Logica Docens and Relevance

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I have been asked to make some "opening remarks" concerning the role of formal logic in a liberal arts curriculum. My plan is to talk around the topic rather than make a direct assault upon it; I shall say something brief about the various types of logic courses and then fasten attention on what might be called "the freshman logic course." Also the logic I love best from a research point of view is relevance logic; so I shall certainly say something about the relation, or want of relation, between freshman logic and relevance logic.

Naturally what one thinks about the teaching of logic depends on one's vision of the nature of what is to be taught. Let me begin with a distinction, namely, between logic as an art and logic as a science. When I ask you to consider logic as an art I want you to picture it as an "organon" in Aristotle's sense; that is, as a tool which we can use for various purposes. It is interesting, incidentally, that contemporarily we have changed the idea of what the tool is for. It used to be that logic was for reaching conclusions, and if you look at the problems that were set by Aristotle or by the early modern logicians, you will find that one was given fifteen premises by Lewis Carroll involving alligators eating babies and doctors who are both healthy and rich, with the problem being always to "use logic" to find the conclusion. In our day we tend to think that logic is not for that purpose but rather for assaying arguments. We are given fifteen premises (usually the same ones) and a conclusion, and we are asked to say whether the argument is Good or Bad. Maybe this change in our view of the purpose of logic arose because we are presently more pessimistic than we were about the powers of rational thinking to discover the truth; or perhaps the change derived from the theme in the philosophy of science summed up in the title of one of Popper's books, *Conjectures and Refutations*, the idea being that all we can do is to make conjectures and try to refute them; there is no "method" of finding conclusions, except by luck.

The art of logic as we now understand it is therefore the art of separating the Good arguments from the Bad ones; logic is the tool we use in carrying out this task. So I suppose the *science* of logic reduces to the old Socratic axiom, know thy tool. Since it is clearly impractical to try to teach more than a suspicion of logic *qua* science in a freshman logic course, I want to ask about the science only one superficial question: Who is to learn or listen to logic as a science? Who should study formal grammar, proof theory, semantics, and the theoretical questions raised by the application of logic to natural language? Who should study completeness proofs, limitation theorems, questions of definability, decidability, and recursive unrealizability? Surely some philosophers should, because philosophy is the discipline which defines itself as self-reflective, as living always by the maxim of Socrates; some mathematicians should, a few because they are interested in the foundations of their topic, and a few because logic has led to a number of theories interesting from a purely mathematical point of view; probably some theoretically oriented linguists and computer scientists should; and maybe from a professional point of view logic as a science should be studied only by philosophers and mathematicians and linguists and computer scientists, although certainly anybody can and should be interested in the topic. Acquaintance with the great limitation theorems of logic, for instance, can profitably be a part of the liberal education of each of us. There is a prominent psychoanalyst in Pittsburgh who spent his freshman year at Yale mastering Goedel's proof of the consistency of the continuum hypothesis. What superb training for a psychoanalyst, especially when one thinks that he had as his roommate and guide one of the finest logic teachers ever: Alan Ross Anderson.

Consider now logic as an art. I mean: clear thinking. Students come to logic teachers all the time saying, "teach me to think clearly." A sad thing. I don't mean that clear thinking is sad, but that the request to be taught it is one which sadly harbors the profound foolishness that it is good to do a lot of thinking. Perhaps Whitehead defended the view as well as anyone that one ought to think as little as possible. In preaching the virtues of a good notation, Whitehead points out that what is good about a well-designed notation is precisely that it allows one to get by without thinking, and then he sums up his point by saying, "operations of thought are like cavalry charges; they must be carefully planned and they require fresh horses, and should be reserved for decisive moments."

That's clearly right; but as a matter of fact I nevertheless do think that logic as an art is something that can and should be taught to anyone who is willing to take the time to learn it. The thing is, I do not think that one can learn enough of the art of logic to make it worthwhile in the span of the single term ordinarily devoted to the first logic course. I think one needs a solid year. Certainly I am not sure of what I say, and it may be that new techniques (computers?) can compress the time required, but it is nearly equally certain that the typical freshman one-term course (I mean to include any freshman course of a wide range of types) does not and cannot impart skill in the art of logic. Those who get an A are still unable to tell the Good arguments from the Bad when the going gets rough.

What then does the freshman course do? Let me say a word about the phrase "freshman logic." I am not thinking so much about the level at which the course is taught; at any level, I have in mind a course syllabus like that presented in Irving Copi's *Introduction to Logic* (Macmillan, Fifth Edition,

1979) where one does some syllogisms, a little propositional logic, some very elementary quantification theory, maybe a few fallacies and special topics, etc. In short, a few elementary techniques for the evaluation of arguments. Cocchiarella calls this "the logic appreciation course," a pharse to which I shall return. But I first want to say that I think that this sort of course can in fact have a practical spin-off; one can teach students to evaluate short arguments, those with a few premises and a conclusion that's pretty close up.

Which is the catch-14: If one thinks about the practical value of such a course, short arguments are precisely where we don't use *logica docens*; for example, *logica utens* already pretty well has in mind, without fussing too much, how to get from the premises of the Barbara syllogism to her conclusion. When we need *logica docens* is, evidently, when we are evaluating long and complex chains of reasoning. And as Whitehead was partly suggesting, we *do not* do that very often, and as a matter of fact we probably *should not* do it very often. Evaluation of long arguments takes a great deal of time. This is true for even purely truth functional problems; in fact, even speaking technically, long truth functional problems are what the computer scientists call Hard or Intractable.

What then should a freshman logic course teach? I do believe it should teach some elementary techniques; we can do a little better on our short range arguments. But I think there is something even more important we can teach, and I want to remind you of Cocchiarella's phrase, "logic appreciation course." One thing I believe every elementary course should teach, above all else, is: *the ideal of rigor*. I want to emphasize the word "ideal" here, and to give it an explicitly platonic import. In a sufficiently light sense, every elementary logic course should be a course in beginning Platonism. I have in mind two elements: it should in the first place teach *patterns* of reasoning. It should be abstract; this is one of the key elements of the platonic tradition: the emphasis on patterns and the abstract level. And in the second place, freshman logic courses should continually emphasize the *norms* of reasoning, the *value* of careful cogitation, the *ideal* of rigor. Let me remind you of my belief that we cannot teach students to be rigorous in a complex way in a single term. But what we can do is to teach them to *appreciate* what it would be like to be rigorous. We can train them to be able to tell with respect to their own reasoning, and with respect to the reasoning of others, when there is rigor and when there is not.

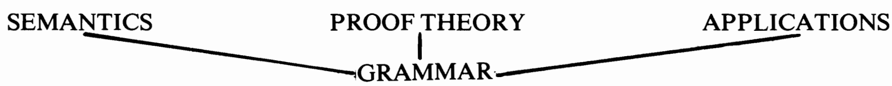
That, I submit, is a pearl of considerable price. And a price well worth paying a term for, by a large number of students: to be able to evaluate their own reasoning with respect to how they measure up to the ideal of rigor (especially when involved in long and complicated arguments). Part of the job has to do with formalizing, that is, moving from the hurly-burly of natural language to the carefully articulated constraints of a formal language. One can and should teach that the job is difficult, and one can and should convey some sense of when you know how to do it and when in fact you do not. Teachers of logic ought freely to confess that most of the time they themselves are not being rigorous and don't know how to be, even when talking about logic itself.

Or at least that they don't care to take the time to be rigorous. That's the other side of this particular coin; one can present the ideal of rigor as a distant goal which requires an enormous investment even to approach. As anyone knows who has tried it at all, formalizing is extremely expensive, expensive of our time, expensive of our energy, and certainly expensive of our patience. The point is obvious: costs are important. Having realized this, one is then in a position to do a little preliminary planning as to whether some particular rational enterprise is the sort into which one wants to bring the fresh horses of formalization. Covey remembers his wood shop course in school: in the few hours available he didn't learn how to make cabinets; but he did learn the difference between a hammer and a saw, and he did learn to appreciate what it takes to make a finely finished cabinet. I agree with Covey: the freshman logic course is like that.

There are in the liberal arts curriculum a couple of other courses which enshrine the ideal of rigor in the sense at issue: mathematics, and computer science. It seems to me that neither of these courses have quite the sort of merit that logic does for the job at hand. It is true and important that mathematics teaches rigorous thinking and that computer science teaches one to be rigorous (to get a program to run you have to get all the pieces in the right place). But both mathematics and computer science courses have the flavor of being special; in contrast, logic, when it is properly taught, can have the flavor of being an omni-applicational discipline. I'm not speaking philosophically here; it is surely true that both computer science and mathematics are topic-neutral in the way that logic is, but it is only logic that can easily be presented in such a way that shows it to apply anywhere that one is trying to be rational. To make similar claims for either mathematics or computer science is to be pretentious.

So much agin' sin. I now want to fight in favor of virtue by remarking on things I would like to see in at least some elementary courses.

One feature that is not much emphasized in most such courses is grammar. Let me make my point against the background of the following picture of the elements of logic.



By "semantics" I refer to formal semantics, be it truth tables or whatever. And by "proof theory" I refer to the various ideas of proofs, derivations, axioms, rules, and the like. My picture correctly indicates that these two disciplines have grammar as their common foundation; it is to emphasize this that I abandon the Charles Morris term, "syntax," which included both grammar and proof theory. By "applications" I refer to the connections between the henscratches and the rest of our life, with "translation" being perhaps the most important component; for example, the hookup between the horseshoe and "if-then." All of these have grammar as their foundation, but each is independent of the other two: given grammar, you can quite independently discuss formal semantics, or a deductive system, or how to translate the henscratches into English (or vice versa).

By "grammar" I mean a sort of formal or purified or idealized grammar; my recommendation is that such a grammar can and should be taught as part of elementary logic courses. For example the idea of a "connective" is almost always badly treated: students are seldom told that at bottom a connective is a grammatical operation which takes sentences as input and produces sentences as output, or that the logician's "predicate" is a grammatical operation mapping singular terms into sentences. We should teach students the fundamental ideas of formalistic grammar. And maybe, if Richard Thomason is right, even Montague grammar. Thomason has a view that there is a continuum between formal languages over there and English over here, and that one might well design a course in which this continuum is made explicit, so that by the time the course winds up the student is looking at, to use Montague's title, "English as a formal language."

Part of this new material would reduce the role of intuition in the techniques of translation which we all teach and would move us more into a rigorous theory of translation of English into henscratches, to the extent that this is possible. I know that students, for example, find it illuminating—I find it illuminating myself—that there is a routine way to translate "at least two advisors have been consulted by each president." 1) One first finds the major term, where "term" is used technically in Montague's sense (roughly) either for a proper name or for a combination of quantifier-word and common noun phrase which can sensibly be substituted for a proper name. Let me underline the term at issue: "at least two advisors have been consulted by *each president*." 2) One then sees that one ought to view the sentence as constructed by substituting the term, "*each president*" into the open sentence, "at least two advisors were consulted by *x*." 3) Then, looking at the term, one discovers that "*each president*" is made up of a quantifier word and a common noun phrase. The common noun phrase leads to a second open sentence, "*x* is a president," and 4) the quantifier word "each" tells us, in a uniform and *routine* way, how to combine these two open sentences ("*x* is a president," "at least two advisors were consulted by *x*") into a sentence. Then one can go on to apply exactly the same routine techniques to the open sentence, "*at least two advisors* were consulted by *x*"; (1,2) one first thinks of the sentence as generated by substituting the term "*at least two advisors*" for the variable *y* in the open sentence "*y* was consulted by *x*" (switching from plural "were" to singular "was") and then (3) gets from the term both an open sentence "*y* is an advisor" and a quantifier phrase, "at least two," (4) the latter coding routine and uniform directions for combining the two open sentences. And we thus finish the translation without using intuition at all.

Another thing which would be good to have in the freshman logic course would be the logic of questions and answers; I think no one since Henry Leonard has treated the topic in such a context in a respectable way. Also, modal and deontic logic should be introduced. A few weeks ago I was working through the IRS instructions with an eye to formulating an exercise for beginning logicians having had some truth functional and quantificational logic, only to be struck by a number of subtle modal and deontic distinctions. The paragraphs I was looking at were filled with "may be required," and with uses of "should" which were sometimes prudential and sometimes deontic.

But what about relevance logic? Permit me an autobiograpical note. I first became interested in relevance logic under the tutelage of Canon Robert Feys over twenty years ago, and I have continued to work on it off and on ever since. When I teach the freshman logic course, I do not teach relevance logic; but although you may not know it, some people do teach relevance logic in such courses. With such people in mind, and in spite of the fact that you may think it odd to defend the omission of relevance logic, I propose to do so.

As background, let me enter as a datum that relevance logic is True: the standard position defending material implication and the classical account of consequence is False. First, classical consequence. Copi gives the following argument to our beginning students:

> If the airplane had engine trouble, it would have landed at Bridgeport. If the airplane did not have engine trouble, it landed at Cleveland. The airplane did not land at either Bridgeport or Cleveland. So it landed in Denver.[1]

When given this argument, the students say: "Denver?" And so do I: Denver! Now Copi, of course, tries to de-shock us (the freshmen and me) by pointing out that the premises are inconsistent, and that "any argument with inconsistent premises is valid, regardless of what its conclusions may be."

Is Copi correct? But the question is improper, because what he says is ambiguous. There is on the one hand the mathematically defined notion of a truth functionally valid argument: no row with premises all T and conclusion F ("no counterexample"). That is a concept of formal semantics, and surely Copi's argument is "valid" in that sense, where I use shudder-quotes with which to shudder. But there remains on the other hand the question as to whether the argument is a Good one. Suppose a Tenure Committee is constrained to use only a certain batch of testimony, and that the chairman's contribution says that six articles were published, while the candidate's *curriculum vitae* claims seven. Should the committee deny tenure on the grounds that it "follows" from this (contradictory) testimony that the candidate traded grades for sexual favors?

I don't mean for a minute to suggest that the dialectic should end at that point of absurdity, but only to bring to your attention that there is an intuitive, normative notion of "good" or "valid" argument, and that it is a philosophical question as to whether the classical notion of consequence in terms of "no counter example" correctly explicates this notion. Why is it that "no counter example" is a sufficient condition for arguing well? It does *not* go without saying that relevance is not a requirement with equal claims. There may be "no counter example" to moral turpitude if you start with testimony to both six and seven publications, but it seems too much to deny tenure as a consequence of that "conclusion" when "derived" using classical "logic."

So does it follow that we should teach freshmen an alternate account of consequence? No, but let me come back to that, first taking up another point at which relevance logicians complain about their classical counterparts, namely, the routine translation of English "if-thens" into the horseshoe of material implication. I deny that if some astronomers are blind then all of them are. But I do not thereby mean to assert that some astronomers are blind, nor even to deny that all of them are, which is of course what I would be doing were the ingredient "if-then" to be the material conditional. In fact, contrary to Grice and others, I don't believe that any "if-thens" in English are material conditionals. I also do not believe that all "if-thens" in English assert relevance as part of their meaning; but I think that some of them do. Nevertheless, I do not recommend the teaching of the logic of relevant implication to freshmen.

What I recommend instead, with regard to both the choice of a consequence concept and the choice of conditionals, is to make an occasion for some philosophy of logic which includes a defense of the classical choices on strictly practical grounds. Consider the Plumber from J. L. Synge's *Kandelman's Krim*. The Plumber is sitting around discussing the philosophy of mathematics with the blue Goddess, the Unicorn, the Orc, and the Kea. It's a question of how to calculate the volume of water which will move through a pipe, and at one point the Plumber says, "I am of course perfectly well aware of the irrationality of π, but on the job, π is 3 1/7, or 3 if I am in a hurry." The Plumber is dead right: there is π up in Plato's heaven, while down here we have 3 1/7 or even 3. What is laid up in heaven is in fact the ratio of the diameter to the circumference; but what we've got down here is quite good enough when what is at issue is the installation of a drain.

The theme is common enough in the philosophy of science that on the one hand there are schematic and idealized "models" while on the other there is the chaos of our booming, buzzing confusion. And what is worth saying is that applied logic presents us with a strictly analogous situation. There are real "if-thens" in English, and as you know, I think that many of them involve a relevance component; but a decent approximation to confused reality is to be found in the truth-functional horseshoe. And what do I mean by a decent approximation? I do not mean, as logic texts sometimes suggest, that the horseshoe catches part of the meaning of all "if-thens," or all of the meaning of some of them. I think such claims are false, just as I think that π is not 3 1/7; but I still want to defend using truth-functional logic as a good approximation. A good approximation for what? The Plumber wished to install a drain, and analogously, I want you to think of logic as an organon or as a tool. It seems to me it's true as a *practical* matter that using 3 1/7 will deliver you the right drain pipe, and it seems to me as a *practical* matter that if you translate English "if-thens" into horseshoes, and if you restrict yourself to the arguments that we actually want to evaluate, you will find out that the intuitively valid ones by and large come out being formally valid, and vice versa. Classical logic comes out with the "right answer" in most cases. Anyone can make counter examples in either way: we can get intuitively valid arguments that come out formally bad (John and Mary each wanted a beer; so there is something John and Mary each wanted) and we can get intuitively invalid ones that come out formally good (the one about the Tenure Committee, or the one about the blind astronomers) — but by and large classical logic works. And that's a strictly practical justification, having no more mathematical content than the Plumber's justificaiton for using 3 1/7 when installing drain pipes. Furthermore, the analogy even extends to the "hurry": 3 1/7 is simpler to use than π, the horseshoe is simpler than any relevance connective or other competitor, and the classical consequence relation is simpler than relevant consequence.

Lest the point not be obvious, let me make it explicit that the above remarks contain no "defense of classical logic," but only a defense of teaching it in freshman logic courses. For it is practicality that is at issue, not truth; and to assess what is practical we must know purposes. Certainly I side with the Plumber when it is a matter of choosing a pipe; but not with the Indiana legislature, who, with no purpose in mind, wished to define π as some easily usable rational number (I forget which one). That was crazy, and so would be any effort by establishment logicians to legislate classical logic. Classical logic is fine when you are in a hurry, as noted; but when you have the means and the time to stroll along the road to inquiry in a more leisurely fashion, by all means tread relevantly.

So, I don't recommend teaching relevance logic in freshman courses. On the other hand, I do suggest talking about it. I do think it is healthy to point out that the classical notion of logical consequence is an "explication," in Carnap's sense, of the normative notion of a good argument; and that it needs philosophical reflection to determine how good an explication it is. One can use something like the standard counter examples to argue that the explication is not totally adequate; but at the same time one can use carts full of examples as evidence that it goes a long way in the right direction. (Occasionally if we finish our work a little early, I will spend in my own course an hour discussing relevance logic in its own terms, to illustrate that there are formally worked out alternatives to the received views.)

I have spoken of this and that, and the remainder of the papers in this volume, which I commend unto you, will doubtless dwell in an organized way on topics I have either touched only lightly or not at all. But permit me to close my portion of these proceedings by highlighting two points, one of them positive, one of them negative. The freshman logic course is given to hundreds of students, term after term after term, many of the students bored, perhaps because they are takng this course as one among a required set of options. The most important thing to do fo these students, I think, is somehow to convey the "ideal of rigor," with emphasis on both of these words equally. My second point comes to this: I don't recommend teaching transcendental concepts in the first term; and though relevance logic is True, it's Transcendental.

Notes

This paper emerges as the revision of some thoroughly informal "Opening Remarks" initiating the conference to which the reports constituting this volume were made. I am

grateful to Preston Covey for argle-bargle leading to a number of improvements. An introduction to relevance logic can be found in *Entailment: The Logic of Relevance and Necessity*, Vol. I, by Alan Ross Anderson and myself, Princeton University Press, 1975.

1. Irving Copi, *Introduction to Logic*, Fourth Edition, Macmillan, 1972, p. 309.

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