace a term a term by its definition; but see RPL, 227 for an alternative suggestion. In many other texts Leibniz does not hedge his account of demonstration in this way (HC, n. 6.).

[9] Their conditional claim is supported by their view that we can (i) associate each constituent of a concept with a natural number and (ii) suppose that our analysis uncovers those constituent concepts in the order of the natural numbers. If we can do both (i) and (ii), then *if* the step "Peter is a denier of Jesus and …" is a legitimate step and *could by luck* be the first step, that step is *guaranteed* to be associated with one of the natural number and thus reached after a finite number of steps.

Kris McDaniel objected to (i) in correspondence, on the grounds that there are non-denumerably many things true of some individuals. (Proof: for each real number, it is true that I am distinct from that number.) Giovanni Merlo (2012) objects to (ii): even if each constituent concept is associated with a natural number and each step of the analysis uncovers one constituent concept, the concepts need not be uncovered in the order of the natural numbers. One way this could happen is if the analysis uncovered all the even-numbered concepts before the odd-numbered ones; even if the concept 'denier of Christ' is numbered '1', there would still be no finite proof of 'Peter denies Christ' (ibid. 14). Another way is if the constituent concepts were also associated with *positive rational* numbers and uncovered in the order of the rationals. (Since the naturals and the positive rationals are equinumerous, any set of constituent concepts that can be paired one-to-one the naturals can also be paired one-to-one with the positive rationals.) Since there are infinitely many positive rational numbers less than any given positive rational number, it would take an infinite number of steps to uncover *any* constituent concept (ibid. n. 20).

[10] It could also replace "Jesus" or "denies." But replacing "denies" will not help us get closer to an identity, and an argument like the one in the text shows that "Jesus," like "Peter," cannot be replaced by its definition or even by part of it.

[11] Rodriguez-Pereyra and Lodge (231ff.) think that this is *not* an identity, because they think that identities must be stated in primitive terms. I return to this issue later in this section.

[12] A 6.3.158 (this text contains Leibniz's Galilean argument for his view); G II, 314-5; A 5.6.157/NE II.xvii.1; A 6.3.503. Thanks to Sam Levey for these references.

[13] Kris McDaniel raised two very good questions for my view: (1) if not part-whole, what is the relation between complete concepts and constituent concepts? (2) can finitely complex concepts have parts? The relation is plausibly the *ingredient* relation. In "The Metaphysical Foundations of Mathematics," Leibniz defines a part as a "homogenous ingredient" (GM VII 19/L 668). In his discussion of Leibniz's mereology, Roy T. Cook (2000, 10-11) argues that for geometrical objects, homogeneity is topological equivalence. According to Cook, points are not parts of lines because points are not topologically equivalent to lines (ibid., 10). It seems reasonable that infinite things are never homogenous to finite things, so finite things can only be ingredients, not parts, of infinite things. But to settle questions (1) and (2), we would really need to delve into the proper definition of homogeneity in terms of qualitative and quantitative properties and consider the qualitative and quantitative properties of concepts (see Cook's discussion of homogeneity).

I do want to expose one tempting but confused argument for a negative answer to (2). Suppose that some finitely complex concepts have parts, and we can replace these concepts by their parts in analyses, and the concept "bachelor" has "unmarried" as a part. One might worry that allowing such substitutions would let us infer 'All unmarried things are male' from 'All bachelors are male'. But such inferences aren't genuinely worrying because of the way analysis works. Even if we can infer 'All unmarried things are male' in an analysis, we won't have *proved* something untrue; remember, in analysis the only thing we prove is the *initial* claim, and we prove that only when we reach an identity. To prove "all unmarried things are male" we would have to transform that claim into an identity, and that task is impossible even if we can replace terms by parts of their definitions.

[14] In a similar vein, after his discussion of the Lucky Proof, Adams (1994) says "we may wonder how we can even begin an analysis of the individual concept of any person, as Leibniz seems to imply that we can. For such a concept, being complete, is not our concept but God's, and we do not seem to have a definition with which to begin to replace it" (34).

[15] Thanks to Giovanni Merlo for asking me about this claim.

[16] Better: the concept has infinitely many ingredients. Given what I said earlier, these ingredients are not parts, so, strictly speaking, nothing *decomposes* into them.

[17] Thanks to Giovanni Merlo for pressing this objection.

[18] Another kind of definition might be invoked. Leibniz speaks of *causal* definitions, which say how a thing can be produced (G IV 294, 425). A causal definition of 'circle,' for instance, says that a circle is the figure produced by the rotation of a straight line in a plane around a fixed end point. I do not see how to give a causal definition of 'natural number.' Since the states of individual substances are produced by their prior inclinations, we could view Hawthorne and Cover's account as offering causal definitions.

[19] Here Leibniz seems to mean 'independent of experience' by '*a priori*', but see Adams 109-110.

[20] This paper was drafted in a marvelous Leibniz course taught by Kara Richardson and Kris McDaniel, where Samuel Levey and Jeffrey McDonough lectured on issues related to infinity in Leibniz's physics and mathematics, and Robert Adams led two seminars on his (1994) book on Leibniz. Thanks to the professors, who gave helpful comments on previous drafts, to the guest lecturers and fellow participants, and to audiences at the 2013 conference of the Leibniz Society of North America at Yale University and the 2nd Congreso Iberoamericano Leibniz at Universidad de Granada, especially Giovanni Merlo and Gonzalo Rodriguez-Pereyra. This paper is dedicated to Angela Cordero. My work and my life are much better because of her support and encouragement.